Global and Stochastic Analysis
Vol. 4 No. 1, January (2017), 83-93

MUK PUBLICATIONS
Open Access Publisher

COMPARISON OF ESTIMATION METHODS FOR INVERSE WEIBULL PARAMETERS

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Abstract. This paper compares different methods for estimating the parameters of the two parameters inverse Weibull distribution based on complete sample. The maximum likelihood estimators of the parameters are examined and the observed Fisher information matrix provided, the method of moment estimator, the least square estimator, the weighted least square estimator of the parameters of the Inverse Weibull distribution are derived. The performances of the proposed estimators are compare on the basis of their mean square error by carrying simulation study. Finally, the usefulness of the methods are illustrate by real data.

1. INTRODUCTION

Weibull distribution has more applications in analyzing the life time data due to the flexibility of probability density function (pdf) and hazard function. Its pdf can be monotone or unimodal and the hazard function can be increasing, decreasing or constant depending on the value of the shape parameter. Besides its merits, the Weibull distribution has some limitations. When the hazard function of a lifetime data is non-monotone and unimodal, use of Weibull distribution is inappropriate. In the study of mortality of lung and breast cancer patients, initially the mortality function increases, it reaches peak and then it declines slowly. It indicates that the hazard function is non-monotone and it is unimodal. For this kind of data, the Weibull distribution cannot be used. This situation as mentioned in Langlands (1979), Bennett (1983) and Kundu and Howladu (2010) under these circumstances, the inverse Weibull distribution is found as an appropriate model

If a random variable Y is distributed according the Weibull (α, λ) distribution, then the probability density function of $X = 1/Y$ can be derived as

$$\mathrm{f}(\mathrm{x}|\alpha,\lambda) = \begin{cases} \alpha\lambda \mathrm{x}^{-(\alpha+1)}\mathrm{e}^{-\lambda \mathrm{x}^{-\alpha}} & , \text{ if } \mathrm{x}>0,\ \alpha>0,\ \lambda>0 \\ 0 & , \text{ otherwise} \end{cases} \tag{1.1}$$

The random variable X, is called as the Inverse Weibull(IW) random variable. The expression, given in (1.1), is the probability density function of the IW (α, λ) distribution, where α is the shape parameter and λ is the scale parameter.

2000 *Mathematics Subject Classification.* Primary 65C40, 62H20; Secondary 62M10.
Key words and phrases. Inverse Weibull distribution; Maximum Likelihood; Moment Estimators; Fisher Information Matrix; Least Square Estimator; Weighted Least Square Estimator.
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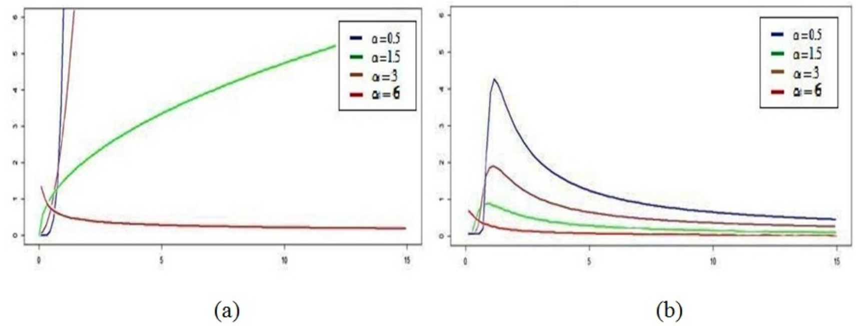

FIGURE 1. (a) hazard function of the Weibull distribution
(b) hazard functions of the inverse Weibull distribution for fixed value λ=1 and different value of α

The cumulative distribution function of IW distribution can be derived as

$$F_X(x|\alpha,\lambda) = e^{-\lambda x^{-\alpha}} \quad ,\ x > 0. \tag{1.2}$$

The mean and variance are

$$E\,(X) = \lambda^{\frac{1}{\alpha}}\,\Gamma\left(1-\frac{1}{\alpha}\right)$$

$$Var\,(X) = \lambda^{\frac{2}{\alpha}}(\Gamma(1-\frac{2}{\alpha}) - (\Gamma(1-\frac{1}{\alpha}))^{2})$$

It can be note that the mean and variance of IW (α,λ) distribution exist only when $\alpha > 1$ and $\alpha > 2$ respectively.

