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FUZZY TIME SERIES MODELING FOR WHEAT PRODUCTION

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ABSTRACT. In this paper investigates the predictive performance of fuzzy time series method for wheat production data set. Fuzzy time series deals with forecasting under the fuzzy environment that contains uncertainty, ambiguity, vagueness and imprecision of the data set. It is a time invariant method for modeling rather than classical method. There is no prerequisite like stationarity and normality. The aim of present work is to design and implement the competent of fuzzy time series model for forecasting of wheat production data set. The accuracy of the forecasting result is compared based on some residual methods.

1. Introduction

Wheat is the second most important food-grain of India and is the staple food of millions of Indians, particularly in the northern and north-western parts of the country. It is rich in proteins, vitamins, carbohydrates and provides balanced food. India is the fourth largest producer of wheat in the world after Russia, USA and China and accounts for 8.7 percent of the worlds total production of wheat. Conditions of growth for wheat are more flexible than those of rice. In contrast to rice, wheat is a Rabi crop which is sown in the beginning of winter and is harvested in the beginning of summer. The time of sowing and harvesting differs in different regions due to climatic changes. The sowing of wheat crop normally begins in the September-October in Karnataka, Maharashtra, Andhra Pradesh, Madhya Pradesh and West Bengal; October-November in Bihar, Uttar Pradesh, Punjab, Haryana and Rajasthan Nov-December. in Himachal Pradesh and Jammu Kashmir.The harvesting is done in Jan - Feb in Karnataka, Andhra Pradesh, M.P., and in West Bengal; March-April in Punjab, Haryana, U.P and Rajasthan and in April-May in Himachal Pradesh the growing period is variable from one agro climatic zone to other that effects the vegetative and reproductive period leading to differences in potential yield. The important factors affecting the productivity are seeding time and methodology, crop establishment and climatic conditions during the growing season. Wheat is primarily a crop of mid-latitude grasslands and requires a cool climate with moderate rainfall. The ideal, wheat climate has winter temperature 10C to 15C and summer temperature varying from 21C to 26C. The temperature should be low at the time of sowing but as the harvesting time approaches higher temperatures are required for proper ripening

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of the crop.But sudden rise in temperature at the time of maturity is harmful. Wheat thrives well in areas receiving an annual rainfall of about 75 cm. Annual rainfall of 100 cm is the highest limit of wheat cultivation. A time series is a sequence of observations collected at regular time intervals.Time series analysis comprises methods for analyzing time series data in order to extract meaningful statistics and other characteristics of the data. Time series forecasting is the use of a model to predict future values based on previously observed values time series model is used to discover the pattern and predict future values of real data. In the present work, we have implemented to forecast the wheat production in India.

2. Methodology

This section, proposed a new method for wheat production forecasting by using actual production as the universe of discourse and mean based partitioning. The data was taken from Ministry of Statistics Programme Implementation (MOSPI) India.

2.1. ARIMA. The main stages in setting up a Box-Jenkins forecasting model are as follows

- Identification
- Estimating the parameters
- Diagnostic checking and
- Forecasting

The ARIMA methodology developed by Box and Jenkins (1976) has gained enormous popularity in many areas and research practices confirmed its power and flexibility (Hoff, 1983; Pankratz, 1983; Vandaele, 1983). In general, an ARIMA model is characterized by the notation ARIMA (p, d, q) where, p, d and q denote orders of auto-regression, integration (differentiation) and moving average, respectively. ARIMA technique comprises of linear time series function of past actual values and random shocks.. For instance, given a time series process $\{Y_t\}$, a first order autoregressive process is denoted by ARIMA (1,0,0) or simply AR(1) and is given by

$$\widehat{Y_t} = \pi + \Phi Y_{t-1} + \varepsilon_t$$

and first order moving average process is denoted by ARIMA (0,0,1) or simply MA(1) and is given by

$$\widehat{Y} = \mu - \theta_1 \varepsilon_{t-1} + \varepsilon_t$$

Alternatively, the ultimately derived model may be a mixture of these processes and of higher orders as well. Thus a stationary ARMA (p, d, q) process is defined by the equation:

$$Y_t = \mu + \Phi_1 Y_{t-1} + \Phi_2 Y_{t-2} + \cdots + \Phi_p Y_{t-p} - \theta_1 \varepsilon_{t-1} - \varepsilon_2 \Phi_{t-2} + \cdots + \varepsilon_q \Phi_{t-q} \varepsilon_q$$

where ε_t's are independently and normally distributed with zero mean and constant variance σ^2 for t = 1,2,...,n. Note that the values of p and q, in practice lies between 0 and 3.

