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ALMOST HEPTIC SPLINES FOR MODIFIED (0,2,4,6) INTERPOLATORY PROBLEM

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ABSTRACT. In this paper we consider a modified lacunary interpolation problem in which we have function values, second, fourth and sixth derivatives at node points, for which we derive a heptic spline belongs to a set of spline functions $s_{n,7}$ which approximates the given function. First we have shown that such spline function exists uniquely. Then the theorem of convergence has also been proved.
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1. Introduction

There are several communications in the field of lacunary interpolation [1],[2], Recently K. Singh [10] has obtained a quartic spline function.

On the same lines In this present paper we consider a birkhoff interpolation problem which we call modified (0,2,4,6) problem, using almost heptic splines s(x) ε $s_{n,7}$ where for the given partition

$$P: 0=x_0<x_1<\ldots<x_{n-1}<x_n=1$$

Of the unit interval [0,1], $S_{n,7}$ denotes the class of spline functions (x) such that

(1) $S(x)\ \varepsilon\ \Pi_7$, $k=1,2,\ldots,n-2$,

(2) $S(x)\ \varepsilon\ \Pi_8$, $k=0,1\ldots n-1$; Where $x\ \varepsilon\ [x_k,x_{k+1}]$

(3) $S(x)\ \varepsilon\ C^2(I)$

Also we denote $x_{k+1}-x_k=h_k$ for all k=0,1,2…n-1.

We prove the existence and uniqueness of spline functions and then show that they converge to the given function $f(x)\ \varepsilon\ C^7(I)$ up to derivative of order 7.

2. Theorem 2.1

Given P and the real numbers $y_m,y_m^{(2)},y_m^{(4)}$ and $y_m^{(6)}$, m=0,1…n; $y_0^{(1)},y_n^{(1)}$ there exist a unique spline function $s_p(x)\ \varepsilon\ S_{n,7}$ such that

(2.1) $$s_p^{(q)}(x_m)=y_m^{(q)},\ m=0,1,2\ldots n;\ q=0,2,4,6$$

(2.2) $$s_p^{(1)}(x_0)=y_0^{(1)},\ s_p^{(1)}(x_n)=y_n^{(1)}$$

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Proof: We set (2.3) $s_p(x) = s_0(x)$ when $x\ \varepsilon\ [x_0,x_1]$

$S_p(x) = s_m(x)$ when $x\ \varepsilon\ [x_m,x_{m+1}]$, $m=1,2\ldots n-2$

$S_p(x) = s_{n-1}(x)$ when $x\ \varepsilon\ [x_{n-1},x_n]$,

Using interpolatory conditions (2.1),(2.2) and (2.3) we have

(2.4) $$s_0(x) = y_0+ (x-x_0)\, y_o^{(1)} + (x-x_0)^2/2!\, y_0^{(2)} + (x-x_0)^3/3!\, a_{0,3} + (x-x_0)^4/4!\, y_0^{(4)} + (x-x_0)^5/5! a_{0,5} + (x-x_0)^6/6! y_0^{(6)} + (x-x_0)^{(7)}/7! a_{0,7} + (x-x_0)^{(8)}/8! a_{0,8}$$

(2.5) $$s_m(x) = y_m + (x-x_m)a_{m,1} + (x-x_m)^2/2! y_m^{(2)} +(x-x_m)^3/3! a_{m,3} + (x-x_0)^4/4! y_m^{(4)} + (x-x_m)^5/5! a_{m,5} + (x-x_0)^6/6! y_m^{(6)} + (x-x_m)^7/7! a_{m,7}$$

(2.6) $$s_{n-1}(x) = y_{n-1} + (x-x_{n-1})a_{n-1,1} + (x-x_{n-1})^2/2! y^{(2)}_{n-1} + (x-x_{n-1})^3/3! a_{n-1,3} + (x-x_{n-1})^4/4! y_{n-1}^{(4)} + (x-x_{n-1})^5/5! a_{n-1,5} + (x-x_{n-1})^6/6! y_{n-1}^{(6)} +(x-x_{n-1})^7/7! a_{n-1,7} +(x-x_{n-1})^8/8! a_{n-1,8}$$

