Stochastic Modeling and Applications
Vol. 25 No. 2 (July-December, 2021)
ISSN: 0972-3641

Received: 02nd July 2021 *Revised: 27th August 2021* *Selected: 16th September 2021*

ANALYSIS AND PREDICTION OF ACTIVE INFECTION CASES FOR COVID-19 VIRUS IN INDIA, ITALY AND USA

DEEPAK B. PACHPATTE, GAJANAN S. SOLANKE* AND HARIBHAU L. TIDKE

Abstract. The main objective of this paper is to analyze the spread of COVID-19 active infection cases in three countries India, Italy and United States Of America(USA). The actual observed data for from March 01, 2020 to May 09, 2020 in the interval of ten days is taken and compared with data obtained from the basic SIR mathematical model. The actual observed data is taken from available on World Health organization and Ministry of health and family welfare Government of India websites. The spread of infection factor β is obtained from calculation and compared with the actual data. The basic reproduction ratio R_0 and number of infections, are predicated for subsequent values in the interval of ten days.

1. Introduction

In the year 1927 SIR model for mathematical modeling of epidemics was introduced by W. O. Kermack, A. G. McKendrick see[9]. Since then many mathematicians, biologists and medical professionals have developed and studied models on various types of infectious diseases. The model is formulated for the infectious diseases which helps in predicting the infections in population. Study of model will help in eradication or control of the disease. Basic information of Epidemiology and various models can be found in [4, 3, 6, 10, 15].

Recently the outbreak of corona virus in China have studied by the authors [1, 12]. Forecast and Analysis of spread of Covid 19 virus in various countries such as China, France, Italy etc. have been done in [13, 8, 14]. For study of Covid-19 outbreak for eight countries authors in [2] has used data driven approach.

As the Covid-19 virus is spreading all over the world since December 2019, in this paper we have analyzed the growth of the Covid-19 virus over seventy days from first week March 2020 to second week of May 2020 in the interval of ten days. In this we have done analysis of data by using SIR Model for various countries and comparison is done for the spread of number of infections during certain period.

2. Preliminaries

According to World Health Organization (WHO) and Ministry of health and family welfare [11, 16] Corona viruses are large number of viruses causing the illness. In humans Corona viruses causes respiratory infections such as Middle East Respiratory Syndrome(MERS) and Severe Acute Respiratory Syndrome (SARS).

2010 *Mathematics Subject Classification.* 34D05, 92D30.
Key words and phrases. Epidemic model, SIR, COVID-19, Pandemic .

COVID -19 is the most recently discovered Coronavirus. This disease (COVID-19) is an infectious disease caused by the coronavirus discovered in 2019. On January 30,2020 outbreak of Covid-19 virus is declared as health emergency of International concern by WHO. Italian authorities have reported the cases of Covid-19 virus in third week of February. In India the first case of Covid-19 was found in January.

The COVID-19 virus spreads due to droplets of saliva through the coughs or sneezes of infected persons. The spread of virus can be prevented by keeping social distancing, wearing a masks, avoiding public gathering places and washing the hands with alcohol based sanitizer. The symptoms for COVID-19 according to Centers for Disease Control and prevention(CDC)[5] are cough, shortness of breath or difficulty breathing, fever, chills, muscle pain, soar throat, loss of test or smell.

The main aim of developing Mathematical model for infectious disease is to study how the disease infection is spread. Mathematical model plays an important role in controlling of contagion and understanding the disease dynamics. In studying the infectious diseases various parameter such as population, contact rate, transmission rate, recovery rate and death rate are important. The infectious individual is introduced to the population called as susceptible and the disease spread to other individuals. Infected person may be symptomatic or asymptomatic, most of patients do not have symptoms at early stage, symptoms are developed after some period. If the number of cases are above the average for the short time then it is said that disease outbreak has occurred. If the spread of virus is not for particular region or country but for the global world then it is said to be pandemic. Thus as the Covid-19 virus has affected globally it is declared as pandemic.

3. SIR Model

The basic and simple model for study of infectious disease is the Susceptible, Infective and Recovered cases known as SIR model which was originally developed by Kermack and McKendrick in 1927 [9]. The model describes the spread of infection in population of susceptible, infected and recovered. This model is compartment model in which the population is divided into three compartments and the population progresses in compartments. At the beginning there are very few number of individuals infected in the population, the spread or rate of transmission depends on patterns of contact of various individual in population.

