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NONLINEAR STOCHASTIC EQUATION IN EPIDEMIOLOGY

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Abstract. This paper is dedicated to the initial value problem for a system of nonlinear differential equations of fractional deterministic and stochastic epidemic models. These types of problems arise when we analyze the spread of the COVID-19 pandemic. In future work, we will focus on some properties of stochastic epidemic models that do not take place in deterministic cases. Important properties of fractional stochastic models are the probability of a pandemic outbreak, quasi-stationary probability distributions, the finite dimension of the spread and the expected duration of the epidemic. The properties listed here are dependent on the stochastic nature of the process. Better understanding and evaluation of the presence, stability and management of infectious diseases can be achieved through mathematical epidemic models. Unfortunately, the classic models mentioned above are not accurate enough to model such diseases. In this regard, it is important to involve fractional differential equations..

1. Introduction

An article published in 1927 by W. O. Kermack, A.G. McKendrick [9] served as a theoretical basis for further research in the field of mathematical modeling of epidemics. In that article, for the first time in the field of epidemiology, the so-called law of mass action", according to which the quantity of newly infected in a population is directly proportional to the product of current numbers of susceptible and infected individuals was applied. This law induced the widespread use of deterministic SIR models (Susceptible - Infected - Recovered) for which, with the help of a system of differential equations, we can describe the dynamics of groups of susceptible, infected and recovered individuals.

Research on the stochastic SIR model under the environmental conditions in Bartlett [2] and Bailey [3] laid the foundation for stochastic models of epidemic processes.

Elveback T. et al. [6] published results on the first individual-oriented model of the spread of disease. This new direction, considering the development of computing technologies and the predictive power of stochastic models, played a decisive role in the development of mathematical epidemiology.

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Over the last few years, the study of the problem has become more real due to the COVID-19 pandemic.

The SARS-COV-2 virus has led to COVID-19 infection. The first cases were documented in Wuhan, China [5], and the virus subsequently spread to all continents and led to the outbreak of the pandemic. The total number of countries affected by this global challenge is 212. The antiviral drugs and vaccines available in the initial period of the pandemic were not effective against the new virus. Through the efforts of scientists in various countries in subsequent stages of the pandemic, new vaccines were developed, and large populations were vaccinated.

Better understanding and evaluation of the presence, stability and management of infectious diseases can be achieved through mathematical models of epidemics [10]. Unfortunately, the classic models mentioned above are not accurate enough to model such diseases. In this regard, it is important to involve fractional differential equations. The adaptation of Caputo fractional derivatives to the description of infectious diseases in biological communities is a new step in the analysis of the epidemic model. It is worth noting that the efficiency of fractional analysis methods has already been noted by scientistsspecialists in other fields, such as acoustics, rheology, and polymer chemistry [1]. In [7], considering positive vaccination factor v as a vaccine parameter of a susceptible population, a system of differential equations with a Caputo fractional derivative of order $\alpha, 0 < \alpha < 1$, was introduced in the form of

$$\begin{cases} {}^cD_t^\alpha S(t) = -vS(t) - \beta\frac{J(t)S(t)}{N_0}, \\ {}^cD_t^\alpha J(t) = \beta\frac{J(t)S(t)}{N_0} - (\gamma + k)J(t), \\ {}^cD_t^\alpha R(t) = vS(t) + \gamma J(t), \\ {}^cD_t^\alpha D(t) = kJ(t), \end{cases} \tag{1.1}$$

with the positive initial conditions

$$S(0) = S_0, J(0) = J_0, R(0) = R_0, D(0) = D_0. \tag{1.2}$$

Here, the total population N is divided into the following epidemiological classes:

S - susceptible class, J - infected class, R - recovering class, D - deceased class.

The parameters in system (1.1) are described as follows:

β the average number of contacts of one person during time t; γ - the level of recovery; k the mortality rate.

This work is devoted to the analysis of a system of stochastic differential equations and SIRD stochastic models.

2. Fractional Integrals and Derivatives

There are many definitions of fractional integration and differentiation. Here, we need definitions of the Riemann-Liouville fractional integral and Caputo fractional derivative, first introduced in [13], [8].

Let $L([0,T],R)$ be the space of the Lebesgue integrable scalar functions in $[0,T], T < \infty$.

Definition 2.1. Fractional Riemann - Liouville integral of order $\alpha, 0 < \alpha < 1$, of a function $f \in L([0,T],R)$ is given by the equation

$$I_t^\alpha f(t) = \frac{1}{\Gamma(\alpha)} \int\limits_0^t (t-s)^{\alpha-1} f(s) ds. \tag{2.1}$$

.

