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A STOCHASTIC MODEL FOR FACE PERCEPTION IN THE FFA

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ABSTRACT. Functional Magnetic Resonance Imaging (fMRI) studies have shown that the fusiform face area (FFA) is sensitive to both face parts and face configurations. In this note it is postulated that the FFA is partitioned into clusters that respond to specific facial features, and that in each cluster the intensity of the blood-oxygen-level dependent (BOLD) response can be measured. A probability function supported on the clusters is then defined. Among the properties of this function are: 1) response magnitude invariance, 2) individuation of faces and an economical procedure to quantify and store the neural representation of a face; 3) repurposing of clusters to define new probability functions on the FFA partition; 4) dynamics between clusters in the FFA produce the emergent property that accounts for a unified face; 5) stability of probability functions under perturbations.

1. Probability Function Model of Faces

It is well accepted that the neural correlates of face perception are in the FFA [Kanwisher and Yovel (2006)]. As well, the FFA is implicated in "extracting the perceptual information used to distinguish between faces" [Kanwisher and Yovel (2006)]. In this note we assume that sensory neurons that perceive different facial features terminate at different locations in the FFA [Liu et al. (2009)] where they begin the process of activating the respective clusters [Wandel et al. (2005)]. Let us assume there are n features such as eyes, nose, mouth, shape, symmetry, smoothness of skin, expression, spacing between features etc. We label these features: $F_1, ..., F_n$. The clusters in the FFA corresponding to these features are labeled $A_i, i = 1, ..., n$. The collection of clusters defines a partition $\mathcal{P}$ of the FFA as shown schematically in Figure 1. It is reasonable to assume that $\mathcal{P}$ will be defined once higher resolution is achieved in the fMRI experiments. Presently each pixel in the fMRI spans hundreds of thousands of neurons, implying that the actual selectivity in the FFA is finer than what is being observed [Kanwisher (2001), Kanwisher (2006)].

Next we postulate that the intensity of the BOLD responses in the clusters $A_i, i = 1, ..., n$, can be measured. Let us label these intensities: $M_i, i = 1, ..., n$. As a first approximation, we assume that M_i is constant on the corresponding A_i, as shown in Figure 2. We define the magnitude of the ith face feature by $S_i = M_i \cdot A_i$. If M_i is not uniform on A_i, then M_i, as a function of position inside A_i, is integrated over A_i to determine S_i. Calculation of the magnitude of the BOLD responses has been performed in [Liu et al. (2009)]. We collect the values $S_1, ..., S_{n.}$ and let $S = \sum_{i=1}^{n} S_i$. Define $P_i = \frac{S_i}{S}$. The collection $P_i, ..., P_n$ defines a probability

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function on FFA since each $P_i \geq 0$ and $\sum_{i=1}^{n} P_i = 1$. We shall refer to this as the facial probability function, $\mathbf{P}$, on the partition $\mathcal{P}$ of the FFA. It is in effect the activation pattern in the FFA, as depicted in Figure 2.

It is known that "the FFA is engaged both in detecting faces and in extracting the necessary perceptual information to recognize them" [Kanwisher and Yovel (2006)]. We suggest that the facial probability function contains the perceptual information (cluster shape and their respective activations) to recognize a face, that is, $\mathbf{P}$ encodes face identity information. This model is consistent with the work of [Nestor et al. (2011), Nichols et al. (2010)] which provides evidence that the FFA "responds with distinct patterns of activation to different face identities."

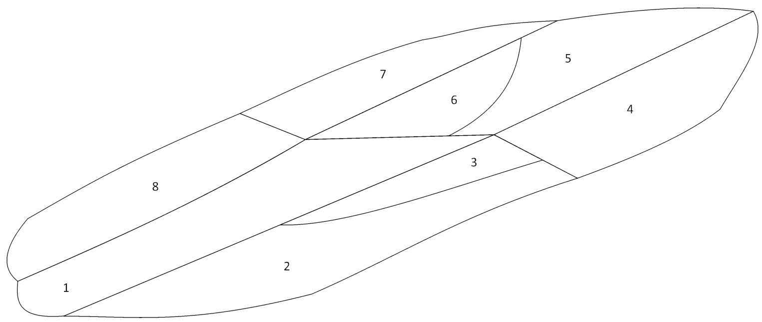

FIGURE 1. Partition of FFA.

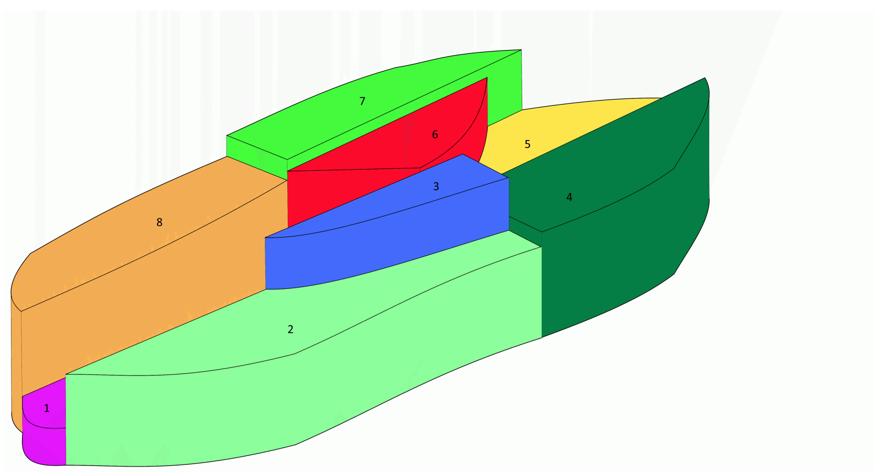

FIGURE 2. Sketch of FFA activation.