The annual production data of wheat during the period 1958 to 2015 were used to fit an appropriate ARIMA model. The Box-Jenkins methodology is used for analyzing and modeling a time series data it involves following steps, model identification, parameter estimation and model validation. Identification of suitable ARIMA model with lowest AIC (Akaike Information Criterion) and parameter estimation for forecasting is tedious. It is not feasible to simply fit every potential model and choose the one with the lowest AIC. To overcome the above situation the function *auto.arima* () (Hyndman and Khandakar, 2008) available for the projections to be more precise and realistic, we need to specify certain preconditions considered for time series analysis. This includes absence of abnormal climatic conditions and unusual developments such as wide spread flood, viral attacks, effects of pest and diseases.

2.2. Fuzzy Time Series. The forecasting process of Fuzzy time series models are described as follows:

Step - 1: Firstly, define the universe of discourse U and Partition U into equally length intervals.

Step - 2: Define fuzzy sets A_i, and apply fuzzification.

Step - 3: Define fuzzy numbers the number of intervals (fuzzy numbers), m is computed by $m = \dfrac{D_{max} + D_2 - D_{min} + D_1}{1}$ thus, there are m interval and m fuzzy numbers, which are $u_1, u_2, \cdots, u_m$, and $A_1, A_2, \cdots, A_m$, respectively assume that the m interval are $u_1 = [d_1, d_2], u_2 = [d_2, d_3], u_3 = [d_3, d_4], \cdots, u_{m-2} = [d_{m-2}, d_{m-1}], u_{m-1} = [d_{m-1}, d_m]$, and $u_m = [d_m, d_{m+1}]$. The fuzzy numbers $A_1, A_2, \cdots, A_m$.

Step - 4: Fuzzify the historical data: If the value of D_{vt} is located in the range of u_j, then it belongs to fuzzy number A_j. All D_{vt} must be classified into the corresponding fuzzy numbers.

Step -5: Generate the fuzzy logical relationship: For all fuzzified data, derive the fuzzy logical Relationship based on definition: 3 The fuzzy logical relationship is like $A_j \rightarrow A_k$ which denotes that if the D_{vt-1} value of time t-is A_j then that of time t is $A_k^{"}$

Step - 6: Establish the fuzzy logical relationship groups: The derived fuzzy logical relationship can be arranged into fuzzy logical relationship group based on the same fuzzy numbers on the left hand sides of the fuzzy logical relationship. The fuzzy logical relationship groups are like the following

$$\begin{array}{c} A_j \rightarrow A_{k1} \\ A_j \rightarrow A_{k2} \\ . \\ . \\ . \\ A_j \rightarrow A_{kp} \end{array}$$

Step - 7: compute the forecasted outputs: The forecasted value at time t, F_{vt} is determined by the following three heuristic rules. Assume the fuzzy number of D_{vt-1} at time t-1 is A_j.

Rule 1: If the fuzzy logical relationship group of A_j empty; $A_j \rightarrow \emptyset$, then the value of F_{vt} is A_j which is $(d_{j-1}, d_j, d_{j+1}, d_{j+2})$

Rule 2: If fuzzy logical relationship group of A_j is one to one; $A_j \rightarrow A_k$, then the value of F_{vt} is A_j, which is $(d_{j-1}, d_j, d_{j+1}, d_{j+2})$,

Rule 3: If the fuzzy logical relationship group of A_j is one to many $A_j \rightarrow A_k, A_j \rightarrow A_{k2}, \ldots.., A_j \rightarrow A_{kp}$, and then the value of $F_{vt}$is calculate as follows.