The Gamma function is defined as

$$\Gamma(\alpha) = \int\limits_0^\infty e^{-t} t^{\alpha-1} dt, \alpha > 0.$$

Let $C^1([0,T],R)$ be the space of continuously differentiable scalar functions defined on $[0,T]$.

Definition 2.2. The Caputo fractional derivative of order $\alpha, 0 < \alpha < 1$, is given by

$${}^cD_t^\alpha f(t) = I_t^{1-\alpha}(\frac{d}{ds} f)(t) = \frac{1}{\Gamma(1-\alpha)} \int\limits_0^t (t-s)^{-\alpha} f'(s) ds, \tag{2.2}$$

where $I_t^{1-\alpha}$ - fractional integral from (2.1), $f \in C^1([0,T],R)$.

The decomposition formula for the Caputo derivative of order α, $0 < \alpha < 1$ follows directly from (2.1) and (2.2)

$$(I_t^{\alpha c} D_t^\alpha f)(t) = \varphi(t) - \varphi(0), \text{ for } \varphi \in C([0,T],R) \tag{2.3}$$

3. Deterministic Fractional Epidemic Model (SIRD)

It is well known that the Caputo fractional differential operator (2.2) is widely used in the analysis of infectious diseases of various biological models, particularly in time-continuous model environments [11]. Based on [11] and considering the positive vaccination factor, v, as a parameter of susceptible populations, we pass to the system of differential equations (1.1) with Caputo fractional derivatives of order $\alpha, 0 < \alpha < 1$

$$\begin{cases} {}^cD_t^\alpha S(t) = -vS(t) - \beta \frac{J(t)S(t)}{N_0}, \\ {}^cD_t^\alpha J(t) = \beta \frac{J(t)S(t)}{N_0} - (\gamma + k) J(t), \\ {}^cD_t^\alpha R(t) = vS(t) + \gamma J(t), \\ {}^cD_t^\alpha D(t) = kJ(t), t \geq 0 \end{cases} \tag{3.1}$$

The transition between populations S, J, R, and D during the transmission process for COVID-19 is described by a digraph (see Figure 1).

In this section, taking into account the results of [4], we present a qualitative analysis of solutions to systems (3.1) and (3.2), find the stability criteria, describe the boundedness of solutions and calculate the equilibrium points of the nonlinear system (3.1), the so-called disease-free states. Next, we set the necessary conditions

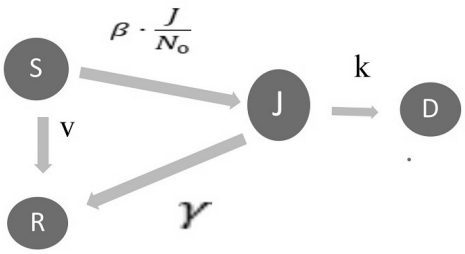

FIGURE 1. Model SIRD - transmission diagram for COVID - 19

for the existence of at least one solution of systems (3.1) - (3.2) and their uniqueness on the basis of well-known theorems on the fixed points of a nonlinear operator.

Lemma 3.1. *The solutions for systems (3.1) and (3.2) are bounded in a feasible domain of the form*

$$U = \{(S, J, R, D) \in \mathbb{R}^4_+, 0 \leq N(t) \leq N_0\}.$$

The pandemic happens when

$$S_0 > \frac{\gamma + k}{\beta} N_0,$$

where $\frac{\gamma+k}{\beta}$ is said to be the threshold or critical size of a pandemic in a community.

Lemma 3.2. *The disease-free equilibrium point of the system (3.1) is*

$$U* = (\frac{\gamma + k}{\beta} N_0, 0, R_0, 0).$$

By definition, $U* = (S*, J*, R*, D*)$ is a solution to a system of fractional differential equations

$$\begin{cases} ({}^cD^\alpha_t S)(t) = 0, \\ ({}^cD^\alpha_t J)(t) = 0, \\ ({}^cD^\alpha_t R)(t) = 0, \\ ({}^cD^\alpha_t D)(t) = 0 \end{cases} \tag{3.2}$$

Considering the elements of fractional differential calculus, from the second equation of system (3.1), we obtain

$$\frac{\beta S(t) - (\gamma + k)N_0}{N_0} \cdot J(t) = 0.$$

and further, we obtain

$$S(t) = S* = \frac{\gamma + k}{\beta} N_0.$$

At the equilibrium point U_*, COVID-19 does not spread, and the mortality class decreases to zero in the absence of new infections.

