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MODELLING FOR OUTBREAK OF SWINE FLU USING FRACTIONAL DERIVATIVE

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Abstract. The main purpose of this paper is to study the outbreak of swine flu using fractional order calculus definition. Boundedness of solution of the system is obtained. Stability properties of solution are also studied. This helps in future prediction of outbreaks.

1. Introduction

The study of infectious data began in 17th century [10]. To describe the transmission of diseases, the useful basic compartmental models are contained in a sequence of few papers by W. Kermack and A. McKendrick [19, 20, 21]. These are few papers which describes epidemic models [10]. Since then many researchers have used these models to study the infectious diseases, which are useful to future prediction in infection and outbreak of various diseases.

Authors in [2, 3, 4, 15, 24] have discussed global properties such as positivity, boundedness, local stability and global stability of SIR, SEIR, SIRS and SEIRS models. In [22], authors have discussed an epidemic model and backward bifurcation with treatment. In [9] authors have divided total population into ten partitions and they discussed their positivity, boundedness and stability of that model. In [1] discussed Lyapunov functions for classical models such as SIR, SIRS and SIS. Some authors [5, 6, 12, 14] have discussed mathematical models with fractional order derivative and they have studied their properties, such as positivity, boundedness, stability, equilibrium points and some had given the numerical simulation. Global dynamics for a class of disease and various properties of SEIRS models with general non-linear incidence have studied in[23].

In 2009 due to swine flu pandemic 11- 21 percent population i.e. nearly 1.4 billion peoples contracted with illness [28]. 18449 lab-confirmed deaths reported to the WHO, but actually it may be 284,000 deaths [30]. Swine flu is due to H1N1 virus, it has common symptoms as cough, weakness, fever, sore throat, chills and body aches. The first case of H1N1 was reported on may 16, 2009 in hyderabad. WHO declared pandemic on 10th August 2010.[28, 27, 29]

Some models are developed on this H1N1 virus [7, 11, 16, 18, 26], in which authors have discussed about symptomatic, asymptomatic, infections, equilibria,

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disease free equilibrium and their stability and global stability, optimal control and numerical simulations.

Motivated from above research, we have discussed a mathematical model for swine flu using fractional calculus and studied the positivity, boundedness and stability of this system.

2. Preliminaries

In this section we give some basic concepts and definitions.

Definition 2.1. [13, 25]. The classical form a fractional calculus is given by the Reimann-Liouville integral given by,

$$ {}_aD_{\mathfrak{t}}^{-\tau}(u(\mathfrak{t})) = {}_a\, I_{\mathfrak{t}}^{\tau}(u(\mathfrak{t})) = \frac{1}{\Gamma(\tau)} \int_a^{\mathfrak{t}} (\mathfrak{t}-\varsigma)^{\tau-1} u(\varsigma) d\varsigma, \tag{2.1} $$

where $\mathfrak{t} > a$.

Definition 2.2. [6, 13, 25]. The Caputo fractional derivative operator of order τ is given by,

$$ D_{\mathfrak{t}}^{\tau}(u(\mathfrak{t})) = \frac{1}{\Gamma(n-\tau)} \int_0^{\mathfrak{t}} (\mathfrak{t}-\varsigma)^{n-\tau-1} \frac{d^n}{d\mathfrak{t}^n} u(\varsigma) d\varsigma, \tag{2.2} $$

where $n \in \acute{\mathrm{N}} \cup \{0\}$, $(\tau \geq 0)$, $n-1 \leq \tau \leq n$.

Definition 2.3. [25]. The regularized Caputo fractional derivative of order τ is,

$$ ({}_0^C D_{\mathfrak{t}}^{\tau} u)(\mathfrak{t}, x) = \frac{1}{\Gamma(1-\tau)} \left(\frac{\partial}{\partial \mathfrak{t}} \int_0^{\mathfrak{t}} (\mathfrak{t}-\varsigma)^{-\tau} u(\varsigma, x) d\varsigma - \mathfrak{t}^{-\tau} u(0, x) \right), \tag{2.3} $$

where $0 \leq \tau \leq 1$.

Definition 2.4. [8] In a system, if for any $\epsilon > 0$ there exists a $\delta > 0$ such that $||x(\mathfrak{t}_0)|| < \delta$ implies $||x(\mathfrak{t})|| < \epsilon$ for all $\mathfrak{t} \geq \mathfrak{t}_0$ then that system is said to stable.