At any given time t the total population is denoted by N, S denotes susceptible class which consists of those people which are currently not infected but likely to get infected, I denotes the class of people having infections and R denotes the number of persons recovered or died or immune to disease. The basic SIR model is given by

$$\frac{dS}{dt} = \mu N - \mu S - \beta I \frac{S}{N},$$

$$\frac{dI}{dt} = \beta I \frac{S}{N} - \mu I - \gamma I,$$

$$\frac{dR}{dt} = \gamma I - \mu R.$$

For the above model following hypotheses holds: The total population size is N, the infections will circulate in this population, μ is the death rate, γ is the recovery rate. Spread of infection from infected person to susceptible persons given by the $\beta I\frac{S}{N}$, where β is contact rate times the probability of transmission. For our calculation purpose we have omitted the natural birth and death during this period taking $\mu = 0$. Here for this model the assumption is $N = S + I + R$ =constant.

4. Results

In this section we give the analysis of the Covid-19 data for the three countries India, Italy and USA. For calculation actual data starting from March, 01,2020 to May 09, 2020 is considered for each day. For the analysis purpose the Covid-19 data for India is taken from the web site of Ministry of health and family welfare, Government of India [11], for United States of America(USA) and Italy data is taken from World health organization web sites [16].

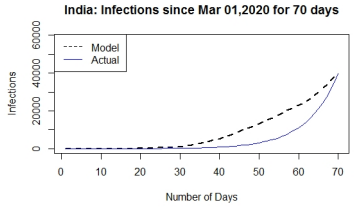
For calculation purpose Mar 1, 2020 is taken as base day and considered it as day one, calculations are carried out for 70 subsequent days upto May 9,2020. For calculation purpose the total population of India on March 1, 2020 is assumed as 1,341,000,000, the number of Covid-19 active infected and recovered patients from actual data were 6 and 3 respectively. Using the SIR model calculated the number of susceptible, infected and recovered persons The individual once is infected then on an average it takes 10 days for him to recover thus $\gamma = \frac{1}{10} = 0.1$.

The Figure 1 shows the graph Covid-19 active infected patients from SIR model calculations versus actual data for country India. For Italy the total population assumed on March 1, 2020 is 60,460,000 and number of Covid-19 active infected and recovered patients according to actual data were 1689 and 82 respectively see Figure 1 (b). For USA the total population assumed on March 1, 2020 is 329,300,000 and number of Covid-19 active infected and recovered patients according to actual data were 62 and 6 respectively see Figure 1 (c).

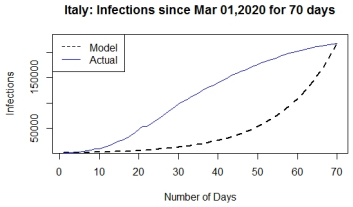
As shown in the Fig 1 the graph is plotted between the calculated value from the SIR model of active infected patients and with the actual cases of Covid-19 patients for from March 1, 2020 to May 9, 2020 for three countries.

In studying the dynamics of the disease the basic reproduction ratio R_0 is the excepted number of secondary cases produced in susceptible population by an infected individual. Hence R_0 is very important and is used as measure of reference for controlling the infectious diseases. This basic reproduction number R_0 which is used also as measure of stability and is given by $R_0 = \frac{\beta}{\gamma+\mu}$, we have assumed birth rate as zero during this period thus $\mu = 0$ which implies $R_0 = \frac{\beta}{\gamma}$. If the value of $R_0 < 1$ then model state is said to be unstable and if $R_0 > 1$ model state is stable. From calculation using the SIR model it is found that the values from calculations matches with actual values for R_0 as $2.27548, 1.70684301$ and 2.4375594 for India Italy and USA respectively on 70th day.

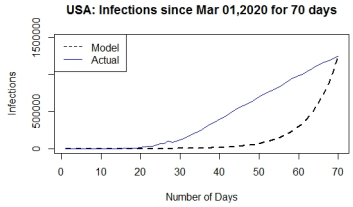
From the Figure 1 the spread of infections for the three countries India, Italy and USA can be compared. It is observed that for India initially the number of cases are increasing slowly as compared to the other two countries. In case of India the rate at which the infections are increasing are less than the value predicated



(A) India



(B) Italy



(C) USA

FIGURE 1. SIR Calculated data versus actual Data

predicated from model calculation with $\beta = 0.227548$. This may be because in India the Government has declared the total lockdown only essential services were operating. Because of this even though the India is such a densely populated country with large population the spread was less compared to other countries. Another reason might be compared to other countries average number of youth population in India is more and due to heard immunity in youths they are less likely to get infected.

In case of Italy according to European Center for Disease Prevention and Control [7] in the third week of February Italian authorities have reported the cluster of cases from several parts of the country. The spread has started from the various parts of the country simultaneously. In Italy the average number of of persons with older age is more compared with other countries. As older persons are more likely to get infected by the virus due to less immunity or they may be suffering from blood pressure, diabetes, respiratory diseases or any other illness. Due to this reasons if we compare the data with other two countries the number of actual infections are more than the values calculated from the calculation through SIR model.