Definition 3.3. An equilibrium point U_*, free from a pandemic, is locally asymptotically stable if the class of infected population $S(t)$ satisfies the inequality $S(t) \leq S_*$, and it is unstable if $S(t) > S_*$.

Furthermore, in this section, using the fixed point theorem, we formulate a result on the existence of at least one solution of the system (3.1). Let $U = (S, J, R, D) \in E$, where $E = [C([0, T], R_+)]^4$ is a Banach space with norm

$$\|u\|_E = \|S\|_c + \|J\|_c + \|R\|_c + \|D\|_c$$

and let

$$\begin{cases} f_1(t, u(t)) = -vS(t) - \beta \frac{J(t)S(t)}{N_0} \\ f_2(t, u(t)) = \beta \frac{J(t)S(t)}{N_0} - (\gamma + k)J(t) \\ f_3(t, u(t)) = vS(t) + \gamma J(t) \\ f_3(t, u(t)) = KJ(t). \end{cases}$$

It is obvious that the function $f = (f_1, f_2, f_3, f_4) \in ([0, T] \times E)^4$ is continuous.

Applying the fractional integral (2.2) to both parts of the system (3.1) and using (2.3), we obtain

$$S(t) = S_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f_1(\tau, u(\tau)) d\tau$$

$$J(t) = J_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f_2(\tau, u(\tau)) d\tau$$

$$R(t) = R_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f_3(\tau, u(\tau)) d\tau$$

$$D(t) = D_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f_4(\tau, u(\tau)) d\tau$$

Choosing $u_0 = (u_1, u_2, u_3, u_4) = (S_0, J_0, R_0, D_0)$ we obtain

$$u(t) = u_0 + \frac{1}{\Gamma(\alpha)} \int_0^t (t - \tau)^{\alpha-1} f(\tau, u(\tau)) d\tau. \tag{3.3}$$

The following statements are true.

Theorem 3.4. *Let* $\beta, v, \gamma, k, \alpha, t \in \mathbb{R}_+, \alpha \in (0, 1)$ *and*

$$T < \left(\frac{\Gamma(\alpha + 1)}{4(\beta + v + \gamma + k + 3)}\right)^{1/\alpha}.$$

Then, there exists at least one solution of the systems (3.1) - (3.2) defined on $[0, T]$.

Theorem 3.5. *Let* $\alpha \in (0,1)$ *and* $\beta, v, \gamma, k, \mu \in \mathbb{R}_+$, *such that*

$$\mu = max\{\beta + v + 2, \beta + \gamma + k + 2, v + \gamma + 3, k + 3\}.$$

If

$$\frac{4\mu T^{\alpha}}{\Gamma(\alpha+1)} < 1$$

then the systems (3.1), (3.2) have unique solutions.

4. Stochastic integral and stochastic differential equation

Markov processes play an important role in the various classes of stochastic processes. A Markov process should be understood as a process whose value at time t_0 completely determines its future behavior, regardless of the past. The Wiener process is an example of a Markov process. We can assume that the solution of the ordinary deterministic differential equation is also a Markov process. Markov processes play an important role in the various classes of stochastic processes. A Markov process should be understood as a process whose value at time t_0 completely determines its future behavior, regardless of the past. The Wiener process is an example of a Markov process. We can assume that the solution of the ordinary deterministic differential equation is also a Markov process. Using the Wiener process $W(t)$, one can construct a wide class of Markov processes with continuous trajectories, defined as solutions of the stochastic differential equation

$$\begin{gathered} dX(t) = a(t, X(t))dt + \sigma(t, X(t))dW(t), t > 0, \\ X(0) = x, X \in \mathbb{R}^n, \xi \in \mathbb{R}^m. \end{gathered} \tag{4.1}$$

Here, x is a deterministic or random vector of initial conditions, vector function $a(t, X) \in \mathbb{R}^n$ and matrix $\sigma(t, X) \in \mathbb{R}^n \times \mathbb{R}^m$ are given, and W is a standard Wiener process. Equation (4.1) is a symbolic representation of the following integral identity:

$$X(t) = x + \int_0^t \alpha(s, X(s))ds + \int_0^t \sigma(s, X(s))dW(s). \tag{4.2}$$

The last integral on the right side of equality (4.2) is the It stochastic integral.