Definition 2.5. [8] If a system is stable and $x(\mathfrak{t}) \to 0$ as $\mathfrak{t} \to \infty$ then that system is known as asymptotically stable.

Definition 2.6. [8] A system is said to be unstable, if it is not stable, i.e. there exists an $\epsilon > 0$ such that for every $\delta > 0$ there exists an $x(\mathfrak{t}_0)$ with $||x(\mathfrak{t}_0)|| < \delta, ||x(\mathfrak{t}_1)|| \geq \epsilon$, for some $\mathfrak{t}_1 \geq \mathfrak{t}_0$.

Definition 2.7. [8] A system is said to be completely unstable, if there exists an $\epsilon > 0$ such that for every $\delta > 0$ and for every $x(\mathfrak{t}_0)$ with $||x(\mathfrak{t}_0)|| < \delta, ||x(\mathfrak{t}_1)|| \geq \epsilon$ for some $\mathfrak{t}_1 > \mathfrak{t}_0$.

The average number of new infections due to single infectious individual when contact with susceptible population is called The basic reproduction number. [9, 17] Now we give the Routh test theorem given in [8].

Lemma 2.8. *(The Routh Test)* [8]

A polynomial $a(\lambda)$ (whose coefficients are real numbers) have all roots with negative real parts when the below conditions are satisfied

- $\lambda^2 + a_1\lambda + a_2$ *: all the coefficients are positive*
- $\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$ *: all the coefficients are positive and* $a_1a_2 > a_3$

- $\lambda^4 + a_1\lambda^3 + a_2\lambda^2 + a_3\lambda + a_4$ *: all the coefficients are positive,* $a_1a_2 > a_3$ *and* $a_1a_2a_3 > a_1^2a_4 + a_3^2$.

3. Formulation of Mathematical Model

In this section we present a SIR model with with fractional derivative.
Total population $\acute{\mathrm{N}}(\mathfrak{t})$ is given by,
$\acute{\mathrm{N}}(\mathfrak{t}) = S_s(\mathfrak{t}) + S_I(\mathfrak{t}) + S_R(\mathfrak{t})$.
$S_s(\mathfrak{t})$- Class of individuals Susceptible at time $\mathfrak{t}$,
$S_I(\mathfrak{t})$-Class of individuals infected at time $\mathfrak{t}$,
$S_R(\mathfrak{t})$- Class of individuals recovered at time $\mathfrak{t}$.

Λ is the birth rate, μ is the natural death rate, $\mu + d$ is the death rate occurred due to infection of swine flu. β is effective contact rate with infected peoples, γ is recovery rate and b is rate of infection.

The model can be formulated as,

$$\frac{dS_S(\mathfrak{t})}{d\mathfrak{t}} = \Lambda - (\mu + \beta b)S_S(\mathfrak{t}), \tag{3.1}$$

$$\frac{dS_I(\mathfrak{t})}{d\mathfrak{t}} = \beta b S_S(\mathfrak{t}) - (\mu + d + \gamma)S_I(\mathfrak{t}), \tag{3.2}$$

$$\frac{dS_R(\mathfrak{t})}{d\mathfrak{t}} = \gamma S_I(\mathfrak{t}) - \mu S_R(\mathfrak{t}), \tag{3.3}$$

with initial conditions,

$$S_S(0) \geq 0, S_I(0) \geq 0, S_R(0) \geq 0. \tag{3.4}$$

By using the fractional calculus definitions the above system can written as follows

$$D_{\mathfrak{t}}^{\tau}(S_S(\mathfrak{t})) = \Lambda - (\mu + \beta b)S_S(\mathfrak{t}), \tag{3.5}$$

$$D_{\mathfrak{t}}^{\tau}(S_I(\mathfrak{t})) = \beta b S_S(\mathfrak{t}) - (\mu + d + \gamma)S_I(\mathfrak{t}), \tag{3.6}$$

$$D_{\mathfrak{t}}^{\tau}(S_R(\mathfrak{t})) = \gamma S_I(\mathfrak{t}) - \mu S_R(\mathfrak{t}), \tag{3.7}$$

with the initial conditions,

$$S_S(0) \geq 0, S_I(0) \geq 0, S_R(0) \geq 0. \tag{3.8}$$

4. Properties of model

Now in this section we prove the results on positivity and boundedness of the solution of the system. Let $\{(S_S, S_I, S_R) \in \mathbb{R}_+^3\}$ be any solution of model 3.5-3.7 with initial conditions 3.8. Now, let us assume the region
$\Omega = \{(S_S, S_I, S_R) \in \mathbb{R}_+^3 : 0 \leq \acute{\mathrm{N}}(\mathfrak{t}) \leq \frac{\Lambda}{\mu+\beta b}\}$.

