Stochastic Modeling and Applications
Vol. 25 No. 1 (January-June, 2021)
ISSN: 0972-3641

Received: 21st March 2021 Revised: 23rd April 2021 Selected: 29th June 2021

STUDY OF ANTI-FUZZY GK SUB ALGEBRA AND ANTI-FUZZY GK IDEAL

J.KAVITHA AND R.GOWRI

Abstract. In this paper, we establish the theory of Anti-fuzzy GK sub algebra and Anti-fuzzy GK ideals. We defined lower-level set of GK algebra and discussed some of its aspects in this paper.

1. INTRODUCTION

In 1991, the fuzzification of BCK algebras was introduced by O.G. Xi [10] discussed its characteristics and its properties. In 1993, the concept of Fuzzy BCI algebra was introduced by B. Ahamed [1], in this study he explored the properties of Fuzzy BCI algebras. In 2003, Ahn and Bang [2] introduced fuzzified B algebra and in this article, they classified the sub algebras by their family of level sets. Many authors [3-7] have introduced new algebraic structures and fuzzified the same and obtained many interesting results and also derived new concepts of that new algebraic structure. Inspiring by these kinds of articles, we introduced new algebraic structure namely GK algebra [8]and fuzzified [9] it. In this paper we discuss about Anti-fuzzy GK sub algebra and Anti-fuzzy GK ideal and brought very interesting results.

2. ANTI-FUZZY GK SUB ALGEBRA AND ANTI-FUZZY GK IDEAL

Definition 2.1. A fuzzy set ρ_{gk} in GK algebra T is said to be an anti-fuzzy sub algebra of T if

$$\rho_{gk}(i \circledast j) \leq max\{\rho_{gk}(i), \rho_{gk}(j)\}, \text{ for all } i, j \in T.$$

Theorem 2.2. *Let ρ_{gk} is an anti-fuzzy sub algebra of GK algebra. Prove that $\rho_{gk}(1) \leq \rho_{gk}(i)$, for any i in T.*

Proof. We know that $i \circledast j = 1$ from the definition of GK algebra
Now, $\rho_{gk}(1) = \rho_{gk}(i \circledast j)$
$\leq max\{\rho_{gk}(i), \rho_{gk}(j)\} \leq \rho_{gk}(i)$
Therefore $\rho_{gk}(1) \leq \rho_{gk}(i)$. □

Definition 2.3. Let ρ_{gk} be any fuzzy subset of a GK algebra and let $q \in [0, 1]$. The set $\Gamma(\rho_{gk}, q) = \{i \in T : \rho_{gk} \leq q\}$ is called a lower-level subset of ρ_{gk} in T.

2000 *Mathematics Subject Classification.* 08A72; 16Y80.

Key words and phrases. Fuzzy GK sub algebra, Anti-Fuzzy GK sub algebra, Anti-Fuzzy GK ideal.

Theorem 2.4. *A fuzzy set ρ_{gk} in GK algebra is an anti-fuzzy sub algebra if and only if for every q in $[0,1]$, $\Gamma(\rho_{gk},q)$ is either ϕ or a sub algebra of T.*

Proof. Let us assume ρ_{gk} is an anti-fuzzy sub algebra of T and also lower-level subset is non-empty. Then for any $i,j \in \Gamma(\rho_{gk},q)$
we have, $\rho_{gk}(i \circledast j) \leq max\{\rho_{gk}(i), \rho_{gk}(j)\} \leq q$.
Therefore, $i \circledast j \in \Gamma(\rho_{gk},q)$.
Hence $\Gamma(\rho_{gk}$ is a sub algebra.
Conversely,
Now,Consider $i,j \in T$.
Take $q = max\{\rho_{gk}(i), \rho_{gk}(j)\}$.
Since $\Gamma(\rho_{gk},q)$ is a sub algebra of T,
$\Rightarrow i \circledast j \in \Gamma(\rho_{gk},q)$
Therefore $\rho_{gk}(i \circledast j) \leq q = max\{\rho_{gk}(i), \rho_{gk}(j)\}$
Hence ρ_{gk} is an anti-fuzzy sub algebra. □

Definition 2.5. Let T be a GK algebra. A fuzzy set ρ_{gk} in T is called anti-fuzzy GK ideal of T if it satisfies the following conditions.

(i) $\rho_{gk}(1) \leq \rho_{gk}(i)$

(ii) $\rho_{gk}(i \circledast k) \leq max\{\rho_{gk}(j \circledast k), \rho_{gk}(j \circledast i)\}$

for all $i,j,k \in T$.

Definition 2.6. Let $(T, \circledast_T, 1)$ and $(P, \circledast_P, 1')$ be a GK algebra. Then the mapping $\sigma : T \rightarrow P$ of GK algebra is called anti-homomorphism if $\sigma(i \circledast_T j) = \sigma(j) \circledast_P \sigma(i)$ for all $i,j \in T$.