The coefficient involved in the above equation determine by remaining interpolatory conditions and the continuity requirement that $s_p(x)\ \varepsilon\ c^2(I)$. Applying these conditions we get following equations: (2.7)

$$y_1 =y_0 + h_0y_0^{(1)} +h_0^2/2! y_0^{(2)} +h_0^3/3! a_{0,3}+h_0^4/4! y_0^{(4)} +h_0^5/5! a_{0,5} +h_0^6/6! y_0^{(6)} +h_0^7/7! a_{0,7} +h_0^8/8! a_{0,8}$$

$$y^{(2)}_1 = y_0^{(2)} +h_0\, a_{0,3} + h_0^2/2! y_0^{(4)} + h_0^3/3! a_{0,5} +h_0^4/4! y_0^6 +h_0^5/5! a_{0,7}+h_0^6/6! a_{0,8}$$

$$y_1^{(4)} = y_0^{(4)} + h_0a_{0,5}+h_0^2/2! y_o^6 +h_0^3/3! a_{0,7}+ h_0^4 a_{0,8}$$

$$y_1^{(6)} = y_0^{(6)} +h_0a_{0,7} +h_0^2/2! a_{0,8}$$

(2.8) $$y_{m+1} = y_m +h_ma_{m,1} +h_m^2/2! y_m^{(2)} +h_m^3/3! a_{m,3}+h_m^4/4! y_m^{(4)} +h_m^5/5! a_{m,5}+h_m^6/6! y_m^{(6)} + h_m^7/7! a_{m,7}$$

$$y_{m+1}^{(2)} = y_m^{(2)} +h_ma_{m,3}+h_m^2/2! y_m^{(4)}+h_m^3/3! a_{m,5}+h_m^4/4! y_m^{(6)}+ h_m^5/5! a_{m,7}$$

$$y_{m+1}^{(4)} = y_m^{(4)} + h_ma_{m,5} +h_m^2/2! y_m^{(6)} +h_m^3/3! a_{m,7}$$

$$y_{m+1}^{(6)} = y_m^{(6)} + h_ma_{m,7}$$

(2.9) $$y_n = y_{n-1} + h_{n-1}a_{n-1}+h_{n-1}^2/2! y_{n-1}^{(2)} +h_{n-1}^3/3! a_{n-1,3} +h_{n-1}^4/4! y_{n-1}^{(4)}+ h_{n-1}^5/5! a_{n-1,5} +h_{n-1}^6/6! y_{n-1}^{(6)} +h_{n-1}^7/7! a_{n-1,7}+h_{n-1}^8/8! a_{n-1,8}$$

$$y_n^{(2)} = y_{n-1}^{(2)} + h_{n-1}a_{n-1,3} +h_{n-1}^2/2! y_{n-1}^{(4)}+h_{n-1}^3/3! a_{n-1,5}+h_{n-1}^4/4! y_{n-1}^{(6)} +h_{n-1}^5/5! a_{n-1,7} + h_{n-1}^6/6! a_{n-1,8}$$

$$y_n^{(4)} = y_{n-1}^{(4)} + h_{n-1}a_{n-1,5} +h_{n-1}^2/2! y_{n-1}^{(6)} + h_{n-1}^3/3! a_{n-1,7}+h_{n-1}^4/4! a_{n-1,8}$$

$$y_n^{(6)} = y_{n-1}^{(6)} + h_{n-1}a_{n-1,7} +h_{n-1}^2/2! a_{n-1,8}$$

From these equations we obtained $a_{0,3}, a_{0,5}, a_{0,7}, a_{0,8}, a_{m,1}, a_{m,3}, a_{m,5}, a_{m,7}, a_{n-1,1}, a_{n-1,3}, a_{n-1,5}, a_{n-1,7}$ and $a_{n-1,8}$.

The unique existence of these coefficients shows unique existence of the spline function $S_p(x)$.