Keller *et.al.* (1982) applied the inverse Weibull distribution as a suitable probability model to describe degradation phenomena of mechanical components such as pistons and crank shafts of diesel engines. More applications of the inverse Weibull distribution can be found in Calabria and Pulcini (1990), Nelson (1982), Maswadah (2003), Murthy *et al.* (2004). The rest of the paper is organized as follows. Section 2 estimate the parameters α and λ of IW distribution using different estimation method. In section 3 comparison of estimation methods is done based on simulation study are presented. In section 4 discussion about the result. In section 5 a real data is used to illustrate the results obtained from section 2. Finally Conclude in the last section

2. Estimation of Parameters based on Uncensored Complete Sample

2.1. Fisher Information Matrix. Let x be a random sample having the pdf specified in by taking natural lograithm

$$\log \mathrm{f}\,(\mathrm{x}|\lambda,\alpha) = \log\alpha + \log\lambda - (\alpha+1)\log\mathrm{x} - \lambda\mathrm{x}^{-\alpha}$$

The fisher information matrix corresponding to this PDF is given by

$$I(\lambda,\alpha) = -\begin{bmatrix} E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial^2\lambda}\right) & E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial\lambda\,\partial\alpha}\right) \\ E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial\alpha\,\partial\lambda}\right) & E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial^2\alpha}\right) \end{bmatrix}$$

$$E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial^2\lambda}\right) = -\frac{1}{\lambda^2}$$

$$E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial\lambda\,\partial\alpha}\right) = \int_0^\infty \left(-x^{-\alpha}\log\left(\frac{1}{x}\right)\right)\alpha\lambda x^{-\alpha-1}e^{-\lambda x^{-\alpha}}dx$$

where,
$u = \lambda x^{-\alpha}$

$$= \frac{1}{\lambda\alpha}\left(\Psi(2) - \log(\lambda)\right)$$

$$E\left(\frac{\partial^2 \log f(x|\lambda,\alpha)}{\partial^2\alpha}\right) = \int_0^\infty \left(-\frac{1}{\alpha^2} - \lambda x^{-\alpha}\left(\log\left(\frac{1}{x}\right)\right)^2\right)\alpha\lambda x^{-(\alpha+1)}e^{-\lambda x^{-\alpha}}dx,$$

where,
$u = \lambda x^{-\alpha}$

$$= -\frac{1}{\alpha^2}\left[1 + \psi'(2) - 2\psi(2)\log\lambda + (\log\lambda)^2\right]$$

$$I(\lambda,\alpha) = \begin{bmatrix} \frac{1}{\lambda^2} & \frac{1}{\lambda\alpha}\left(\psi(2) - \log\lambda\right) \\ \frac{1}{\lambda\alpha}\left(\psi(2) - \log\lambda\right) & \frac{1}{\alpha^2}\left[1 + \psi'(2) - 2\psi(2)\log\lambda + (\log\lambda)^2\right] \end{bmatrix} \quad (2.1)$$

Where

$$\psi(x) = \int_0^\infty e^{-u}u^{x-1}\log u\,du \quad \text{and} \quad \psi'(x) = \int_0^\infty e^{-u}u^{x-1}(\log u)^2\,du$$

are digamma and trigamma function

2.2. Method of Maximum likelihood. Let X_n=(X_1,. . .,X_n)be an uncensored complete sample of n observations drawn from IW distribution.

The log-likelihood can be obtained by taking natural logarithm as

$$\ell(\alpha,\lambda|x) = n\log\alpha + n\log\lambda - (\alpha+1)\sum_{i=1}^{n}\log(x_i) - \lambda\sum_{i=1}^{n}x_i^{-\alpha}$$

Calculating the first partial derivatives of $\ell(\alpha,\lambda|x)$ with respect to α and λ and equating them to zero, the likelihood equations are obtained as

$$\frac{\partial\ell(\alpha,\lambda|x)}{\partial\alpha} = \frac{n}{\alpha} - \sum_{i=1}^{n}\log x_i - \lambda\sum_{i=1}^{n}x_i^{-\alpha}\log(1/x_i) = 0 \quad (2.2)$$

$$\frac{\partial\ell(\alpha,\lambda|x)}{\partial\lambda} = \frac{n}{\lambda} - \sum_{i=1}^{n}x_i^{-\alpha} = 0 \quad (2.3)$$

The estimate $\overset{\wedge}{\lambda}$ of the parameter λ can be obtained using equation 2.3. The MLE of α can be obtained by solving the above non linear equation. Numerical methods such as Fisher-Scoring method can be applied for solving the equation 2.2 by using the equation 2.1. In order to have a guess about the initial value of α the value of $\ell\,(\alpha)$ is calculated for various value of α and a curve of the log likelihood of α was drawn, when the curve reaches its peak at a neighborhood of α, this is considered as the initial value of α.

