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TOTALLY UMBILICAL LIGHTLIKE HYPERSURFACES OF SASAKIAN SPACE FORM

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Abstract: Object of present paper is to study the properties of totally umbilical lightlike hypersurfaces of Sasakian space form with (l, m) –type connection.
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1. Introduction

A linear connection $\overline{\nabla}$ on a semi-Riemannian manifold (M, G) is called an (l, m) –type connection [7] if $\overline{\nabla}$ and its torsion tensor $\overline{T}$ satisfy

(1.1)1.

$$(\overline{\nabla}_{\overline{X}}\overline{g})(\overline{Y},\overline{Z})=l\{\theta(\overline{Y})\overline{g}(\overline{X},\overline{Z})+\theta(\overline{Z})\overline{g}(\overline{X},\overline{Y})\}$$
$$-m\{\theta(\overline{Y})\overline{g}(J\overline{X},\overline{Z})+\theta(\overline{Z})\overline{g}(J\overline{X},\overline{Y})\}$$ and

$$\overline{T}(\overline{X},\overline{Y})=l\{\theta(\overline{Y})\overline{X}-\theta(\overline{X})Y)\}$$
(1.2) $$+m\{\theta(\overline{Y})J\overline{X}-\theta(\overline{X}),J\overline{Y})$$

where l and m are two smooth functions on $\overline{M}$, J is a tensor field of type (1,1) and θ is a 1-form associated with a smooth unit vector field ζ which is called the characteristic vector field of $\overline{M}$, given by $\theta(\overline{X})=\overline{g}(\overline{X},\zeta)$.

By direct calculation it can be easily seen that a linear connection $\overline{\nabla}$ on M is an (l, m) –type connection if and only if $\overline{\nabla}$ satisfies

(1.3) $\overline{\nabla}_{\overline{X}}\overline{Y}=\nabla_{\overline{X}}\overline{Y}+\theta(\overline{Y})\{l\overline{X}+mJ\overline{X}\}$, where ∇ is Levi-Civita connection of a semi-Riemannian manifold ($\overline{M}$, $\overline{g}$) with respect to $\overline{g}$.

In case (*l*, *m*) = (1, 0): The above connection $\overline{\overline{\nabla}}$ becomes a semi-symmetric non-metric connection. The notion of semisymmetric non-metric connection on a Riemannian manifold was introduced by Ageshe-Chafle [1,2] and later, studied by several authors [12,11]. In case (*l*, *m*) = (0, 1): The above connection $\overline{\overline{\nabla}}$ becomes a non-metric φ-symmetric connection such that $\varphi(\overline{X},\overline{Y})=\overline{g}(J\overline{X},\overline{Y})$. The notion of the non-metric φsymmetric connection was introduced by Jin [9, 10, 11]. In case (*l*, *m*) = (1, 0) in (1.1) and (*l*, *m*) = (0, 1) in (1.2): The above connection $\overline{\overline{\nabla}}$ becomes a quarter-symmetric non-metric connection. The notion of quarter-symmetric non-metric connection was introduced by Golab [5] and then, studied by Sengupta-Biswas [14] and Ahmad-Haseeb [3]. In case (*l*, *m*) = (0, 0) in (1.1) and (*l*, *m*) = (0, 1) in (1.2): The above connection $\overline{\overline{\nabla}}$ becomes a quarter-symmetric metric connection. The notion of quarter-symmetric metric connection was introduced Yano-Imai [15]. In case (*l*, *m*) = (0, 0) in (1.1) and (*l*, *m*) = (1, 0) in (1.2): The above connection $\overline{\overline{\nabla}}$ becomes a semi-symmetric metric connection. The notion of semi-symmetric metric connection was introduced by Hayden [6].

2. Introduction

Let M be an almost contact manifold equipped with an almost contact metric structure $\{J,\zeta,\theta,\overline{g}\}$ consisting of a (1,1) tensor field J, a vector field ζ, a 1-form θ and a compatible Riemannian metric $\overline{g}$ satisfying

(2.1)
$$J^2\overline{X}=-\overline{X}+\theta(\overline{X})\zeta,\quad \overline{g}(J\overline{X},J\overline{Y})=\overline{g}(\overline{X},\overline{Y})-\theta(\overline{X}),\theta(\overline{Y}),\theta(\zeta)=1,$$

From this, we also have

$$J\zeta = 0,\ \theta o J = 0, \overline{g}(\mathrm{J}\,\overline{X}, \overline{Y}) = -\overline{g}(\overline{X}, \mathrm{J}\overline{Y}), \theta(\overline{X}) = \overline{g}(\overline{X}, \zeta).$$

for all $X, Y \in \chi(M)$.