The coefficients of this polynomial are given below

$$a_{0,3} = 168/17h^3[(y_1-y_0-hy_0^{(1)}-h^2{}_0/2!y_0^{(2)}-h_0^4/4!y_0^{(4)}-h_0^6/6!y_0^{(6)}) -11h_0^2/168\,(y_1^{(2)}-y_0^{(2)}-h_0^2/2!y_0^{(4)}-h_0^4/4!y_0^{(6)}) + 13/7h_0^4(y_1^{(4)}-y_0^{(4)}-h_0^2/2!y_0^{(6)}) - 137/60480h_0^6\,(y_1^{(6)}-y_0^{(6)})]$$

$$a_{0,5} = 10h^{-3}\,[-168/17h^2(y_1-y_0-h_0y_0^{(1)}-h^2{}_0/2!y_0^{(2)}-h^4{}_0/4!y_0^{(4)}-h_0^6/6!y_0^{(6)}) +28/17(y_1^{(2)}-y_0^{(2)}-h^2/2!y_0^{(4)} -h_0^4/4!y_0^{(6)}) + 7h/170(y_1^{(4)}-y_0^{(4)}-h^2{}_0/2!y_0^{(6)}) +h^5{}_0/5!(y_1^{(6)}-y_0^{(6)}\,)\,]$$

$$a_{0,7} = 12/h^3\,[1680/17h^4(y_1-y_0-h_0y_0^{(1)}-h_0^2/2!y_0^{(2)}-h^4{}_0/4!y_0^{(4)}-h^6{}_0/6!y_0^{(6)}) -280/17h^2(y_1^{(2)}-y_0^{(2)}-h_0^2/2!y_0^{(4)} -h^4/4!y_0^{(6)}) +10/17(y_1^{(4)}-y_0^{(4)}-h^2/2!y_0^{(4)}) -23/204h^2(y_1^{(6)}-y_0^{(6)})]$$

$$a_{0,8} = 2/h^2\,[-20160/17h^6(y_1-y_0-h_0y_0^{(1)}-h_0^2/2!y_0^{(2)}-h^4/4!y_0^{(4)}-h^6{}_0/6!y_0^{(6)}) + 3360/17h^4(y_1^{(2)}-y_0^{(2)} -h_0^2/2!y_0^{(4)}-h_0^4/4!y_0^{(6)}) -120/17h^2(y_1^{(4)}-y_0^{(4)}-h_0^2/2!y_0^{(4)}) + 40/17(y_1^{(6)}-y_0^{(6)}\,)\,]$$

$$a_{m,1}= h_m^{-1}[(y_{m+1}-y_m-h^2{}_m/2!y_m^{(2)}-h_m^4/4!y_0^{(4)}-h_m^6/6!y_0^{(6)}) -h_m^2/3!(y_{m+1}^{(2)}-y_m^{(2)}-h_m^2/2!y_m^{(4)} - h_m^4/4!y_m^{(6)}\,) +7h^4/360(y_{m+1}^{(4)} -y_m^{(4)}-h_m^2/2!y_m^{(6)}) -h_m^6/336(y_{m+1}^{(6)}-y_m^{(6)})$$

$$a_{m,3} = h_m^{-1}[7h_m^4/360(y_{m+1}^{(6)}-y_m^{(6)}) - h_m^2/3!(y_{m+1}^{(4)}-y_m^{(4)} -h_m^2/2!y_m^{(6)}) +(y_{m+1}^{(2)}-y_m^{(2)}-h_m^2/2!y_m^{(4)} h_m^4/4!y_m^{(6)})]$$

$$a_{m,5} = h_m^{-1}[-h_m^2/3!(y_{m+1}^{(6)}-y_m^{(6)}) +(y_{m+1}^{(4)}-y_m^{(4)}-h_m^2/2!y_m^{(6)})]$$

$$a_{m,7} = h_m^{-1}[y_{m+1}^{(6)}-y_m^{(6)}]$$

$$a_{n-1,1} = 170/837[1674/170h_{n-1}(y_n-y_{n-1}-h_{n-1}^2/2!y_{n-1}^{(2)}-h^4{}_{n-1}/4!y_{n-1}^{(4)}-h_{n-1}^6/6!y_{n-1}^{(6)}) -2511h_{n-1}/340 (y_n^{(2)}-y_{n-1}^{(2)}-h_{n-1}^2/2!y_{n-1}^{(4)}-h_{n-1}^4/4!y_{n-1}^{(6)}) + 3348h_{n-1}^2/680(y_n^{(4)}-y_{n-1}^{(4)}-h_{n-1}^2/2!y_{n-1}^6) + 5004h_{n-1}^4/850(y_n^{(6)}-y_{n-1}^{(6)}) +h_{n-1}y_{n-1}^{(2)}-h_{n-1}^2y_{n-1}^{(4)}+y_{n-1}^{(1)}]$$