2.3. Method of Moments. The estimators of α and λ can be obtained by applying the method of moments (MME). System of two equations can be constructed from

$$\mu_1^{'}= \mathrm{m}_1^{'}\,,\ \mu_2^{'}= \mathrm{m}_2^{'}$$

Thus, the system of moments equations becomes as

$$\lambda^{\frac{1}{\alpha}}\Gamma\left(1-\frac{1}{\alpha}\right)=\mathrm{m}_1^{'}\,,\ \lambda^{\frac{2}{\alpha}}\Gamma\left(1-\frac{2}{\alpha}\right)=\mathrm{m}_2^{'},$$

It can be derived from these equations as

$$\Gamma\left(1-\frac{2}{\alpha}\right)=\frac{\sum \mathrm{x}_\mathrm{i}^2}{\mathrm{n}}\lambda^{-\frac{2}{\alpha}} \tag{2.4}$$

$$\overset{\wedge}{\lambda}=\left(\frac{\mathrm{m}_1^{'}}{\Gamma\left(1-\frac{1}{\alpha}\right)}\right)^{\alpha} \tag{2.5}$$

The estimate $\overset{\wedge}{\lambda}$ of the parameter λ can be obtained in 2.5. The estimate $\overset{\wedge}{\alpha}$ of the parameter α is obtained by solving 2.4 with respect to α. This equation has no analytical solution and must be solved numerically. Newton Raphson method is used to solve the equation.

2.4. Method of Least Squares. Let X_1, X_2,..., X_n be a random sample from the Inverse Weibull distribution. The cumulative distribution function 1.2 will be transformed to a linear function. The equation.

$$\ln\left[-\ \ln \mathrm{F(X)}\right]=\ln\lambda-\alpha\ln \mathrm{x}$$

Let $\mathrm{Y}=\ln\left[-\ln \mathrm{F(X)}\right], \mathrm{X}=\ln \mathrm{x}$, $\beta_1=-\alpha$,$\beta_0=\ln\lambda$ the equation can be written a

$$\mathrm{Y}=\beta_1\mathrm{X}+\beta_0$$

Now Let $\mathrm{X}_{(1)} < \mathrm{X}_{(2)}<\ldots<\mathrm{X}_{(n)}$ represent the order statistics of X_1, X_2,..., X_n.That is, $\mathrm{X}_{(i)}$ is the i^{th} smallest of X_1, X_2, ..., X_n, for i=1, 2, ..., n. Mean rank is used to estimate the values of the cumulative distribution function F(X). $\overset{\wedge}{\mathrm{F}}\,(\mathrm{x}_{(\mathrm{i})})=\frac{\mathrm{i}}{\mathrm{n}+1}$.

Subsequently, β_0 and β_1 regression parameter is choose to minimize the sum of the square errors. i.e. β_0 and β_1 are chosen to minimize.

$$\mathrm{Q}\left(\beta_0,\beta_1\right)=\sum_{\mathrm{i}=1}^{\mathrm{n}}\left(\mathrm{Y}_\mathrm{i}-\beta_0-\beta_1\ln \mathrm{x}_{(\mathrm{i})}\right)^2$$

To estimate β_0 and $\beta_1$differentiate Q partially with respect to β_0 and β_1 respectively and equate to zero i.e. $\frac{\partial Q}{\partial \beta_0} = 0$ and $\frac{\partial Q}{\partial \beta_1} = 0$ Therefore, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the parameters β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{n \sum_{i=1}^{n} \ln x_{(i)} \ln[-\ln \hat{F}(x_{(i)})] - \sum_{i=1}^{n} \ln x_{(i)} \sum_{i=1}^{n} \ln[-\ln \hat{F}(x_{(i)})]}{n \sum_{i=1}^{n} \ln^2 x_{(i)} - (\sum_{i=1}^{n} \ln x_{(i)})^2}$$

$$\hat{\beta}_0 = \frac{1}{n} \sum_{i=1}^{n} \ln[-\ln \hat{F}(x_{(i)})] + \hat{\alpha} \frac{1}{n} \sum_{i=1}^{n} \ln x_{(i)}$$