$$F_{vt} = \frac{A_{k1}+A_{k2}+\cdots+A_{kP}}{P}$$

$$= \left(\frac{d_{k1-1}+\cdots+d_{kp-1}}{p}, \frac{d_{k1}+\cdots+d_{kp}}{p}, \frac{d_{k1+1}+\cdots+d_{kp+1}}{p}, \frac{d_{k1+2}+\cdots+d_{kp+2}}{p}\right)$$

Where

$$\begin{aligned}
A_{k1} &= (d_{k1-1}, d_{k1}, d_{k1+1}, d_{k1+2}), \\
A_{k2} &= (d_{k2-1}, d_{k2}, d_{k2+1}, d_{k2+2}) \\
\vdots & \quad \vdots \\
A_{kp} &= (d_{kp-1}, d_{kp}, d_{kp+1}, d_{dk+2})
\end{aligned}$$

The following measures of goodness of fit have been used to judge the adequacy of the model developed

$$\text{Root mean square error (RMSE)} = \left[\frac{\sum_{i=1}^{n}\left(Y_i - \hat{Y}_i\right)^2}{n}\right]^{\frac{1}{2}}$$

$$\text{Mean absolute error (MAE)} = \frac{\sum_{i=1}^{n}\left|\left(Y_i - \hat{Y}_i\right)^2\right|}{n}$$

2.3. Concept of Fuzzy Time Series.

Definition 2.1. Fuzzy Set Fuzzy sets are sets which elements consists degree of membership. Let U be the universe of discourse, $U = u_1, u_2, \cdots, u_n$, and let A be a fuzzy set in the universe of discourse U defined as follows:

$$A = f_A(u_1)u_1 + f_A(u_2)u_2 + \cdots + f_A(u_n)u_n$$

where f_A is the membership function of A, $f_A : U \rightarrow [0, 1], f_A(u_i)$ referred the grade of membership of u_i in the fuzzy set A, $f_A(u_i)\varepsilon[0, 1]$, and $1 \leq i \leq n$.

Definition 2.2. Time Series A time series is a collection of sequential data points, measured at successive time span at uniform time intervals

Definition 2.3. Fuzzy Time Series Consecutive sequences of indefinite data are considered as time series with fuzzy data. A time series with fuzzy data is referred to as fuzzy time series. Let $X(_t)(t = \cdots, 0, 1, 2, \cdots)$ be the universe of discourse and be a subset of R, and let fuzzy set $f_i(t)(i = 1, 2\cdots)$ be defined in X(t). Let F (t) be a collection of $f_i(t)(i = 1, 2, \cdots)$. Then, F(t) is called a fuzzy time series of $X(t)(t = \cdots, 0, 1, 2, \cdots)$.

3. RESULTS AND DISCUSSION

The Production of wheat (in million tons) in India is considered for the model building for 57 observations from the years 1958 to 2015 are used for fitting the both fuzzy time series and ARIMA model. The main objective is to identify the more accurate model for forecasting agricultural production.

3.1. ARIMA Model. The production and yield of wheat in India have observed a many fold increase during past 57 years (Fig.1). Increase in wheat productivity represented an increase of year. However, cultivated area merely doubled during this period. Highest production as well as yield of wheat was recorded during 2019-2020. Increase in production of wheat in this, Improved plant protection measures, efficient irrigation facility, etc. As close observation of the production and yield of last ten years showed that overall negative trend was registered over these years. Possible cause of such disparity in production might be abnormal climate change, excessive chemical use in recent years etc.

Estimation of ARIMA model involves transformation of the forecasting variables into a stationary series. The most usual method is to check stationary series through the graph or time plot of data (Mandal, 2005). Fig. 1 revealed that the data is non-stationary. Non-stationary in mean is corrected through suitable differencing of the function auto.arima () in the forecast library of the language worked out that difference of order 1 was sufficient to achieve stationarity in mean for all the cases. Model diagnostics variable production is presented in a lower value of RMSE (root mean square error) and MAE (mean absolute error) define that identified models were accurate enough to forecast above variables. The production results are here under.