Which is meaningful if all state variables are positive. Now we give our next theorem.

Theorem 4.1. *Let* $\{(S_S, S_I, S_R) \in \mathbb{R}_+^3\}$ *be any solution of model 3.5-3.7 with initial conditions 3.8. Consider*

$$\Omega = \left\{(S_S, S_I, S_R) \in \mathbb{R}_+^3 : 0 \leq \acute{N}(t) \leq \frac{\Lambda}{\mu + \beta b}\right\}, \tag{4.1}$$

then $S_S(t) \geq 0, S_I(t) \geq 0, S_R(t) \geq 0$ *for all* $t \geq 0$.

Proof. Suppose that for some point $\dot{\mathfrak{t}} > 0$, the $S_I(\mathfrak{t}) = 0$, i.e. $S_I(\dot{\mathfrak{t}}) = 0$ and $S_S(\mathfrak{t}) \geq 0, S_R(\mathfrak{t}) \geq 0$ (given).

Then from equation 3.6 we have,

$$D^{\tau}_{\mathfrak{t}}(S_I(\mathfrak{t})) > 0, \tag{4.2}$$

which is not true.

Thus, $S_I(\mathfrak{t}) \geq 0$ for all $\mathfrak{t} > 0$ (by contradiction).

Similarly, we can prove that, $S_S(\mathfrak{t}) \geq 0$ and $S_R(\mathfrak{t}) \geq 0$ for all time $\mathfrak{t} > 0$. □

Now in next theorem we will prove a boundedness of our system 3.5-3.7 with initial conditions 3.8.

Theorem 4.2. *The total population is denoted by $\acute{N}(t)$ and given by,*

$$\acute{N}(t) = S_S(t) + S_I(t) + S_R(t),$$

then

$$D^{\tau}_{t}(\acute{N}(t)) \leq \Lambda - \mu\acute{N}(t), \tag{4.3}$$

where, Λ- birth rate, μ- death rate.

Proof. Since,

$$\acute{\mathrm{N}}(\mathfrak{t}) = S_S(\mathfrak{t}) + S_I(\mathfrak{t}) + S_R(\mathfrak{t}),$$

using equations of system 3.5-3.7, we have

$$\begin{aligned}
D^{\tau}_{\mathfrak{t}}(\acute{\mathrm{N}}(\mathfrak{t})) =& D^{\tau}_{\mathfrak{t}}(S_S(\mathfrak{t})) + D^{\tau}_{\mathfrak{t}}(S_I(\mathfrak{t})) + D^{\tau}_{\mathfrak{t}}(S_R(\mathfrak{t})),\\
=& \Lambda - (\mu + \beta b)S_S(\mathfrak{t}) + \beta b S_S(\mathfrak{t}) - (\mu + d + \gamma)S_I(\mathfrak{t}) + \gamma S_I(\mathfrak{t})\\
& - \mu S_R(\mathfrak{t}),\\
=& \Lambda - \mu S_S(\mathfrak{t}) - \beta b S_S(\mathfrak{t}) + \beta b S_S(\mathfrak{t}) - \mu S_I(\mathfrak{t}) - d S_I(\mathfrak{t}) - \gamma S_I(\mathfrak{t})\\
& + \gamma S_I(\mathfrak{t}) - \mu S_R(\mathfrak{t}),\\
=& \Lambda - \mu S_S(\mathfrak{t}) - \mu S_I(\mathfrak{t}) - d S_I(\mathfrak{t}) - \mu S_R(\mathfrak{t}),\\
=& \Lambda - \mu(S_S(\mathfrak{t}) + S_I(\mathfrak{t}) + S_R(\mathfrak{t})) - d S_I(\mathfrak{t}).
\end{aligned}$$