Recall that the clusters in the FFA corresponding to facial features are A_i, $i = 1, ..., n$. We define transition probabilities P_{ij}, $i, j = 1, ..., n$ where for each i the positive numbers P_{ij} sum up to 1 ($\sum_{j=1}^{n} P_{ij} = 1$), and are proportional to the numbers of connections from A_i to A_j activated by sensory input.

The transition probabilities P_{ij} form a transition probability matrix $\mathbb{P} = [\{P_{ij}\}_{1 \leq i,j \leq n}]$ (see example below). The neuronal architecture in the FFA and the initial sensory

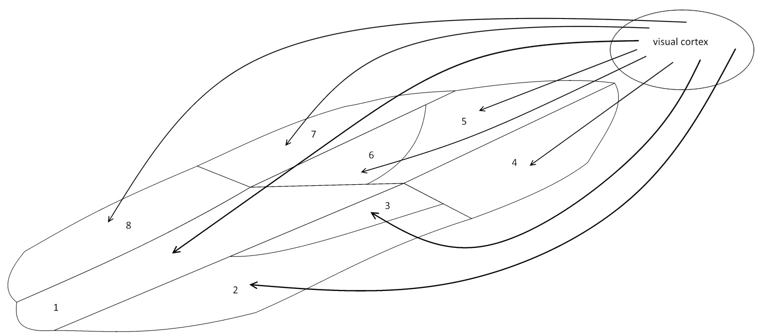

FIGURE 3. FFA dynamics initiated by sensory neurons.

input determine which clusters interact with others and the strength of these interactions. This generates the transition probability matrix $\mathbb{P}$.

This transition matrix generates the stationary probability function (vector) $\mathbf{P} = [p_1, p_2, \ldots, p_n]$. The vector $\mathbf{P}$ is the unique solution of the equation $\mathbf{P}\cdot\mathbb{P} = \mathbf{P}$, which in expanded form is a system of n linear equation with n unknowns:

$$\begin{cases} p_1P_{11} + p_2P_{12} + \cdots + p_nP_{1n} = p_1; \\ p_1P_{21} + p_2P_{22} + \cdots + p_nP_{2n} = p_2; \\ \vdots \quad \vdots \quad \vdots \quad \vdots \\ p_nP_{n1} + p_2P_{n2} + \cdots + p_nP_{nn} = p_n. \end{cases}$$

Example: We assume the structure of the neuronal architecture in the FFA defines the 8×8 transition probability matrix in Figure 4. The stationary probabilities are approximately

$$\mathbf{P} \approx [0.138, 0.102, 0.115, 0.111, 0.128, 0.132, 0.137, 0.137].$$

$$\mathbb{P} = \begin{bmatrix} 0.11 & 0.09 & 0.15 & 0.12 & 0.17 & 0.09 & 0.03 & 0.24 \\ 0.21 & 0.12 & 0.01 & 0.16 & 0.08 & 0.18 & 0.21 & 0.03 \\ 0.25 & 0.11 & 0.24 & 0.08 & 0.02 & 0.04 & 0.15 & 0.11 \\ 0.11 & 0.20 & 0.06 & 0.11 & 0.09 & 0.20 & 0.02 & 0.21 \\ 0.16 & 0.13 & 0.18 & 0.03 & 0.16 & 0.13 & 0.19 & 0.02 \\ 0.04 & 0.06 & 0.14 & 0.07 & 0.21 & 0.15 & 0.18 & 0.15 \\ 0.05 & 0.02 & 0.03 & 0.13 & 0.22 & 0.21 & 0.25 & 0.09 \\ 0.20 & 0.11 & 0.10 & 0.19 & 0.04 & 0.07 & 0.07 & 0.22 \end{bmatrix}$$

FIGURE 4. A Probability Transition Matrix reflecting neuronal architecture in the FFA, for $n = 8$ features.