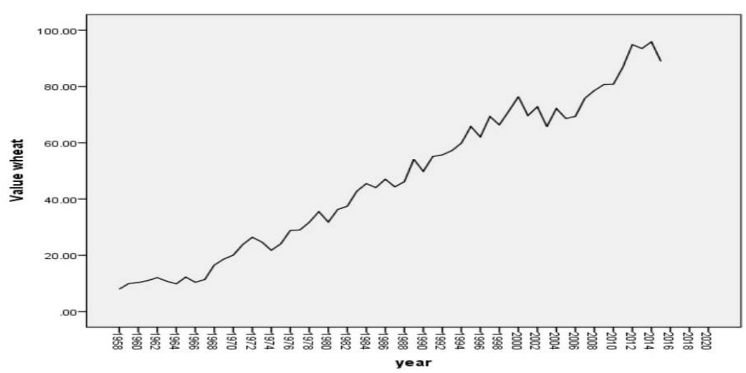

FIGURE 1. non-stationary series of wheat production

Using ARIMA (1, 1, 1) the 5 year advance production and their 95 per cent confidence interval are calculated and presented in Table 1. The production results in wheat production indicated trends in production in the coming years. On the basis of model output an increase in the production from 88.77 million tons in 2010-11 to 103.87 million tons in 2015-16 and 110.47 million tons in 2020 is expected (Table 5). The minimum production with 95 per cent confidence showed that wheat production may decrease in the coming years and will reach 86.97 million tons during 2020 which is at par with the current production level. This may occur due to adverse effect of temperature rise, government attention to the crop and decrease in cultivable land. However, maximum production registered a significant increase in production levels which may reach upto 110.47 million tons by 2020, like wheat production in India. Figure represent the forecasting value of wheat production.

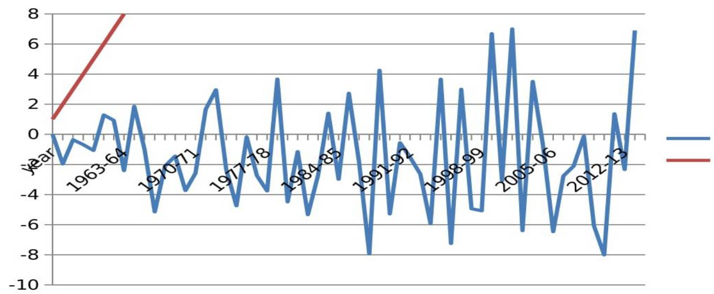

FIGURE 2. stationary series of wheat production

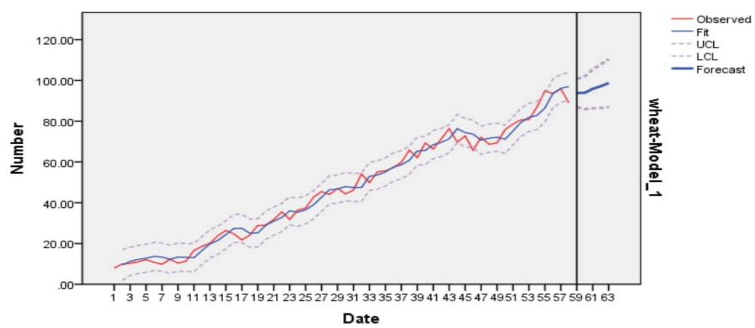

FIGURE 3. Forecasting of wheat production

TABLE 1. Prediction of wheat production

Year	Production	Difference	Prediction	LCL	UCL	Noise Residual
2010	80.8	0.12	81.87	74.97	88.77	-1.07
2011	86.87	6.07	82.8	75.9	89.7	4.07
2012	94.88	8.01	86.46	79.56	93.36	8.42
2013	93.51	-1.37	93.65	86.75	100.55	-0.14
2014	95.85	2.34	96.09	89.19	102.99	-0.24
2015	88.94	-6.91	96.97	90.07	103.87	-8.03
2016			93.81	86.91	100.71	
2017			93.92	85.9	101.95	
2018			95.91	86.36	105.47	
2019			97.17	86.49	107.84	
2020			98.71	86.94	110.47	