The estimate $\hat{\lambda}$ and $\hat{\alpha}$ of the parameters λ and α are given by

$$\hat{\alpha} = -\left[\frac{n \sum_{i=1}^{n} \ln x_{(i)} \ln[-\ln \hat{F}(x_{(i)})] - \sum_{i=1}^{n} \ln x_{(i)} \sum_{i=1}^{n} \ln[-\ln \hat{F}(x_{(i)})]}{n \sum_{i=1}^{n} \ln^2 x_{(i)} - (\sum_{i=1}^{n} \ln x_{(i)})^2}\right]$$

$$\hat{\lambda} = \exp\left[\frac{1}{n} \sum_{i=1}^{n} \ln[-\ln \hat{F}(x_{(i)})] + \hat{\alpha} \frac{1}{n} \sum_{i=1}^{n} \ln x_{(i)}\right]$$

2.5. Method of Weighted least squares. The estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the regression parameters β_0 and β_1 minimize the function

$$Q(\beta_0, \beta_1) = \sum_{i=1}^{n} w_i (Y_i - \beta_0 - \beta_1 \ln x_{(i)})^2$$

Where w_i is the weight factor, i $=1, 2, \ldots, n$. The weight factor proposed by Bergman(1986) for weibull distribution. The weight factor of inverse weibull distribution is

$$W_i = \left[\hat{F}(x_{(i)}) \ln \hat{F}(x_{(i)})\right]^2$$

Therefore, the estimates $\hat{\beta}_0$ and $\hat{\beta}_1$ of the parameters β_0 and β_1 are given by

$$\hat{\beta}_1 = \frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln x_{(i)} \ln[-\ln \hat{F}(x_{(i)})] - \sum_{i=1}^{n} w_i \ln x_{(i)} \sum_{i=1}^{n} w_i \ln[-\ln \hat{F}(x_{(i)})]}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln^2 x_{(i)} - (\sum_{i=1}^{n} w_i \ln x_{(i)})^2}$$

$$\hat{\beta}_0 = \frac{\sum_{i=1}^{n} w_i \ln[-\ln \hat{F}(x_{(i)})] + \hat{\alpha} \sum_{i=1}^{n} w_i \ln x_{(i)}}{\sum_{i=1}^{n} w_i}$$

The estimate $\hat{\lambda}$ and $\hat{\alpha}$ of the parameters λ and α are given by

$$\alpha = -\left[\frac{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln x_{(i)} \ln[-\ln \hat{F}(x_{(i)})] - \sum_{i=1}^{n} w_i \ln x_{(i)} \sum_{i=1}^{n} w_i \ln[-\ln \hat{F}(x_{(i)})]}{\sum_{i=1}^{n} w_i \sum_{i=1}^{n} w_i \ln^2 x_{(i)} - (\sum_{i=1}^{n} w_i \ln x_{(i)})^2}\right]$$

$$\lambda = \exp\left[\frac{\sum_{i=1}^{n} w_i \ln\left[-\ln \hat{F}(x_{(i)})\right] + \overset{\wedge}{\alpha} \sum_{i=1}^{n} w_i \ln x_{(i)}}{\sum_{i=1}^{n} w_i}\right]$$

3. Comparison of estimation methods using Simulation Study

In this section, the performance of the proposed estimators under four different estimation methods are investigate on the basis of their MSE through a simulation study. The simulation are carry out using R software. Random samples of various sizes are generated from the IW (α, λ) distribution by applying the algorithm present below. To view the performance of the estimators, eight different parameters for the scale parameter λ is taken such as 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3. The shape parameter α=6 is fix. For each value of λ, 1000 samples are generate by sample sizes n= 25, 50, 100, 150, 200, 250, 300, 350, 400, 450 and 500. In the same way, the scale parameter λ =3 is fixed, eight different values are taken for the shape parameter α such as 0.25, 0.5, 0.75, 1, 1.5, 2, 2.5, 3. The performance of the methods are compare based on the mean square error (MSE). To carry out this comparative study, the following steps are followed.

Algorithm:
Step 1: Fixed the value of n, α and λ.
Step 2: Generate n standard uniform variate i.e U$\sim$Uniform(0,1) distribution
Step 3: Generate n samples from IW distribution by using the following formula
$X = \left(-\frac{1}{\lambda}\log(u)\right)^{-\frac{1}{\alpha}}$
Step 4: Obtain the estimates of α and λ as $\hat{\lambda}$ and $\hat{\alpha}$
Step 5: Repeat steps 2-4 for 1000 times and obtained $\hat{\lambda}_1, \hat{\lambda}_2, ..., \hat{\lambda}_{1000}$ and $\hat{\alpha}_1, \hat{\alpha}_2, ..., \hat{\alpha}_{1000}$
Step 6: The mean square error (MSE) for each method was calculated. The results are displayed in Table 1 and 2 for the different parameters given by
$\mathrm{MSE}(\hat{\lambda}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\lambda}_i - \lambda)^2$ and $\mathrm{MSE}(\hat{\alpha}) = \frac{1}{1000}\sum_{i=1}^{1000}(\hat{\alpha}_i - \alpha)^2$.