Let us recall a number of properties of Ito stochastic integrals. Let us define on the main probability space $(\Omega, \mathfrak{F}, P)$, a family of σ - algebras $\mathfrak{F}_t$ generated by the Wiener process $W(t)$ satisfying the conditions:

a) for any t and $s, t < s$, the inclusion $\mathfrak{F}_t \subset \mathfrak{F}_s$ holds;

b) the Wiener process $W(t)$ is measurable with respect to $\mathfrak{F}_t$; The latter means that for any Borel set $A \in \mathbb{R}^n$, the event $\{\omega : W(t) \in A\}$ belongs to $\mathfrak{F}_t$;

c) For any nonnegative t and s, the process $W(t + s) - W(t)$ does not depend on any of the events of the σ - algebra $\mathfrak{F}_t$.

The stochastic Ito integral

$$\xi(t) = \int_0^t \alpha(s) dW(s) \tag{4.3}$$

is defined for any random processes $\alpha(t)$ satisfying the requirements:

1) the process $\alpha(t)$ is measurable with respect to $\mathfrak{F}_t$ for any t;

2) the probability that the following integral is finite equal to one.

$$J(\alpha) = \int_0^T E(|\alpha(t)|^2/\mathfrak{F}_0) dt.$$

Considered as a function of the upper limit, the stochastic integral (4.3) defines some random process $\xi(t)$ with zero mathematical expectation and correlation matrix

$$E\xi(t) = E \int_0^t \alpha(s) dW(s) \Big|^{\mathfrak{F}_0} = 0$$

$$E\xi(t_1)\xi^1(t_2) = \int_0^{min(t_1,t_2)} E(\alpha(s)\alpha^{'}(s)\big|\mathfrak{F}_0) ds$$

The solution of equation (4.1) is a Markov process whose transition probability $P(t, x, t, A), t_1 > t, A \in \mathbb{R}^n$, is defined by the equality

$$P(t, x, t, A) = P(X_{t,x}(t_1) \in A). \tag{4.4}$$

In (4.4), the process $X_{t,x}(s)$ is a solution of equation (4.1) for $s > t$ with the initial condition $X_{t,x}(t) = x$.

In mathematical epidemiology modeling, it becomes necessary to calculate the average values of several functionals of solutions of equation (4.1). Sometimes this issue can be reduced to solving a boundary value problem for partial differential equations. Let, for example, calculate the expected value

$$EF(X_{t,x}(s)), s \geq t. \tag{4.5}$$

Here, $F(x)$ is a given continuous and bounded scalar function. The value of functional (4.5) depends on the initial moment t and the initial vector x. Let

$$u(t, x) = EF(X_{t,x}(s)), s \geq t, \tag{4.6}$$

where s is fixed.

Let us assume that the coefficients $a(t, x)$ and $\sigma(t, x)$ of equation (4.1) are defined for $0 \leq t \leq T, x \in \mathbb{R}^n$ and have continuous, bounded second derivatives with respect to x in this region. Then, the function $u(t, x)$ from (4.6) satisfies the equation

$$\frac{\partial u}{\partial t} + a^{'} \frac{\partial u}{\partial x} + \frac{1}{2} Tr\sigma\sigma^{'} \frac{\partial^2 u}{\partial x^2} = 0, t \leq s, x \in \mathbb{R}^n. \tag{4.7}$$

The initial condition for $u(t, x)$ follows from (4.6)

$$\lim_{t \to s} u(t,x) = F(x), x \in \mathbb{R}^n. \tag{4.8}$$

After solving equations (4.7) and (4.8), we obtain the value of functional (4.5) for arbitrary (t,x). Equation (4.7) is called the inverse Kolmogorov equation.

5. Stochastic fractional SIRD model

The results presented in Sections 3 and 4 allow us to state that the stochastic fractional SIRD model follows from the subdiffusion process. This conclusion is based on the nature of the time-fractional processes presented in system (4.1). Random variables $S(t), J(t)$ are continuous and bounded, i.e.,

$$0 \leq S(t) \leq N, 0 \leq J(t) \leq N.$$

We will consider a heuristic derivation of fractional stochastic differential equations for the fractional SIRD epidemic model. As noted in the introduction, SIRD gives a good description of the spread of COVID-19.

Let us present a high-resolution finite-difference approximation of the fractional Caputo derivative based on the expansion of the integrand in a Taylor series with subsequent replacement of derivatives by finite difference relations.