TABLE 2. Forecasting of wheat production

Model		59	60	61	62	63
wheat-Model_1	Forecast	93.81	93.92	95.91	97.17	98.71
	UCL	100.71	101.95	105.47	107.84	110.47
	LCL	86.91	85.90	86.36	86.49	86.94

3.2. Computation of Fuzzy Time Series model.

Step 1: Define the universe of discourse to accommodate the time series data. It needs the minimum and maximum production and set as D min and D max. Thus universe of discourse U is defined as [Dmin – D_1, Dmax - D_2], here D_1, and D_2 are two proper positive numbers. In the present case of production forecasting universe of discourse computed is U = [7.99 – 95.85]

step 2: Partition the universes of discourse into 20 equal length intervals U_1, U_2,...., $U_{20.}$

step 3: Calculate the number of intervals (fuzzy numbers) as follow.

Intervals	*Fuzzy Numbers*
U_1 =[5, 10], U_{11}=[55, 60]	A_1= (0, 5, 10, 15)
U_2=[10, 15], U_{12}=[60, 65]	A_2= (5, 10, 15, 20)
⋮	⋮
U_9=[45, 50], U_{19}=[95, 100]	A_{19}= (85, 90, 95, 100)
U_{10}=[50, 55], U_{20}=[100,105]	A_{20}= (90, 95, 100, 105)

step 4: Fuzzify the productions, for example, the wheat production in year 1958 is 7.99, which is located at the range of $U_2 = [10, 15]$. Thus, the corresponding fuzzy number of year 1958 is assigned as A_i. Table 1 lists the corresponding fuzzy number for the wheat production of each year.

step 5: According to table 3, we can derive the fuzzy logical relationships as shown in table 4. Notice that the repeated relationships are counted only once.

step 6: Based on the same fuzzy numbers on the left hand side of the fuzzy logical relationships in table 4, 18 fuzzy logical relationship groups are generated as shown in table 5.

step 7: According to tables 5 and 6, we can calculate the forecasted wheat productions. For instance, the forecasted wheat productions of years 1959 and 1961 can be illustrated below:

Forecasting 1958: The fuzzified wheat production of year 1958 in table 3 is A_1, and from table 6, we can find that there are three fuzzy logical relationships in group 1. A2→A2, A2→A3, A2→A3.

According to *Rule 2,*the forecasted wheat productions of year 1958 is A_1.Thus, $F_{V1958} = (9.27, 11.17, 11.17) = 10.53$

Forecasting 1959: According to table 6, we can find that there are five fuzzy logical relationships in group 2. A3→A3, A3→A3, A3→A2, A3→A3, A3→A4. The forecasted wheat production of year 1959 is A_6. Thus, $F_{V1959} = (11.17, 11.17, 11.17, 9.27, 17.60) = 12.76$

Forecasting 1960: Because the fuzzified wheat production of 1960 in table 5 is A_1, and from table 6, we can find that there are two logical relationships in group 3. A4→A4, A4→A5

According to *Rule 3*, the forecasted wheat production of year 1960 is computed as follows:

$$F_{V1960} = \frac{A_4 + A_5}{2} = \frac{17.60 + 22.91}{2} = 20.22$$

In figure, established that observed value and estimated value of wheat production based on fuzzy time series model.