The Simulation results are summarized in Table 1 and Table 2 as follows :

- Table 1 provide the MSE values for the estimation of λ.
- Table 2 provide the MSE values for the estimation of α.

3.1. Results and Discussion.

From Table 1 and Table 2, and we conclude that:

- The MSEs of all the estimators (MLEs, MMEs, LSEs and WLSEs) decrease as sample size n increases.
- It is observe that the MLE are much closed to the real parameter values.
- From the results in the above table it observed that the performance of MLE is very close to the performance of WLSE in all sample size.
- Compared to the other estimator MME perform worst and MME also have some limitation that does not exist when α lies between $0 < \alpha \leq 2$, due to this reason, in the case of estimating MME cannot be considered in estimating α value.

TABLE 1. Simulation Results (MSE of the different value of λ for fixed α=6 based on 1000 iterations)

n	Methods	0.25	0.50	0.75	1	1.5	2	2.5	3
25	MLE	0.00284	0.01179	0.02555	0.05238	0.10267	0.21009	0.29741	0.46487
	WLSE	0.00383	0.01491	0.03246	0.06443	0.13166	0.26165	0.38296	0.57299
	LSE	0.00527	0.02270	0.04794	0.08996	0.20719	0.31921	0.62270	0.76754
	MME	0.00796	0.02699	0.07054	0.11104	0.27597	0.40561	0.67466	1.12592
50	MLE	0.00140	0.00531	0.01190	0.02210	0.04566	0.08219	0.13945	0.19533
	WLSE	0.00161	0.00632	0.01547	0.03133	0.06184	0.11509	0.18178	0.25184
	LSE	0.00257	0.00948	0.02227	0.04005	0.08040	0.15779	0.26781	0.32336
	MME	0.00303	0.01228	0.02866	0.04768	0.13107	0.21154	0.32519	0.46215
100	MLE	0.00070	0.00263	0.00593	0.01063	0.02293	0.04225	0.06630	0.09159
	WLSE	0.00086	0.00391	0.00745	0.01346	0.03261	0.06138	0.08921	0.13167
	LSE	0.00107	0.00454	0.01056	0.01768	0.03814	0.06966	0.10911	0.15938
	MME	0.00139	0.00554	0.01414	0.02316	0.05325	0.09840	0.15050	0.20352
150	MLE	0.00040	0.00163	0.00403	0.00664	0.01592	0.02771	0.03939	0.06164
	WLSE	0.00054	0.00200	0.00473	0.00834	0.02020	0.03435	0.05392	0.08553
	LSE	0.00069	0.00297	0.00676	0.01202	0.02580	0.04554	0.07753	0.10427
	MME	0.00094	0.00386	0.00864	0.01603	0.03456	0.05808	0.09329	0.13345
200	MLE	0.00035	0.00128	0.00298	0.00535	0.01176	0.02023	0.03349	0.04483
	WLSE	0.00042	0.00154	0.00385	0.00725	0.01609	0.02758	0.04317	0.06167
	LSE	0.00057	0.00213	0.00493	0.00824	0.01828	0.03504	0.05517	0.08411
	MME	0.00067	0.00313	0.00661	0.01099	0.02662	0.04638	0.07645	0.10079
250	MLE	0.00025	0.00103	0.00256	0.00428	0.00917	0.01572	0.02465	0.03681
	WLSE	0.00032	0.00129	0.00305	0.00508	0.01100	0.02055	0.03279	0.04708
	LSE	0.00043	0.00164	0.00378	0.00645	0.01547	0.02912	0.03996	0.05757
	MME	0.00060	0.00235	0.00529	0.00910	0.02067	0.03703	0.05743	0.08187
300	MLE	0.00020	0.00081	0.00203	0.00319	0.00763	0.01336	0.02162	0.03227
	WLSE	0.00026	0.00108	0.00270	0.00440	0.00931	0.01816	0.02793	0.03832
	LSE	0.00036	0.00134	0.00320	0.00601	0.01220	0.02280	0.03533	0.04885
	MME	0.00048	0.00194	0.00404	0.00690	0.01883	0.03095	0.04868	0.07181
350	MLE	0.00019	0.00072	0.00169	0.00294	0.00677	0.01169	0.01696	0.02695
	WLSE	0.00024	0.00089	0.00207	0.00350	0.00863	0.01420	0.02322	0.03397
	LSE	0.00030	0.00122	0.00254	0.00500	0.01085	0.01897	0.02794	0.04259
	MME	0.00041	0.00168	0.00342	0.00613	0.01422	0.02437	0.04014	0.05610
400	MLE	0.00015	0.00061	0.00151	0.00241	0.00565	0.01001	0.01561	0.02272
	WLSE	0.00020	0.00083	0.00178	0.00333	0.00760	0.01339	0.01983	0.02858
	LSE	0.00026	0.00108	0.00240	0.00428	0.01030	0.01861	0.02593	0.03373
	MME	0.00035	0.00142	0.00316	0.00602	0.01223	0.02285	0.03634	0.05626
450	MLE	0.00014	0.00054	0.00121	0.00221	0.00493	0.00872	0.01482	0.02093
	WLSE	0.00018	0.00074	0.00160	0.00280	0.00650	0.01229	0.01907	0.02517
	LSE	0.00025	0.00101	0.00209	0.00384	0.00908	0.01576	0.02200	0.03373
	MME	0.00031	0.00126	0.00276	0.00465	0.01077	0.02002	0.03190	0.04602
500	MLE	0.00013	0.00050	0.00111	0.00205	0.00426	0.00845	0.01255	0.01919
	WLSE	0.00016	0.00064	0.00135	0.00243	0.00576	0.01037	0.01642	0.02227
	LSE	0.00020	0.00082	0.00176	0.00313	0.00749	0.01344	0.02063	0.02754
	MME	0.00026	0.00114	0.00250	0.00447	0.01118	0.01887	0.02821	0.03986
1000	MLE	0.00006	0.00025	0.00052	0.00094	0.00214	0.00422	0.00622	0.00927
	WLSE	0.00007	0.00033	0.00075	0.00134	0.00290	0.00522	0.00895	0.01262
	LSE	0.00010	0.00041	0.00090	0.00171	0.00378	0.00691	0.01100	0.01540
	MME	0.00013	0.00062	0.00129	0.00226	0.00467	0.00857	0.01544	0.02081