In case of USA the number of actual infections are more than the values calculated from the calculation through SIR model. The individuals in infected population may be asymptotic are symptomatic depending on whether they shows that symptoms or not. Many individuals may be asymptotic where they are infected and carrying the virus but not showing any symptoms. It is likely that such individual may not be tested but can infect another individual. In case of USA from initial period average number of tests performed were more thus more number of

asymptotic individual infected are also get added into the number of infected active individuals thus increasing the total count. This should also be considered as one the the reason that number of actual infections are seen more than the values calculated from the calculation through SIR model.

In order to reduce the difference between the number of infections from actual data and the values calculated from the calculation through SIR model we have simulated the values in the interval of ten days. After each interval of ten days we have varied the value of β and calculated its value which matches the actual infected individual from the data.

The values of β and R_0 are calculated from the SIR model in the interval of every ten days where the number of active cases from the SIR model and actual data cases matches and are given in Table 1,Table 2 and Table 3 respectively for India, Italy and USA.

Number of Days	Infections Actual	Infections SIR model	β	R_0
10	44	44.00	0.32139	3.2139
20	196	196.0287	0.28350	2.8350
30	1118	1118.068	0.280262	2.80262
40	5218	5218.72	0.273545	2.73545
50	13295	13295.38	0.257213	2.57213
60	22904	22904.41	0.239786	2.39786
70	39834	39834.08	0.227548	2.27548
80	58802	58802.49	0.2163333	2.163333

TABLE 1. India data

Number of Days	Infections Actual	Infections SIR model	β	R_0
10	10149	10149.04	0.29928	2.9928
20	47021	47021.19	0.27517	2.7517
30	97689	97689.16	0.2400756	2.400756
40	139422	139422.00	0.213366	2.13366
50	175925	175925.00	0.195058547	1.95058547
60	199414	199414.01	0.181141524	1.81141524
70	217185	217185.00	0.170684301	1.70684301
80	225886	225886.12	0.16229009	1.6229009

TABLE 2. Italy data

5. Discussions and Conclusions

From above table it can be seen that the value of R_0 is decreasing from day 10 to day 70. If we compare the values of R_0 for first 70 days then using the

Number of Days	Infections Actual	Infections SIR model	β	R_0
10	696	696.68	0.3688	3.688
20	15219	15218.58	0.389643	3.89643
30	122653	122653.82	0.361748	3.61748
40	395030	395030.50	0.32466885	3.2466885
50	695353	695353.74	0.2904074	2.904074
60	983457	983457.86	0.26405872	2.6405872
70	1245874	1245874.31	0.24375594	2.4375594
80	1477459	1477459.68	0.22776052	2.2776052

TABLE 3. USA data

Days	India R_0	Italy R_0	USA R_0
80	2.1531	1.60227078	2.2345316
90	2.03072	1.497698550	2.03150380
100	1.9083399	1.393126320	1.8254760
110	1.7859599	1.288554090	1.6254481
120	1.6635800	1.183981859	1.42242040

TABLE 4. Predication R_0 data

	Days	India	Italy	USA
Actual Value	80	58802	225886	1477459
Estimated value	80	56764	234956	1508291

TABLE 5. Prediction of active infected cases on 80th day 19 May 2020

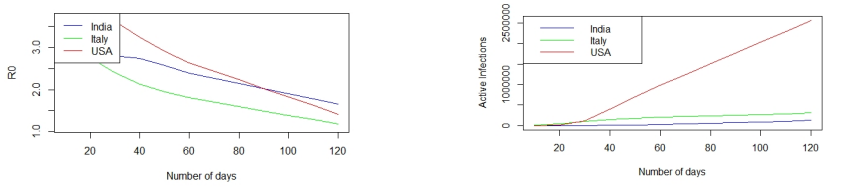
extrapolation values after the 10 days interval can be estimate for 80, 90, 100, 110 and 120 days as given in the tables. From that table if we compare the value for 80th days obtained form model calculation value of R_0 is 2.163333 and the estimated value of R_0 is 2.1531 with the actual values which nearer. Similarly if we compare the number of active patients for 80th day obtained from model calculation is $56,764$, $234,956$ and $1,508,291$ where as actual values recorded on 80th day 19 May 2020 is 58802, 225886 and $1,477,459$ for India, Italy and USA respectively as given in table 5. Similarly values after ten days that is on 90, 100, 110, 120 in the interval of ten days have been estimated in table 6.