$$a_{n-1,3} = 4250/7533h_{n-1}^3(y_n-y_{n-1} -h_{n-1}^2/2!y_{n-1}^{(2)}-h_{n-1}^4/4!y_{n-1}^{(4)}-h_{n-1}^6/6!y_{n-1}^{(6)})-9207/2720h_{n-1}^2 (y_n^{(2)}-y_{n-1}^{(2)}-h_{n-1}^2/2!y_{n-1}-h_{n-1}^4/4!y_{n-1}^{(6)}) -7553/2380h_{n-1}(y_n^{(4)}-y_{n-1}^{(4)}-h_{n-1}^2/2!y_{n-1}^{(6)}) 10044h_{n-1}^4/3360(y_n^{(6)}-y_{n-1}^{(6)}) +279h_{n-1}/34y_{n-1}^{(2)}-3h_{n-1}/2y_{n-1}^{(2)}-5/8h_{n-1}^2y_{n-1}^{(4)} +7h_{n-1}/3y_{n-1}^{(1)}]$$

$$a_{n-1,5}= 5022/1870h_{n-1}^5(y_n-y_{n-1} -h_{n-1}^2/2!y_{n-1}^{(2)}-h_{n-1}^4/4!y_{n-1}^{(4)}-h_{n-1}^6/6!y_{n-1}^{(6)})+3348/850h_{n-1}^3 (y_n^{(2)}-y_{n-1}^{(2)}-h_{n-1}^2/2!y_{n-1}-h_{n-1}^4/4!y_{n-1}^{(6)}) -5859/2890h_{n-1}(y_n^{(4)}-y_{n-1}^{(4)}-h_{n-1}^2/2!y_{n-1}^{(6)}) +11718h_{n-1}^4/2380(y_n^{(6)}-y_{n-1}^{(6)}) +55h_{n-1}/81y_{n-1}^{(2)}+106h_{n-1}^4/68y_{n-1}^{(4)}+11h_{n-1}/5y_{n-1}^{(1)}+ h_{n-1}^2\,y_{n-1}^{(6)}$$

$$a_{n-1,7}= 15066/2720h_{n-1}^{7}(y_n-y_{n-1} -h_{n-1}^{2}/2!y_{n-1}^{(2)}-h_{n-1}^{4}/4!y_{n-1}^{(4)}-h_{n-1}^{6}/6!y_{n-1}^{(6)}) +13392/2210h_{n-1}^{5}$$
$$(y_n^{(2)}-y_{n-1}^{(2)}-h_{n-1}^{2}/2!y_{n-1}-h_{n-1}^{4}/4!y_{n-1}^{(6)})-14229/2380h_{n-1}^{3}(y_n^{(4)}-y_{n-1}^{(4)}-h_{n-1}^{2}/2!y_{n-1}^{(6)})$$
$$11718\ h_{n-1}^{4}/2890(y_n^{(6)}-y_{n-1}^{(6)}) +4400h_{n-1}/567y_{n-1}^{2} + 742h_{n-1}^{4}/340y_{n-1}^{(4)}+264h_{n-1}/625y_{n-1}^{(1)}$$
$$+ 12h_{n-1}^{2}/65y_{n-1}^{(6)}$$

$$a_{n-1,8}= 18414/4250h_{n-1}^{8}(y_n-y_{n-1} -h_{n-1}^{2}/2!y_{n-1}^{(2)}-h_{n-1}^{4}/4!y_{n-1}^{(4)}-h_{n-1}^{6}/6!y_{n-1}^{(6)})+15903/2380h_{n-1}^{7}$$
$$(y_n^{(2)}-y_{n-1}^{(2)}-h_{n-1}^{2}/2!y_{n-1}-h_{n-1}^{4}/4!y_{n-1}^{(6)}) + 17577/3740h_{n-1}^{5}(y_n^{(4)}-y_{n-1}^{(4)}-h_{n-1}^{2}/2!y_{n-1}^{(6)}) +$$
$$19251h_{n-1}^{4}(y_n^{(6)}-y_{n-1}^{(6)}) + 7150h_{n-1}/1134y_{n-1}^{(2)} + 848h_{n-1}^{4}/476\ y_{n-1}^{(4)} +308h_{n-1}/1500\ y_{n-1}^{(1)}$$
$$132h_{n-1}^{2}/325\ y_{n-1}^{(6)}$$

This proves the theorem.