An almost contact metric manifold M is a Sasakian manifold [13] if and only if it satisfies

$$(\overline{\nabla}_{\overline{X}}\ \mathrm{J})\overline{Y} = \overline{g}(\overline{X}, \overline{Y})\zeta - \theta(\overline{Y})\overline{X}.\ X, Y \in \chi(M).$$

where ∇ is Levi-Civita connection of the Riemannian metric g.

With the above equation and (1.3), (2.1) and $\theta(JY) = 0,$ it follows :

(2.2)

$$(\overline{\nabla}_{\overline{X}}\ \mathrm{J})\overline{Y} = \overline{g}(\overline{X}, \overline{Y})\zeta - \theta(\overline{\mathrm{Y}})\overline{X} - \theta(\overline{\mathrm{Y}})\{lJ\,\overline{X} - m\overline{X} + m\theta(\overline{X})\zeta\}.$$

Taking $\overline{Y} = \zeta$ and using $J\zeta = 0$ with $\theta(\overline{\nabla}_X \zeta) = l\theta(X),$ we have

(2.3) $\overline{\nabla}_{\overline{X}}\zeta = (m-1)J\,\overline{X} + l\,\overline{X},$

Consider (M, g) a lightlike hypersurface of $\overline{M}$. The normal bundle T$M^{\perp}$ of M is a subbundle of the tangent bundle TM of M, of rank 1, and coincides with the radical distribution Rad(TM) = TM $\cap$ T$M^{\perp}$. Denote by F (M) the algebra of smooth functions on M and by T(E) the F (M) module of smooth sections of any vector bundle E over M.

A complementary vector bundle S(TM) of Rad(TM) in TM is non-degenerate distribution on M, which is called a screen distribution on M, such that

TM = Rad(TM) $\oplus_{\mathrm{orth}}$ S(TM),

where $\oplus_{\mathrm{orth}}$ denotes the orthogonal direct sum. For any null section ξ of Rad(TM), there exists a unique null section N of a unique lightlike vector bundle tr(TM) in the orthogonal complement S$(TM)^{\perp}$ of S(TM) satisfying

$\overline{g}$ (ξ , N) = 1, $\overline{g}$ (N; N) = $\overline{g}$ (N; X) = 0; $\forall$ 8 X $\in$ T(S(TM)):

We call tr(TM) and N the transversal vector bundle and the null transversal vector field of M with respect to the screen distribution S(TM), respectively.

The tangent bundle T $\overline{M}$ of $\overline{M}$ is decomposed as follow:

T $\overline{M}$ = TM $\oplus$ tr(TM) = {Rad(TM) $\oplus$ tr(TM)} $\oplus_{orth}$ S(TM):

In the sequel, let X, Y, Z and W be the vector fields on M, unless otherwise specified. Let P be the projection morphism of TM on S(TM). Then the local Gauss and Weingartan formulas of M and S(TM) are given respectively by

(2.4) $\overline{\nabla}_X Y = \nabla_X Y + B(X.Y)N,$

(2.5) $\overline{\nabla}_X N = -A_N X + \tau(X)N,$

(2.6) $\nabla_X PY = \nabla^*_X PY + C(X.PY)\xi,$

(2.7) $\nabla_X \xi = -A^*_\xi X - \sigma(X)\xi.$

where ∇ and ∇^* are the induced linear connections on TM and S(TM) respectively, B and C are the local second fundamental forms on TM and S(TM) respectively, A_N and A^*_ξ are the shape operators on TM and S(TM) respectively, and τ and σ are 1-forms on M.

For a lightlike hypersurface M of an almost Hermitian manifold ($\overline{M}$, $\overline{g}$), it is known [3] that J(Rad(TM)) and J(tr(TM)) are subbundles of S(TM), of rank 1 such that J(Rad(TM)) $\cap$ J(tr(TM)) = 0. Thus there exist two non-degenerate almost complex distributions D_o and D on M with respect to J, i.e., J(D_o) = D_o and J(D) = D, such that

S(TM) = J(Rad(TM)) $\oplus$ J(tr(TM)) $\oplus_{orth}$ D_o;

D = {Rad(TM) $\oplus_{orth}$ J(Rad(TM))} $\oplus_{orth}$ D_o;

TM = D $\oplus$ J(tr(TM)).

Consider two null vector fields U and V , and two 1-forms u and v such that

(2.8) $U = -JN$, $V = -J\xi$, $u(X) = g(X, V)$; $v(X) = g(X, U)$.