Since $\acute{\mathrm{N}}(\mathfrak{t}) = S_S(\mathfrak{t}) + S_I(\mathfrak{t}) + S_R(\mathfrak{t})$,

$$D^{\tau}_{\mathfrak{t}}(\acute{\mathrm{N}}(\mathfrak{t})) = \Lambda - \mu(\acute{\mathrm{N}}(\mathfrak{t})) - d S_I(\mathfrak{t}) \leq \Lambda - \mu\acute{\mathrm{N}}(\mathfrak{t}).$$

Therefore, $\acute{\mathrm{N}}(\mathfrak{t})$ is bounded for all $\mathfrak{t} > 0$. Thus every solution of system 3.5-3.7 with initial conditions 3.8 is bounded. □

5. Stability analysis

In this section, we will check the stability of system 3.5-3.7 with initial conditions 3.8. The system 3.5-3.7 with initial conditions 3.8 has disease free equilibrium (No disease) which is given by equating the Right hand side of the system 3.5-3.7 to zero.

$$\begin{gathered}
\Lambda - (\mu + \beta b)S_S(\mathfrak{t}) = 0,\\
\beta b S_S(\mathfrak{t}) - (\mu + d + \gamma)S_I(\mathfrak{t}) = 0,\\
\gamma S_I(\mathfrak{t}) - \mu S_R(\mathfrak{t}) = 0.\\
\Rightarrow S_S(\mathfrak{t}) = \frac{\Lambda}{(\mu + \beta b)}, S_I(\mathfrak{t}) = 0, S_R(\mathfrak{t}) = 0.
\end{gathered}$$

The above equation are often written as,

$$\Sigma_o = (S_{S0}, S_{I0}, S_{R0}) = \left(\frac{\Lambda}{\mu + \beta b}, 0, 0\right). \tag{5.1}$$

The Syndemic Equilibrium is given by,

$$\Sigma_* = (S_{S*}, S_{I*}, S_{R*}), \tag{5.2}$$

with $S_{I*} > 0$, $S_{R*} > 0$ for $R_0 > 1$, where R_0 is the basic reproduction number of system 3.5-3.7 with initial conditions 3.8. Now in our next theorem we give the result on the stability of the system 3.5-3.7 with initial conditions 3.8.

Theorem 5.1. *The disease free equilibrium Σ_0 is locally asymptotically stable if $R_0 < 1$.*

Proof. If $R_0 < 1$ then the disease free equilibrium Σ_0 is locally asymptotically stable, when all the eigenvalues of the Jacobian Matrix of the system of equations 3.5-3.7 computed at the disease free equilibrium Σ_0, given by 5.1 have negative real parts.[8]

The Jacobian Matrix of the system of equations 3.5-3.7 at disease-free equilibrium is given by,

$$J_0 = \begin{pmatrix} -\mu - \beta b & 0 & 0 \\ \beta b & -\mu - d - \gamma & 0 \\ 0 & \gamma & -\mu \end{pmatrix}.$$

The eigenvalues of this matrix can be calculated by solving the determinant $|J_0 - \lambda I| = 0$,

$$|J_0 - \lambda I| = \begin{pmatrix} -\mu - \beta b - \lambda & 0 & 0 \\ \beta b & -\mu - d - \gamma - \lambda & 0 \\ 0 & \gamma & -\mu - \lambda \end{pmatrix} = 0,$$

which gives

$$(-\mu - \beta b - \lambda)\Big[(-\mu - d - \gamma - \lambda)(-\mu - \lambda)\Big] = 0,$$

$$(-\mu - \beta b - \lambda)\Big[\mu^2 + \mu\lambda + d\mu + d\lambda + \gamma\mu + \gamma\lambda + \mu\lambda + \lambda^2\Big] = 0$$

$$(-\mu - \beta b - \lambda)\Big[\mu^2 + 2\mu\lambda + d\mu + d\lambda + \gamma\mu + \gamma\lambda + \lambda^2\Big] = 0,$$

$$-\mu^3 - 2\mu^2\lambda - d\mu^2 - d\mu\lambda - \gamma\mu^2 - \mu\gamma\lambda - \mu\lambda^2 - \beta b\mu^2 - 2\beta b\mu\lambda - \beta bd\mu -$$
$$\beta bd\lambda - \beta b\gamma\mu - \beta b\gamma\lambda - \beta b\lambda^2 - \lambda\mu^2 - 2\mu\lambda^2 - d\mu\lambda - d\lambda^2 - \gamma\mu\lambda - \gamma\lambda^2 - \lambda^3 = 0,$$