TABLE 3. Fuzzy numbers of the wheat production

Year	Production	Fuzzy No	Estimate value	Year	Production	Fuzzy No	Estimate value
1957-58	7.99	A2	9.27	1986-87	44.32	A9	43.73
1958-59	9.96	A2	9.27	1987-88	46.17	A10	47.14
1959-60	10.32	A3	11.17	1988-89	54.11	A11	54.11
1960-61	11	A3	11.17	1989-90	49.85	A10	43.14
1961-62	12.07	A3	11.17	1990-91	55.14	A12	56.97
1962-63	10.78	A3	11.17	1991-92	55.69	A12	56.97
1963-64	9.85	A2	9.27	1992-93	57.21	A12	56.97
1964-65	12.26	A3	11.17	1993-94	59.84	A12	56.97
1965-66	10.4	A3	11.17	1994-95	65.77	A14	67.84
1966-67	11.39	A3	11.17	1995-96	62.1	A13	62.10
1967-68	16.54	A4	17.60	1996-97	69.35	A14	67.84
1968-69	18.65	A4	17.60	1997-98	66.35	A14	67.84
1969-70	20.09	A5	22.91	1998-99	71.29	A15	72.07
1970-71	23.83	A5	22.91	1999-00	76.37	A16	76.92
1971-72	26.41	A6	28.09	2000-01	69.68	A14	67.84
1972-73	24.74	A5	22.91	2001-02	72.77	A15	72.07
1973-74	21.78	A5	22.91	2002-03	65.76	A14	67.84
1974-75	24.1	A5	22.91	2003-04	72.16	A15	72.07
1975-76	28.84	A6	28.09	2004-05	68.64	A14	67.84
1976-77	29.01	A6	28.09	2005-06	69.35	A14	67.84
1977-78	31.75	A7	31.79	2006-07	75.81	A16	76.92
1978-79	35.51	A8	36.42	2007-08	78.57	A16	76.92
1979-80	31.83	A7	31.79	2008-09	80.68	A17	80.74
1980-81	36.31	A8	36.42	2009-10	80.8	A17	80.74
1981-82	37.45	A8	36.42	2010-11	86.87	A18	87.91
1982-83	42.79	A9	43.73	2011-12	94.88	A19	94.20
1983-84	45.48	A10	47.14	2012-13	93.51	A19	94.20
1984-85	44.07	A9	43.73	2013-14	95.85	A20	95.85
1985-86	47.05	A10	47.14	2014-15	88.94	A18	87.91

Table 4. Fuzzy Logic Relationship Group

A2→A2	A2→A3	A3→A3	A3→A3	A3→A2	A2→A3	A3→A3
A3→A4	A4→A4	A4→A5	A5→A5	A5→A6	A6→A5	A5→A5
A5→A5	A5→A6	A6→A6	A6→A7	A7→A8	A8→A7	A7→A8
A8→A9	A9→A10	A10→A9	A9→A10	A10→A9	A9→A10	A10→A11
A11→A10	A10→A12	A12→A12	A12→A12	A12→A12	A12→A14	A14→A13
A13→A14	A14→A14	A14→A15	A15→A16	A16→A14	A14→A15	A15→A14
A14→A15	A15→A14	A14→A14	A14→A16	A16→A16	A16→A17	A17→A17
A17→A18	A17→A18	A18→A19	A19→19	A19→A20	A20→A18	

Table 5. Fuzzy logical relationships groups

Group	Fuzzy logical Relationship
1	A2→A2 ,A2→A3,A2→A3
2	A3→A3,A3→A3,A3→A2,A3→A3,A3→A4
3	A4→A4,A4→A5
4	A5→A5,A5→A6,A5→A5,A5→A5,A5→A6
5	A6→A5,A6→A6,A6→A7
6	A7→A8, A7→A8
7	A8→A7,A8→A9
8	A9→A10, A9→A10, A9→A10
9	A10→A9,A10→A9,A10→A11,A10→A12
10	A11→A10
11	A12→A12,A12→A12,A12→A12,A12→A14
12	A14→A13,A14→A14,A14→A15, A14→A15, A14→A15, A14→A14, A14→A16
13	A15→A14,A15→A14,A15→A16
14	A16→A16,A16→A17
15	A17→A17,A17→A18,A17→A18
16	A18→A19
17	A19→A19,A19→20
18	A20→A18

The production and estimated wheat production data base on fuzzy time series modeling is given in the table 6 and estimate depicted in fig: 3