3. Theorem 3.1

Let $f(x)\ \varepsilon\ C^7(I)$. Then for the unique spline function $s_p(x)$ mentioned in Theorem 2.1with $y_m, y_m^{(1)}$ etc being associated with function $f(x)$, that is $y_m=f(x_m)$, $y_m^{(2)}= f^{(2)}(x_m)$ etc; we have for

$x\varepsilon\ [x_m,x_{m+1}]$,m= 0,1,2,…n-1,

(3.1) $|s_p^{(q)}(x) -f^{(q)}(x)| \leq C_{p,q}\ h^{7-q}\ w_7(h)$, q= 0,1…7.

Here we take $h_m=h$ for all m=0,1,…,n-1 and denote the modulus of continuity of $f(x)=C^7(I)$ by$W_7(h)$ .The coefficient $C_{p,q}$ are different constant mentioned below in the proof below

Proof-:

Let $x\ \varepsilon\ [x_0,x_1]$. Then from equation (2.4) we have

$$S_0^{(7)}(x) -f^{(7)}(x) = a_{0,7}-(x-x_0)a_{0,8}-f^{(7)}(x)$$

$$| 2160/17h^7(y_1-y_0-h_0y_0^{(1)}-h_0^{2}/2!y_0^{(2)}-h^4/4!y_0^{(4)}-h^6{}_0/6!y_0^{(6)}) -3360/17h^5(y_1^{(2)}-y_0^{(2)}\ \ h_0^{2}/2!y_0^{(4)}\ \ -h^4/4!y_0^{(6)}) +120/17h^3(y_1^{(4)}-y_0^{(4)}-h^2{}_0/2!y_0^{(6)}) -23/17h(y_1^{(6)}-y_0^{(6)}) + (x-x_0)\ 2/h^2 [-20160/17h^6(y_1-y_0-h_0y_0^{(1)}-h_0^{2}/2!y_0^{(2)}-h^4/4!y_0^{(4)}- h^6{}_0/6!y_0^{(6)}) + 3360/17h^4(y_1^{(2)}-y_0^{(2)} - h_0^{2}/2!y_0^{(4)}-h_0^{4}/4!y_0^{(6)}) -120/17h^2(y_1^{(4)}-y_0^{(4)}- h_0^{2}/2!y_0^{(4)}) + 40/17(y_1^{(6)}-y_0^{(6)}\ )\ \ -f^7(x)|$$

Therefore

$$|s_0^{(7)}(x)-f^7(x)| \leq |20160/17h^7(h^3{}_0/3!y_0^{(3)}+h_0^{5}/5!y_0^{5}+ h_0^{7}/7!\ f^7(\eta_m)\ )-3360/17h^5(y_0^{(3)}+h^3/3!y_0^{(5)}+\ \ h^5/5!f^7(\varepsilon_m) - 120/17h^3(h_0y_0^{(5)}+h^3/3!f^7(\ \alpha_m)-23/17f^7(\beta_m) -40320/17h^7\ (h^3{}_0/3!y_0^{(3)}+h_0^{5}/5!y_0^{5}+ h_0^{7}/7!\ f^7(\eta_m)\ ) + 6720/17h^5(y_0^{(3)}+h^3/3!y_0^{(5)}+ h^5/5!f^7(\varepsilon_m)) -240/17h^3(h_0y_0^{(5)}+h^3/3!f^7(\ \alpha_m)\ ) +80/17\ f^7(\beta_m)\ |$$

$$\leq|4/17\ f^7(\eta_m) -28/17\ f^7(\varepsilon_m)-20/17\ f^7(\ \alpha_m) -23/17\ f^7(\beta_m) -32/17\ f^7(\eta_m)+56/17\ f^7(\varepsilon_m\ )-40/17\ f^7(\ \alpha_m) +80/17\ f^7(\beta_m)|$$

$$|\ s_0^{(7)}(x)-f^7(x)| \leq 120/17w_7(h).$$

Here $x_{m\leq}\ \eta_m\leq\varepsilon_m\leq\alpha_m\leq\beta_m\leq x_{m+1}$.