$$-\lambda^3 - \big(3\mu + \beta b + d + \gamma\big)\lambda^2 - \big(3\mu^2 + 2d\mu + 2\mu\gamma + 2\beta b\mu + \beta bd + \beta b\gamma\big)\lambda - \big(\mu^3$$
$$+ d\mu^2 + \gamma\mu^2 + \beta b\mu^2 + \beta bd\mu + \beta b\gamma\mu\big) = 0,$$

$$\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 = 0, \tag{5.3}$$

where

$$3\mu + \beta b + d + \gamma = a_1,$$
$$3\mu^2 + 2d\mu + 2\mu\gamma + 2\beta b\mu + \beta bd + \beta b\gamma = a_2,$$
$$\mu^3 + d\mu^2 + \gamma\mu^2 + \beta b\mu^2 + \beta bd\mu + \beta b\gamma\mu = a_3.$$

Here we observed that in 5.3, $a_1 > 0, a_2 > 0, a_3 > 0$ and $a_1 a_2 > a_3$. Therefore from lemma 2.8, all the roots of equation 5.3 have negative real parts. Thus, all the eigenvalues of this Jacobian Matrix are negative.

Thus a mathematical model 3.5-3.7 with initial conditions 3.8 is asymptotically stable at disease free equilibrium for $R_0 < 1$. □

6. Data fitting and Numerical Calculations

In this section we will verify our result with Swine flu data of India to this system. The plot shows the curves for the Infected and Recovered peoples for Swine flu disease in India.

We obtained parameters by using real data given by, [27]

TABLE 1. Parameters.

Symbol	Parameter	Value
Λ	Birth rate	0.0000563447
μ	Death rate	0.0000194123
$\mu + d$	Death rate due to swine flu	0.0563124121
β	Effective contact rate	0.014
b	Rate of infection	0.00004524986
γ	Rate of Recovery	0.14436875879
Ń	Total Population	1380004385
$S_I(0)$	Infected peoples at time t=0	1
$S_R(0)$	Recovered peoples at time t=0	1

From nature of above plots we observed that the rate of recovered population is increasing. And also we can observe, the curve of infected population will become at initial stage means infections will become negligible after some time. From these observations we can conclude that situations will be normal in such outbreaks after some period of time.

7. Conclusion

In this paper we have studied the outbreak of swine flu using fractional order calculus. Boundedness of solution of the system have obtained and also stability properties of the solution are also studied. Which are useful in future prediction of such outbreaks.

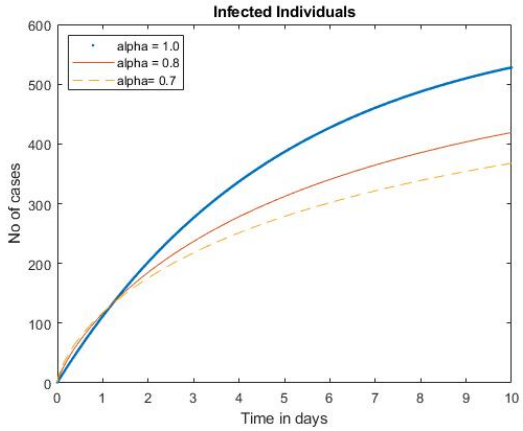

FIGURE 1. **Graph for Infected population**

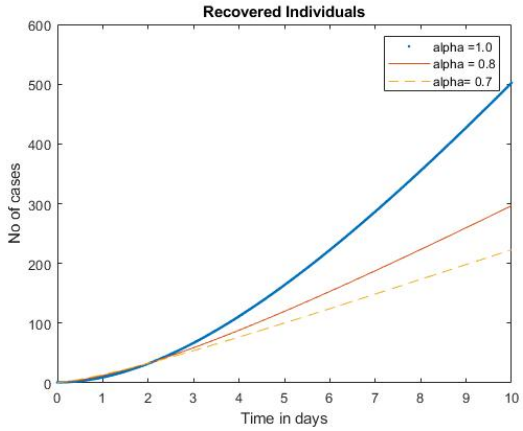

FIGURE 2. **Graph for Recovered population**

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