- It is also observe that the values of λ and α increase, the performance of $\hat{\lambda}$ and $\hat{\alpha}$ for the above declare three method become worse, it can be seen in MSE values.

TABLE 2. Simulation Results (MSE of the different value of α for fixed λ=3 based on 1000 iterations)

n	Methods	0.25	0.50	0.75	1	1.5	2	2.5	3
25	MLE	0.00138	0.00504	0.01149	0.02110	0.05322	0.08450	0.12464	0.18462
	WLSE	0.00221	0.00847	0.01961	0.03787	0.08035	0.13202	0.23618	0.31862
	LSE	0.00269	0.01193	0.02437	0.04507	0.09324	0.15945	0.27143	0.41514
50	MLE	0.00059	0.00254	0.00604	0.00970	0.02228	0.03633	0.06833	0.09352
	WLSE	0.00111	0.00445	0.00977	0.01775	0.03701	0.07172	0.10039	0.16329
	LSE	0.00138	0.00619	0.01315	0.02377	0.05199	0.09711	0.14931	0.20788
100	MLE	0.00031	0.00117	0.00272	0.00464	0.01139	0.01839	0.02987	0.04334
	WLSE	0.00054	0.00201	0.00508	0.00895	0.02019	0.03836	0.05582	0.08376
	LSE	0.00077	0.00291	0.00675	0.01212	0.02724	0.05029	0.07750	0.10765
150	MLE	0.00021	0.00086	0.00180	0.00324	0.00736	0.01234	0.01964	0.02990
	WLSE	0.00035	0.00144	0.00333	0.00560	0.01461	0.02527	0.03850	0.05092
	LSE	0.00049	0.00215	0.00458	0.00800	0.01678	0.03397	0.05343	0.07051
200	MLE	0.00015	0.00061	0.00137	0.00232	0.00514	0.00935	0.01498	0.01974
	WLSE	0.00026	0.00111	0.00258	0.00444	0.01060	0.01822	0.02952	0.03918
	LSE	0.00039	0.00163	0.00372	0.00630	0.01429	0.02376	0.04070	0.05392
250	MLE	0.00011	0.00044	0.00112	0.00187	0.00431	0.00786	0.01234	0.01792
	WLSE	0.00022	0.00088	0.00206	0.00372	0.00784	0.01399	0.02370	0.03294
	LSE	0.00031	0.00122	0.00283	0.00507	0.01235	0.01901	0.03109	0.04271
300	MLE	0.00010	0.00042	0.00078	0.00163	0.00328	0.00630	0.01010	0.01438
	WLSE	0.00018	0.00078	0.00170	0.00321	0.00698	0.01168	0.01850	0.02560
	LSE	0.00024	0.00105	0.00224	0.00407	0.01018	0.01654	0.02510	0.03735
350	MLE	0.00009	0.00032	0.00077	0.00144	0.00290	0.00560	0.00853	0.01167
	WLSE	0.00016	0.00066	0.00150	0.00279	0.00614	0.00981	0.01717	0.02150
	LSE	0.00021	0.00083	0.00194	0.00307	0.00820	0.01397	0.02336	0.03154
400	MLE	0.00008	0.00030	0.00065	0.00130	0.00273	0.00486	0.00711	0.01063
	WLSE	0.00013	0.00057	0.00123	0.00220	0.00519	0.00914	0.01511	0.02019
	LSE	0.00019	0.00078	0.00186	0.00314	0.00754	0.01152	0.01940	0.02731
450	MLE	0.00006	0.00027	0.00061	0.00101	0.00230	0.00399	0.00646	0.01063
	WLSE	0.00012	0.00047	0.00117	0.00203	0.00469	0.00701	0.01385	0.01741
	LSE	0.00016	0.00067	0.00165	0.00286	0.00650	0.01101	0.01842	0.02564
500	MLE	0.00005	0.00023	0.00056	0.00097	0.00227	0.00345	0.00713	0.00883