Denote by S the projection morphism of TM on D. Any vector field X of M is expressed as $X = SX + u(X)U$. Applying J to this form, we have

(2.9) $JX = FX + u(X)N$,

where F is a tensor field of type (1, 1) globally defined on M by $F = JoS$. Applying J to (2.9) and using (1.2) , (1.3) and (2.8), we have

(2.10) $F^2X = -X + u(X)U + \theta(X)\zeta.$

As $u(U) = 1$ and $FU = 0$, the set (F, u, U) defines an indefinite almost contact structure on M and F is called the structure tensor field of M.

3. (*l, m*)-type connection

Let $(\overline{M}, \overline{g}, J)$ be a Sasakian manifold with a semi- symmetric metric connection $\overline{\nabla}$. Using (1.1), (2.1) and (2.7), we obtain

(3.1)
$$\begin{aligned}(\nabla_X g)(Y, Z) &= B(X,Y)\eta(Z) + B(X, Z)\eta(Y)\\ &-l\{\theta(Y)g(X,Z)+\theta(Z)g(X,Y)\}\\ &-m\{\theta(Y)g(JX,Z)+\theta(Z)g(JX,Y)\}\end{aligned}$$

(3.2)
$$\begin{aligned}T(X,Y)=&\,l\{\theta(Y)X-\theta(X)Y\}\\ &+m\{\theta(Y)FX-\theta(X)FY\}\end{aligned}$$

(3.3) $$B(X,Y)-B(Y,X)=m\{\theta(Y)u(X)-\theta(X)u(Y)\}$$

where T is the torsion tensor with respect to ∇ and η is a 1-form such that

$\eta(X) = \overline{g}(X, N)$.

Proposition 1: Let M be a lightlike hypersurface of Sasakian manifold $\overline{M}$ with an (*l*,*m*)-type connection such that ζ is tangent to M. Then if $m = 0$, then B is symmetric and conversely if B is symmetric then $m = 0$.
Proof: If $m = 0$, then B is symmetric by (3.3). Conversely, if B is

symmetric,then replacing X by ζ and Y by U,we get m = 0.

As B(X,Y) = $\bar{g}(\bar{\nabla}_X Y, \xi)$,so B is independent of the choice of S(TM) and satisfies

(3.4) B(X, ξ) = 0, B (ξ , X) = 0.

Local second fundamental forms are related to their shape operators by

(3.5) B(X,Y) = g(A^*_ξ X, Y) + $mu(X)\theta(Y)$,

(3.6) C(X, PY) = g(A_N X, PY) + $\{l\eta(X) + mu(X)\}\theta(PY)$,

(3.7) $\bar{g}$ (A^*_ξ X, N) = 0, $\bar{g}$ (A_N X,N) = 0 , $\sigma = \tau$.

S(TM) is non-degenerate,so using $(3.4)_2$, (3.5), we have

(3.8) $A^*_\xi \xi = 0,$ $\bar{\nabla}_X \xi = -A^*_\xi X - \tau(X)\xi.$.

Taking $\bar{\nabla}_X$ to $\bar{g}(\zeta, \xi) = 0$ and $\bar{g}(\zeta, N) = 0$ and using (1.1),(2.3),(2.5),(3.5),(3.6) and (3.8), we have

(3.9) $g(A^*_\xi X, \zeta) = -u(X),$ $B(X, \zeta) = (m-1)u(X).$

(3.10)

$$g\left(A_N X, \zeta\right) = -v\left(X\right), \; C(X, \zeta) = l\eta(X) + (m-1)v(X)\}.$$

By (2.9),(2.3) and (2.4), we have

(3.11) $\nabla_X \zeta = (m-1)FX + lX.$

Applying $\bar{\nabla}_X$ to (2.8) and (2.9) and using (2.2), (2.4), (2.5), (2.9), (2.10), (3.1), (3.6), (3.8) with $\theta(U) = \theta(V) = 0,$ we have

(3.12) $B(X,U) = C(X,V)$,

(3.13) $\nabla_X U = F\,(A_N X) + \tau\left(X\right)U - \eta(X)\zeta,$

(3.14) $\nabla_X V = F\,(A^*_\xi X) - \tau\left(X\right)V$,

(3.15)
$$\begin{aligned}(\nabla_X F)Y = {} & u\left(Y\right)A_N X - B\left(X, Y\right)U \\ & + \{g(X,Y) - m\theta(X)\theta(Y)\}\zeta \\ & + (m-1)\theta(Y)X - l\theta(Y)FX,\end{aligned}$$