Using interpolatory conditions, we have

$|s_0^{(6)}-f_0^{(6)}| = \int_{x_0}^{x_1} |s_0^7 \ (x) - f^7 \ (x)| \quad dx$

$= (x-x_0)\ w_7(h)$

$\leq 120/7\ h\ w_7(h)$

Again from equation (2.4) using Taylor's theorem, we have

$| s_0^{(5)}(x) - f^5(x)| \leq 50h^2/17w_7(h)$

Further

$|s_0^{(4)}(x)-f^4(x)| \leq \int_{x_0}^{x_1} |s_0^5 \ (x) - f^5 \ (x)|\ dx$

$\leq |x_1-x_0|\ | s_0^{(5)}(x) - f^5(x)|$

$\leq 50h^3/17w_7(h)$

We using again Taylor's theorem from equation (2.4) we have

$|s_0^{(3)}(x)-f^3(x)| \leq 30h^4/17\ w_7(h)$

Again using interpolatory condition, we have

$|s_0^{(2)}(x)- f^2(x)| \leq \int_{x_0}^{x_1} |s_0^3 - f^3|dx$

$\leq 30h^5/17\ w_7(h)$

Similarly,

$|s_0^{(1)}(x)-f^1(x)| \leq 30h^6/17\ w_7(h)$

$|s_0(x)-f(x)| \quad \leq\ 30h^7/17\ w_7(h)$

This proves Theorem 3.1 for $x\varepsilon[x_0,x_1]$

For $x\ \varepsilon\ [x_m,x_{m+1}]$, m= 1,2,…n-2,we have from (2.5),using Taylor's Theorem

$|s_m^{(7)}(x)-f^7(x)\ | \leq |\ a_{m,7}-f^7(x)|$

$\leq(\ h_m^{-1}(y_{m+1}^{(6)}-y_m^{(6)})-f^7(x)\ )$

$\leq(\ h_m^{-1}(h_m f^7(\beta_m))-f^7(x)$

$\leq w_7(h)$

$|s_0^{(6)}(x)-f^6(x)|\ \leq |\int_{x_m}^{x} |s_m^7\ (x) - \ f^7(x)|\,dx$

$\leq |\ x-x_m|\ |s_m^7(x)-f^7(x)|$

$\leq h\, w_7(h)$

$| s_m^{(5)}(x)-f^5(x) | \leq |a_{m,5}+ h_m y_m^{(6)}+h_m^2/2\, a_{m,7} - f^5(x)|$

$\leq | h_m^{-1}[-h_m^2/3!(y_{m+1}^{(6)}-y_m^{(6)}) +(y_{m+1}^{(4)}-y_m^{(4)}-h_m^2/2!y_m^{(6)})] +h_m y_m^{(6)}+$

$h_m^{-1}[y_{m+1}^{(6)}-y_m^{(6)}] - f^5(x) |$

$\leq 3h^2/2w_7(h)$

Further

$|s_m^{(4)}(x)-f^4(x)| \leq | \int_{x_m}^{x} |s_m^5(x) - f^5(x)|dx$

$\leq |x-x_m|\, |s_m^5(x) - f^5(x)|$

$\leq 3h^3/2w_7(h)$

$|s_m^{(3)}(x)-f^3(x)| \leq |a_{m,3}+h_m y_m^{(4)}+h_m^2/2!a_{m,5}+h^3/3!y_m^{(6)}+h_m^4/4!a_{m,7}|$

$\leq 5h^4/2w_7(h)$

In the same way

$|s_m^{(2)}(x)-f^2(x)| \leq 5h^5/2w_7(h)$

$|s_m^{(1)}(x)- f^1(x)| \leq 7h^6/2w_7(h)$

And $|s_m(x)-f(x)| \leq 7h^7/2\, w_7(h)$

This gives the result for x ε $[x_m,x_{m+1}]$
Proof for x ε $[x_{n-1},x_n]$ can also be carried out on similar manner, so we omit the details.

4. Conclusion

As we have seen here that a unique heptic spline function interpolate here a (0,2,4,6) problem .It is also provide spline function converges uniformly. Such type of functions are used to solve special differential equations and finding quadrature formula .In our next communication we will show that this function is applicable for solving boundary value problem of differential equation.

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