2. PROPERTIES OF THE FACIAL PROBABILITY FUNCTION

2.1. **Response Magnitude Invariance.** Let $M_i, i = 1, ..., n$ be activity intensities on the clusters in the FFA for a given face. Suppose the same face is now reduced in size, resulting in weakened signal strengths. We assume that the signal strengths are reduced uniformly by a factor $\alpha < 1$ Then, the new strengths are: $S_i' = \alpha M_i \cdot A_i$, and $S' = \sum_{i=1}^{n} S_i' = \alpha \sum_{i=1}^{n} S_i = \alpha S$. The new probability function $P'_i = \frac{S'_i}{S'} = P_i$. Hence the face probability function P is invariant under a uniform size reduction (or expansion). Face-inversion [Yovel and Kanwisher 2005] may result in a weakening of intensities in the clusters in the FFA. The resulting reduction in cluster activity nonetheless preserves the probability function, resulting in recognition of the original upright face. In [O'Craven and Kanwisher (2000)], it was shown that mental images of faces activate the same FFA area as the actual visual image. The mental image, initiated entirely by interneurons, no doubt has lower response activation than the actual visual image. Yet, both activations yield the same probability function and hence represent the same face.

2.2. **Individuation.** . The information necessary to characterize a face consists of n cluster areas and n activity intensities of these clusters. It is reasonable to assume that the cluster regions in the FFA are fixed by the brain architecture. However, the cluster activations vary from face to face. Thus, defining and storing a facial probability function involves n numbers, the relative magnitudes of facial feature activity. This information may be stored in the amygdala [Herrington et al. (2002)]. or possibly inside the FFA itself as suggested by [Liu et al. (2009)] and see the discussion therein).

2.3. **Repurposing.** can be viewed as the same clusters being activated with a set of different activations. It is noted in [Quiroga et al. (2013)] that the late neuroscientist Jerry Lettvin suggested that as few as 18,000 neurons could be the bases of a conscious experience, such as seeing a face. Even so, the brain may not have enough neurons to represent all possible concepts and faces [Quiroga et al. (2013)]. In our repurposing model, the same partition of a cortical region such as the FFA is reused to define an infinite number of different patterns of activation, each one representing a different face.

2.4. **Stochastic Dynamics in the FFA.** Just as a differential equation requires initial conditions to start the dynamics, so the FFA requires the visual sensory input. Once this occurs, as depicted in Figure 3, the FFA architecture [Klinshov et al. (2014)] determines the connectivity probabilities and with the synaptic weights of the different clusters determines a flow of information, which generates the activation pattern.

The visual sensory input gives not only the initial conditions but also defines which parts of the interconnecting FFA architecture is activated. This defines both the initial probability function and the transition probabilities matrix.

The transition probability function between clusters determines the stationary probability function that characterizes a face.

The continuous sensory input gives all the time new initial conditions and new transition probabilities matrices but this is not of an importance as close transitions matrices have close invariant probability functions. Thus, the stationary probability function is stable under the perturbations of transition probabilities matrix.

2.5. **Stability under perturbations of facial features.** Once we relate a stochastic dynamical system to a face activation on a partition of the FFA, we can discuss the stability of such a system. Consider a small perturbation of a face (say a laughing face or even caricature) resulting in a probability function that is close to the original one. Such properties are proved using stability of the stationary probability functions under small perturbations of the transition matrix. They hold as long as all entries in the matrix are strictly positive, which corresponds to all parts of the FFA communicating between themselves [Kemeny and Snell (1976)].

To illustrate the stability of $\mathbf{P}$ under small perturbations of $\mathbb{P}$ we give an example of a transition matrix $\mathbb{P}_1 \approx \mathbb{P}$, with perturbation ± 0.01, and its stationary probability vector $\mathbf{P}_1$. Let

$$\mathbb{P}_1 = \begin{bmatrix} 0.10 & 0.08 & 0.14 & 0.13 & 0.18 & 0.08 & 0.04 & 0.25 \\ 0.19 & 0.12 & 0.02 & 0.17 & 0.08 & 0.19 & 0.21 & 0.02 \\ 0.25 & 0.12 & 0.23 & 0.07 & 0.03 & 0.05 & 0.15 & 0.10 \\ 0.10 & 0.19 & 0.07 & 0.10 & 0.10 & 0.21 & 0.01 & 0.22 \\ 0.17 & 0.12 & 0.17 & 0.03 & 0.17 & 0.13 & 0.20 & 0.01 \\ 0.03 & 0.06 & 0.13 & 0.06 & 0.21 & 0.16 & 0.19 & 0.16 \\ 0.06 & 0.03 & 0.04 & 0.12 & 0.21 & 0.20 & 0.26 & 0.08 \\ 0.21 & 0.11 & 0.11 & 0.20 & 0.03 & 0.06 & 0.06 & 0.22 \end{bmatrix}$$

FIGURE 5. A Probability Transition Matrix $\mathbb{P}_1$, a perturbation of $\mathbb{P}$ in Figure 4.

Then, the stationary probability vector is

$$\mathbf{P}_1 \approx [0.135, 0.100, 0.115, 0.109, 0.131, 0.133, 0.141, 0.135].$$

We see that the largest difference between the components of $\mathbf{P}_1$ and $\mathbf{P}$ is 0.004 (component number 7).

3. Discussion:

This note proposes a method for encoding facial identity using probability functions. To define such a function requires detailed knowledge of which clusters in the FFA are activated by which features of a face and the relative strengths of these activations. To define a transition probability matrix $\mathbb{P}$ we need to know which clusters interact with other clusters. Once $\mathbb{P}$ is known, the stationary probability function can be calculated, which characterizes a face.

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