(3.16) $(\nabla_X u)Y = -u\left(Y\right)\tau(X) - B\left(X, FY\right) - l\theta(Y)u(X),$

$$(3.17)\quad (\nabla_X \nu)Y = \nu(Y)\tau(X) - g(A_N X, \mathrm{F}Y) - l\theta(Y)\nu(X) + (\mathrm{m}-1)\theta(Y)\eta(X).$$

4. Totally Umbilical distribution:

Let R and R^* be the curvature tensor of induced connection ∇ and ∇^* on M and S(TM).By Gauss-Weingarten formula, the Gauss equations for M and S(TM) are

$$(4.1)\quad \begin{aligned} \overline{R}(X,Y)Z = {} & R(X,Y)Z + B(X,Z)A_N \mathrm{Y} - B(Y,Z)A_N X \\ & + \{(\nabla_X B)(Y,Z) - (\nabla_Y B)(X,Z) \\ & + \tau(X)B(Y,Z) - \tau(Y)B(X,Z) \\ & - l[\theta(X)B(Y,Z) - \theta(Y)B(X,Z)] \\ & - m[\theta(X)B(\mathrm{F}Y,Z) - \theta(Y)B(\mathrm{FX},Z)]\}\mathrm{N}, \end{aligned}$$

$$(4.2)\quad \begin{aligned} R(X,Y)\mathrm{P}Z = {} & R^*(X,Y)\mathrm{P}Z + C(X,\mathrm{PZ})A^*_\xi \mathrm{Y} - C(Y,\mathrm{P}Z)A^*_\xi X \\ & + \{(\nabla_X C)(Y,Z) - (\nabla_Y C)(X,\mathrm{P}Z) \\ & - \tau(X)C(Y,\mathrm{P}Z) + \tau(Y)\mathrm{C}(X,\mathrm{P}Z) \\ & - l[\theta(X)C(Y,\mathrm{P}Z) - \theta(Y)C(X,\mathrm{P}Z)] \\ & - m[\theta(X)C(\mathrm{F}Y,\mathrm{P}Z) - \theta(Y)C(\mathrm{FX},\mathrm{P}Z)]\}\xi. \end{aligned}$$

Definition: On a Sasakian manifold $\overline{M}$ is called Sasakian space form of constant holomorphic sectional curvature c, if we have

(4.3)

$$\vec{R}(\bar{X},\bar{Y})\bar{Z} = \frac{(c+3)}{4}\{\bar{g}(\bar{Y},\bar{Z})\bar{X} - \bar{g}(\bar{X},\bar{Z})\bar{Y}\} + \frac{(c-1)}{4}\{\bar{g}(\bar{X},J\bar{Z})J\bar{Y} - \bar{g}(\bar{Y},J\bar{Z})J\bar{X} + 2\bar{g}(\bar{X},J\bar{Y})J\bar{Z} + \theta(\bar{X})\theta(\bar{Z})\bar{Y} - \theta(\bar{Y})\theta(\bar{Z})\bar{X} + \bar{g}(\bar{X},\bar{Z})\theta(\bar{Y})\zeta - \bar{g}(\bar{Y},\bar{Z})\theta(\bar{X})\zeta,$$

where $\vec{R}$ is the curvature tensor of the (l, m)-type connection $\bar{\nabla}$ on $\bar{M}$.

By (1.2), (1.3) and (2.2), we have

$$\bar{R}(\bar{X},\bar{Y})\bar{Z} = \vec{R}(\bar{X},\bar{Y})\bar{Z} + (\nabla_{\bar{X}}\theta)(\bar{Z})\{l(\bar{Y} + mJ\bar{Y}\} - (\nabla_{\bar{Y}}\theta)(\bar{Z})\{l(\bar{X} + mJ\bar{X}\} + \theta(\bar{Z})\{(\bar{X}l)\bar{Y} - (\bar{Y}l)\bar{X} + (\bar{X}m)J\bar{Y}) - (\bar{Y}m)J\bar{X}) - \mathrm{m}[\theta(\bar{Y})\bar{X} - \theta(\bar{X})\bar{Y}]\}. \tag{4.4}$$

Taking the scalar product with ξ and N and then substituting (4.1), (3.2) and (4.3) and using (4.2) and (3.7), we have