In the same way values in interval of ten days can similarly be estimated which may be closer to the actual value. The curve in Fig 2 A shows how the value of R_0 is changing. For all the three countries the value of reproduction ratio R_0 is greater than 1 therefore model state is said to be unstable. The primary aim should be such that to make the value of R_0 less than 1 to make it stable, that is reduction in number of infected cases. If we are able to achieve the value of $R_0 < 1$ then the spread of disease will be under control. The Fig 2 B shows the

Days	India	Italy	USA
80	56764	234956	1508291
90	73694	252727	1770708
100	90624	270498	2033125
110	107554	288269	2295542
120	124484	306040	2557959

TABLE 6. Prediction active infected cases

prediction of number of infected individual for 120 days from 01 Mar 2020 with the interval of ten days.



(A) Prediction of R_0 (B) Prediction of active infected cases

FIGURE 2. Prediction

The three important things vaccination, quarantine and reduction of social contacts will change the behavior of epidemic. Vaccination as well as other two things that is quarantine and social distancing should be strictly followed so that β is decreased which decreases the reproduction ratio R_0. This will reduced the Number of Infected individual.

The above model also can be improved for accurate prediction by adding the compartments to the model. Compartments can be added for other things such as age structured population where its affecting to more number older population in some regions than younger generation. Also it is observed that the individual suffering from diabetes, blood pressure or kidney related disease virus is affecting more quickly. Certain part of population has heard immunity because of which virus does not affect their health but they may be the carriers of virus. For each of these category model can have different compartments. In future such compartments can be added which will increase the accuracy of results.

Declaration of Competing Interest The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

References

[1] C. Anastassopoulou, L. Russo, A. Tsakris, C. Siettos, Data-based analysis, modeling and forecasting of the COVID-19 outbreak, *PLOS ONE*, https://doi.org/10.1371/journal.pone.0230405, 2020.

[2] G. Barmparis, G. Tsironis, Estimating the infection horizon of COVID-19 in eight countries with a data-driven approach, *Chaos, Solitons Fractals*, Vol 134, May 2020, 109842.
[3] O. Bjrnstad, Epidemics, Models and Data using R, *Springer*, 2018.
[4] F. Brauer, C. Chavez, Z. Feng, Mathematical Models in Epidemiology. *Springer*, 2019.
[5] Centers for Disease Control and prevention(CDC), https://www.cdc.gov.
[6] D. Daley, J. Gani, Epidemic Modelling :An Introduction, *Cambridge University Press*, 2005.
[7] European Centre for Disease Prevention and Control, https://www.ecdc.europa.eu/en.
[8] D. Fanelli, F. Piazza, Analysis and forecast of COVID-19 spreading in China, Italy and France, *Chaos, Solitons Fractals*, Vol 134, May 2020,109761.
[9] W. Kermack, A. McKendrick, A Contribution to the Mathematical Theory of Epidemics, *Proceedings of the Royal Society A*, 115 (772): 700721.
[10] M. Li, An introduction to mathematical modeling of infectious diseases, *Springer*, 2017.
[11] Ministry of health and family welfare, https://www.mohfw.gov.in, Government of India.
[12] F. Ndarou, I. Area, J. Nieto, F. Torres, Mathematical modeling of COVID-19 transmission dynamics with a case study of Wuhan, *Chaos, Solitons Fractals*, Vol 135, 2020, 109846.
[13] D. Pachpatte, Analysis and forecasting of COVID-19 infections in India using ARIMA model: Computational Intelligence for Managing Pandemics, *De Gruyter*, 2021, pp. 121-130. https://doi.org/10.1515/9783110712254-007.
[14] E. Postnikov, Estimation of COVID-19 dynamics on a back-of-envelope: Does the simplest SIR model provide quantitative parameters and predictions, *Chaos, Solitons Fractals*, Vol 135, May 2020,109841.
[15] G. Solanke and D. Pachpatte, A fractional order differential equation model for tuberculosis, *AIP Conf. Proc.*, **2061**(2019), 020007-1-5.
[16] World Health Organization(WHO), https://www.who.int.

Deepak B. Pachpatte
Dept. of Mathematics, Dr. B. A. M. University, Aurangabad, Maharashtra 431004, (M.S.), INDIA.
E-mail address: pachpatte@gmail.com

Gajanan S. Solanke*
Department of First Year Engineering, CSMSS, Chh. Shahu College of Engineering, Aurangabad- 431002, (M.S), INDIA.
E-mail address: gssolanke@gmail.com

Haribhau L. Tidke
Dept. of Mathematics, School of Mathematical Sciences, Kavayitri Bahinabai Chaudhari North Maharashtra University, Jalgaon, Maharashtra-425 001, (M.S.), INDIA.
E-mail address: tharibhau@amail.com