Definition 2.7. Let $\sigma : T \rightarrow T$ be an endomorphism and ρ_{gk} be a fuzzy set in T. We define fuzzy set in T by $(\rho_{gk})_\sigma$ in T as $(\rho_{gk})_\sigma(i) = (\rho_{gk})(\sigma(i))$ for every $i \in T$.

Theorem 2.8. *Let ρ_{gk} be an anti-fuzzy GK ideal of GK algebra of T and if $i \leq j$, then $\rho_{gk}(i) \leq \rho_{gk}(j)$, for all $i,j \in T$.*

Proof. Let us consider $i \leq j$, then $i \circledast j = 1 = j \circledast i$,
and $\rho_{gk}(i \circledast 1) = \rho_{gk}(1) \leq max\{\rho_{gk}(j \circledast 1), \rho_{gk}(j \circledast i)\}$
$= max\{\rho_{gk}(j), \rho_{gk}(1)\} = \rho_{gk}(j)$.
Hence $\rho_{gk}(x) \leq \rho_{gk}(y)$.

Theorem 2.9. *Let ρ_{gk} be an anti-fuzzy GK-ideal of GK algebra T. If the inequality $j \circledast i \leq k$ carry in T, then $\rho_{gk}(i) \leq \max\{\rho_{gk}(j), \rho_{gk}(k)\}$.*

Proof. Let us consider the inequality $j \circledast i \leq k$ carry in T.
By theorem 2.8, $\rho_{gk}(j \circledast i) \leq \rho_{gk}(k)$ ....(1)
By definition of anti-fuzzy ideal of GK algebra
$\rho_{gk}(i \circledast k) \leq max\{\rho_{gk}(j \circledast k), \rho_{gk}(j \circledast i)\}$
Put $k = 1$, then $\rho_{gk}(i \circledast 1) = \rho_{gk}(i) \leq max\{\rho_{gk}(j \circledast 1), \rho_{gk}(j \circledast i)\}$
$= max\{\rho_{gk}(j), \rho_{gk}(j \circledast i)\}$.............(2)
From (1) and (2), we get
$\rho_{gk}(i) \leq \max\{\rho_{gk}(j), \rho_{gk}(k)\}$, for all $i,j,k \in T$.

Conclusion. In this article, we defined and discussed about Anti-fuzzy GK sub algebra and Anti-fuzzy GK ideal and also derived some important results. In future we planned to work the concept of algebraic structure of GK algebra with soft set, Neutrosophic set for obtaining new kind of results.

References

1. Ahmad, B.: Fuzzy BCI-algebras, *Journal of Fuzzy Mathematics* **2** (1993), 445-452.
2. Ahn, S.S. and Bang, K.: On fuzzy subalgebras in B-algebras, *Communications of the Korean Mathematical Society* **18(3)** (2003) 429-437.
3. Ahn, S.S. and Kim, H.S.: On QS algebras, *J. Chungcheong Math. Soc* **12** (1999) 33-41.
4. Ahn, S.S., Kim, Y.H. and So, K.S.: Fuzzy BE-algebras, *Journal of Applied Mathematics and Informatics* **29** (2011) 1049-1057.
5. Ahn, S.S. and So, K.S.: On generalized upper sets in BE algebras, *Bull.Korean Math. Soc.* **46(2)** (2009) 281-287.
6. Ahn, S.S. and So, Y.H.: On ideals and upper sets in BE algebra, *Sci. Math. Jpn* **Online e-2008 (2)** (2008) 279-285.
7. Akram, M. and Dar, K.H.: On fuzzy d-algebras, *Punjab University, Journal of Mathematics* **37** (2005) 61-76.
8. Gowri, R. and Kavitha, J.: The structure of GK algebras, *International journal for research in applied Science & Engineering Technology* **6(4)** (2018) 1208-1212.
9. Gowri, R. and Kavitha, J.: fuzzy sub algebra and fuzzy ideals of GK algebra, *Journal of Shanghai Jiaotong University* **16(7)** (2020) 919-927.
10. Xi, O.G.: Fuzzy BCK-algebras, *Math. Japonica* **36(5)** (1991) 935-942.

J.Kavitha: Assistant Professor, Department of Mathematics, Dwaraka Doss Goverdhan Doss Vaishnav College(Autonomous), Chennai, India., Research Scholar, Department of Mathematics, Government College for Women (Autonomous), Kumbakonam, India (Affiliated to Bharathidasan University)

E-mail address: jkavitha@dgvaishnavcollege.edu.in

R.Gowri: Assistant Professor, Department of Mathematics,Government College for Women (Autonomous), Kumbakonam, India

E-mail address: dr.r.gowri@gcwk.ac.in