(4.5)

$$\nabla_X B(Y,Z) - \nabla_Y B(X,Z) + \{\tau(\mathrm{X}) - 1\theta(\mathrm{X})\}B(Y,Z) - \{\tau(Y) - 1\theta(Y)\}B(X,Z) - m\{\theta(\mathrm{X})\,\mathrm{B(FY,Z)} - \theta(Y)\,\mathrm{B(F}X,\mathrm{Z)}\} - \mathrm{m}\{(\bar{\nabla}_X\theta)(Z)u(Y) - (\bar{\nabla}_Y\theta)(Z)u(X) - \theta(Z)\{[\mathrm{Xm} + \mathrm{m}\,\theta(\mathrm{X})]\mathrm{u}(Y) - [Ym + m\theta(Y)]\mathrm{u(X)}\} = \frac{(c-1)}{4}\{u(Y)\bar{g}(X,\mathrm{JZ}) - u(X)\bar{g}(Y,\mathrm{JZ}) + 2u(\mathrm{Z})\bar{\mathrm{g}}(X,JY)\},$$

(4.6)

$$
\begin{aligned}
(\nabla_X C)(Y, PZ) - (\nabla_Y C)(X, PZ) - \{\tau(X) + l\theta(X)\}C(Y, PZ) \\
+ \{\tau(Y) + l\theta(Y)\}C(X, PZ) \\
- m\{\theta(X)C(FY, PZ) - \theta(Y)C(FX, PZ)\} \\
- (\overline{\nabla}_X\theta)(PZ)\{l\eta(Y) + m\nu(Y)\} + (\overline{\nabla}_Y\theta)(PZ)\{l\eta(X) + m\nu(X)\} \\
- \theta(PZ)\{(Xl + m\theta(X))\eta(Y) - (Yl + m\theta(Y))\eta(X) \\
+ (Xm)\nu(Y) - (Ym)\nu(X)\} \\
= \frac{(c+3)}{4}\{g(Y, PZ)\eta(X) - g(X, PZ)\eta(Y)\} \\
+ \frac{(c-1)}{4}\{\nu(Y)\overline{g}(X, JPZ) - \nu(X)\overline{g}(Y, JPZ) \\
+ 2\nu(PZ)\overline{g}(X, JY) + \theta(X)\eta(Y)\theta(PZ) - \theta(Y)\eta(X)\theta(PZ)\}.
\end{aligned}
$$

Definition: A lightlike hypersurface M is said to be totally umbilical [4] if there exist a smooth function ρ on u such that

(4.7) $$B(X, Y) = \rho g(X, Y).$$

If $\rho = 0$, then M is totally geodesic.

Theorem 1: Let M be a lightlike hypersurface of Sasakian space form $\overline{M}$ (c) with an (*l*, *m*)-type connection such that ζ is tangent to M. Then if M is totally umbilical, then c = 1.

Proof: If M is totally umbilical, then B is symmetric. Hence m = 0 by the Theorem in section 3. Hence (3.9) reduces to

$$\rho\theta(X) = -u(X).$$

Taking ξ in place of X to this equation, we have $\rho = 0$. Hence M is totally geodesic.

Taking $Z = U$ to (4.5) and using B = 0 with $m = 0$, we get

$$\frac{(c-1)}{4}\{u(Y)\eta(X) - u(X)\eta(Y) + 2\overline{g}(X, JY)\} = 0.$$

Taking $X = \xi$ and Y= U to this equation, we get

$$\frac{(c-1)}{4}=0$$

Therefore $c=1$.

Definition: A screen distribution S(TM) is said to be totally umbilical [4] in M if there exist a smooth function γ on u such that

(4.8) $$C(X,PY)=\gamma g(X,Y).$$

If $\gamma=0$, then S(TM) is totally geodesic.

Theorem 2: Let M be a lightlike hypersurface of Sasakian space form $\overline{M}$ (c) with an (*l*, *m*)-type connection such that ζ is tangent to M. Then if screen distribution S(TM) is totally umbilical, then $c=1$..

Proof: If S(TM) is totally umbilical, then (3.10) reduces to

$$\gamma\theta(X)=l\eta(\mathrm{X})+(\mathrm{m}-1)\nu(\mathrm{X}).$$

Taking ζ , ξ and V in place of X one by one to this equation, we have

$\gamma=0,\qquad l=0,\qquad m=1,$

respectively.As $\gamma=0,\ \ C(X,V)=0.$

Applying $\overline{\nabla}_X$ to $\theta(V)=0$ and using (2.4), (3.13) with the fact $\theta oJ=\theta oF=\theta(N)=0,$ we have

(4.9) $$(\overline{\nabla}_X\theta)(V)=0$$

Replacing PZ by V in (4.6) and using (4.9) with $C(X,V)=0,$ we get

$$[\frac{(c+3)}{4}+1]\{u(Y)\eta(X)-u(X)\eta(Y)\}+2\frac{(c-1)}{4}\overline{g}(X,JY)\}=0.$$

Taking $X=\xi$ and Y= U to this equation, we get

$$\frac{(c-1)}{4}=0$$

Therefore $c=1$.

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