Table 6. Estimated production and forecasting of Wheat Production

Year	Production	Fuzzy No	Estimate	Error	Year	Production	Fuzzy No	Estimate	Error
1957-58	7.99	A2	9.27	0.00	1986-87	44.32	A9	43.73	0.02
1958-59	9.96	A2	9.27	0.12	1987-88	46.17	A10	47.14	0.04
1959-60	10.32	A3	11.17	0.14	1988-89	54.11	A11	54.11	0.00
1960-61	11	A3	11.17	0.03	1989-90	49.85	A10	43.14	0.23
1961-62	12.07	A3	11.17	0.13	1990-91	55.14	A12	56.97	0.06
1962-63	10.78	A3	11.17	0.06	1991-92	55.69	A12	56.97	0.04
1963-64	9.85	A2	9.27	0.10	1992-93	57.21	A12	56.97	0.01
1964-65	12.26	A3	11.17	0.15	1993-94	59.84	A12	56.97	0.08
1965-66	10.4	A3	11.17	0.13	1994-95	65.77	A14	67.84	0.05
1966-67	11.39	A3	11.17	0.03	1995-96	62.1	A13	62.10	0.00
1967-68	16.54	A4	17.60	0.11	1996-97	69.35	A14	67.84	0.04
1968-69	18.65	A4	17.60	0.10	1997-98	66.35	A14	67.84	0.04
1969-70	20.09	A5	22.91	0.24	1998-99	71.29	A15	72.07	0.02
1970-71	23.83	A5	22.91	0.07	1999-00	76.37	A16	76.92	0.02
1971-72	26.41	A6	28.09	0.11	2000-01	69.68	A14	67.84	0.01
1972-73	24.74	A5	22.91	0.13	2001-02	72.77	A15	72.07	0.05
1973-74	21.78	A5	22.91	0.09	2002-03	65.76	A14	67.84	0.02
1974-75	24.1	A5	22.91	0.09	2003-04	72.16	A15	72.07	0.05
1975-76	28.84	A6	28.09	0.05	2004-05	68.64	A14	67.84	0.00
1976-77	29.01	A6	28.09	0.05	2005-06	69.35	A14	67.84	0.02
1977-78	31.75	A7	31.79	0.00	2006-07	75.81	A16	76.92	0.04
1978-79	35.51	A8	36.42	0.04	2007-08	78.57	A16	76.92	0.03
1979-80	31.83	A7	31.79	0.00	2008-09	80.68	A17	80.74	0.04
1980-81	36.31	A8	36.42	0.01	2009-10	80.8	A17	80.74	0.00
1981-82	37.45	A8	36.42	0.05	2010-11	86.87	A18	87.91	0.00
1982-83	42.79	A9	43.73	0.04	2011-12	94.88	A19	94.20	0.02
1983-84	45.48	A10	47.14	0.06	2012-13	93.51	A19	94.20	0.01
1984-85	44.07	A9	43.73	0.01	2013-14	95.85	A20	95.85	0.01
1985-86	47.05	A10	47.14	0.00	2014-15	88.94	A18	87.91	0.00

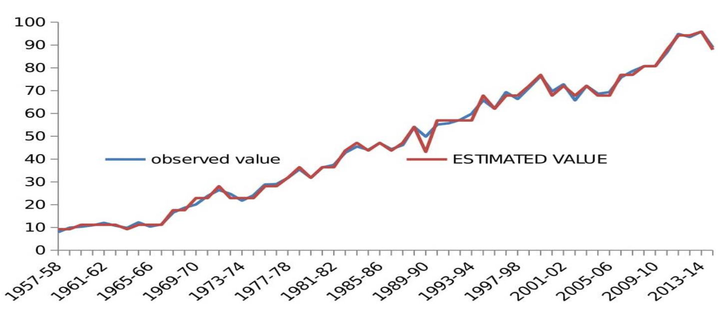

Figure 4. wheat production based on fuzzy time series modeling

Table 7. Characteristics of Performance Parameters

	RMSE	MAE
ARIMA Model	3.44	2.63
FUZZY Model	1.43	1.01

4. CONCLUSIONS

This study discussed about the application of the fuzzy time series model and compared with the ARIMA model. Clearly the Fuzzy time series approach is justified to be a suitable model to predict the wheat production. It was compared with error rate of ARIMA model. From table: 7 established that the low values of RMSE and MAE. It is desirable Error rate of Fuzzy time series model. This approach is less than the error rate of ARIMA model.

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