Following [10], we introduce a finite difference grid $\Omega_{\triangle t} = \{t_j = j\triangle t, j = 0, ..., N\}$, on which approximations of the fractional derivative function of one variable X will be written as

$$^c\triangle_t^\alpha X(t_j) = \frac{\triangle t_j^{-\alpha}}{\Gamma(1-\alpha)} \sum_{i=0}^{j=1} [W_{1,j} - e(X_{l+1} - X_{l-1}) +$$

$$+W_2, j - l(X_{i+l} - 2X_l + X_{l+1})] + 0(\triangle t^{3-\alpha}) \tag{5.1}$$

where $\triangle$ - is time step , $X_{l+1} = X(t_{l+1})$, and $j = 1, ..., N, 0 < \alpha < 1$.

The weight functions in (5.1) are defined as follows:

$$W_{1,j-l} = \frac{2-\alpha}{2}[(j-l)^{1-\alpha} - (j-l-1)^{1-\alpha}]$$

$$W_{2,j-l} = (j-l)^{2-\alpha} - (j-l-1)^{2-\alpha} - (2-\alpha)(j-l-1)^{1-\alpha},$$

for $j = 1, ..., N, 1 = 0, ..., j-1$.

Let

$$\sum_{j=1}^{n} \triangle t_j^{-\alpha} = \triangle t^\alpha, \triangle^\alpha X(t) = \sum_{j=1}^{n} \triangle^\alpha X(t_j),$$

and

$$\triangle^\alpha X(t) = (\triangle^\alpha S(t), \triangle^\alpha J(t))^T.$$

For small enough $\triangle t_i$, it is reasonable to assume that the random variables $\{\triangle^\alpha X(t_i)\}$ are independent and identically distributed on the interval $\triangle^\alpha t$. For

large enough n, it follows from the central limit theorem that $\triangle^\alpha X(t)$ has an approximate normal distribution with mean value $E(\triangle^\alpha X(t))$ and covariance matrix $Var(\triangle^\alpha X(t))$ (see, e.g., L.J.S. Allen [1], [12]).

Thus,

$$\triangle^\alpha X(t) - E(X(t)) = \mathfrak{N}(\mathbf{0}, Var(\triangle^\alpha X(t))),$$

where $\mathbf{0}$ is the null vector. The expected value $\triangle X$ of order $\triangle t$ is the change in the probability time (+1 or -1) i.e.,

$$\mathbb{E}(\triangle^\alpha X) = \begin{pmatrix} -\beta SJ/N \\ BSJ/N - \gamma t \end{pmatrix} \triangle t = f \triangle t$$

and covariance matrix

$$Var(\triangle^\alpha X) = \mathbb{E}(\triangle^\alpha (\triangle^\alpha)^T) =$$

$$= \mathbb{E} \begin{pmatrix} (\triangle^\alpha S)^2 & \triangle^\alpha S \triangle^\alpha J \\ \triangle^\alpha S \triangle^\alpha S & (\triangle^\alpha J)^2 \end{pmatrix} = \begin{pmatrix} \beta SJ/N & -\beta SJ/N \\ -\beta SJ/N & \beta SJ/N + \gamma J \end{pmatrix} \triangle t = Var \triangle t^\alpha.$$

To write a stochastic differential equation for the SIRD stochastic process, either the square root of the covariance matrix Var or a matrix G such that $GG^T = Var$ is needed. The matrix G given below has the required property

$$G = \begin{pmatrix} -\sqrt{\beta SJ/N} & 0 \\ \sqrt{\beta SJ/N} & -\sqrt{\gamma J} \end{pmatrix}.$$

Then

$$\triangle X(t) = f(X(t)) \triangle t + G(X(t)) \triangle W(t),$$

where $\triangle W(t) = (\triangle W_1(t), \triangle W_2(t))^2$ and $\triangle W(t) = N(0, \triangle t)$.

When $\triangle t \to 0$ we obtain the system of stochastic differential equations

$$DX(t) = f(X(t))dt + G(X(t)dW(t)), \tag{5.2}$$

Let us rewrite (5.2) in terms of the random variables $S(t)$ and $I(t)$ and obtain the following system of fractional stochastic differential equations:

$${}^c D_t^\alpha S(t) = -\frac{\beta}{N} SJ + B_{11} \frac{dW_1}{dt} + B_{12} \frac{dW_2}{dt}$$

$${}^c D_t^\alpha J(t) = \frac{\beta}{N} SJ - \gamma J + B_{21} \frac{dW_1}{dt} + B_{22} \frac{dW_2}{dt}$$

where W_1, W_2 - are independent Wiener processes, and elements of the matrix $B = (B_{ij}), i, j = 1, 2$ may be dependent on S and I.

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