	WLSE	0.00011	0.00046	0.00107	0.00176	0.00417	0.00738	0.01190	0.01683
	LSE	0.00014	0.00061	0.00138	0.00250	0.00555	0.00992	0.01546	0.02432
1000	MLE	0.00003	0.00012	0.00026	0.00053	0.00104	0.00183	0.00298	0.00402
	WLSE	0.00005	0.00023	0.00050	0.00087	0.00220	0.00367	0.00573	0.00782
	LSE	0.00007	0.00028	0.00066	0.00117	0.00279	0.00484	0.00707	0.01104

- As the sample size increase, MSE of the estimated parameters (λ and α) decreases. This represent that the MLE, WLSE, LSE and MME provide asymptotically normally distributed and consistent estimators for the parameters.
- From the table it is observe that MSE of the estimated parameters decreases when the sample size increase. It represents that estimator of four methods more efficient for the parameters by increasing sample size.

From the Table 1 & 2, it can be clearly noted that MLE and WLSE perform better than LSE, MME estimator. It can be note that computation part of WLSE is very easy to compare computation of MLE. Here MLE and WLSE are compare in the case of small sample size n(=5,6,7,8,9,10). MSE of MLE and WLSE are quite close when sample size is small and it shown in the Table 3.

Table 3. The MSE based on 1000 iteration simulates data for fixed values of(i) α= 0.25, λ=6 and (ii) α=2, λ=1

n	**MLE (λ)**	**WLSE(λ)**
5	0.03004	0.03282
6	0.02756	0.02715
7	0.01862	0.01766
8	0.01617	0.01906
9	0.01285	0.01358
10	0.01104	0.01224

n	**MLE (α)**	**WLSE(α)**
5	0.23273	0.22975
6	0.19851	0.15477
7	0.12636	0.13614
8	0.11329	0.11420
9	0.08867	0.09624
10	0.08673	0.08730

4. Real data illustration

In this example, consider a real life data set and illustrate the methods are propose in the previous sections. The data set consists survival times of guinea pigs injected with different amount of tubercle bacilli and was studied by Bjerkedal (1960), Guinea pigs are known to have high susceptibility of human tuberculosis, which is one of the reasons for choosing this species. The regimen number is the common logarithm of the number of bacillary units in 0.5 ml. of challenge solution; i.e., regimen 6.6 corresponds to 4.0×10^6 bacillary units per 0.5 ml. (log $(4.0 \times 10^6) = 6.6$). The data represents the survival times of Guinea pigs in days. The data are given below:
12, 15, 22, 24, 24, 32, 32, 33, 34, 38, 38, 43, 44, 48, 52, 53, 54, 54, 55, 56, 57, 58, 58, 59, 60, 60, 60, 60, 61, 62, 63, 65, 65, 67,68, 70, 70, 72, 73, 75, 76, 76, 81, 83, 84, 85, 87, 91, 95, 96, 98, 99, 109, 110, 121, 127, 129, 131, 143, 146, 146, 175, 175, 211,233, 258, 258, 263, 297, 341, 341, 376.

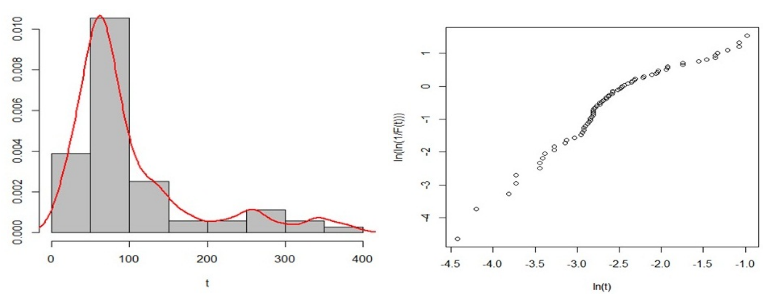

Figure 2. Histogram and IW Probability Plot for guinea pigs failure data

In this case n= 72, the mean, standard deviation and coefficient of skewness are calculate as 99.82, 80.55 and 1.80, respectively. The skewness measure indicates that the data are positively skewed. Histogram drawn to the data is displayed in Figure 2. The histogram and the corresponding frequency curve show that the distribution of given data is a positively skewed distribution. For computational ease, each data point is divided by 1000.

The inverse Weibull probability plot for a complete sample to plot $\ln[-\ln \mathrm{F}(\mathrm{t})]$ against ln (t),seen in Murthy *et al.* (2004). The median rank is used to estimate the values of the cumulative distribution function F(t). $\hat{\mathrm{F}}(\mathrm{t}_{(\mathrm{i})}) = \frac{\mathrm{i}-0.3}{\mathrm{n}+0.4}$, where $\mathrm{t}_{(i)}$ is the i^{th} smallest of t_1, $\mathrm{t}_2, \ldots, \mathrm{t}_n$, for i=1,2,...,n. The inverse Weibull probability plot of the data, Figure 2, shows that it is reasonable to use IW distribution to analyze the data.

The maximum likelihood estimates of λ is 0.01697 and α is 1.40128 is can be obtained by Fisher's scoring method and the initial value is got by plotting log likelihood of α, this shown in Figure 3. The WLS estimates of λ and α are 0.01790 and 1.40824. The LS estimates of λ and α are 0.00935 and 1.60594. Hence the estimate value of MLE and WLSE are quite similar, it can be seen through this real date.

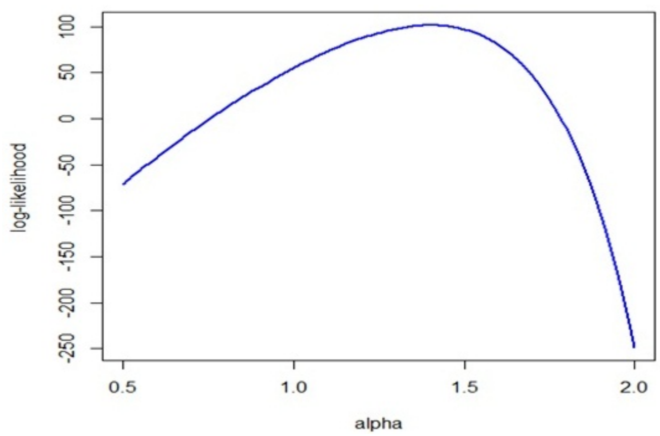

FIGURE 3. Log-likelihood of α for guinea pigs failure Data

5. Conclusion

The performances of the four commonly used methods are studied for estimating the inverse Weibull distribution parameters. Since the moments of the inverse Weibull model does not exist always. Among the four methods MLE, WLSE provide minimum MSE compare to other methods. The WLSE and MLE are nearly similar. It is important to observe that MLEs of the unknown parameters cannot be obtained in explicit form. Further, the estimation of the unknown parameters is obtained by using numerical approximation procedure. Hence WLSE is propose, which is in explicit form. WLSE is simple and easy to implement compared to the MLE. WLSE and MLE are quite similar can be evidently illustrated by the real life example.

Acknowledgment. The authors acknowledge University Grants Commission for providing the infrastructure facility under the scheme of SAP (DRS-I). The second author acknowledges UGC for financial support to carry out this research work under Basic Science Research Fellowship.

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