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SOME OPERATIONS ON BIPOLAR INTUITIONISTICFUZZY α-IDEALAND BIPOLAR INTUITIONISTIC ANTI FUZZY α-IDEALOF A BP-ALGEBRA

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Abstract: The concept of a bipolar intuitionistic fuzzyα-ideal and bipolar intuitionistic anti fuzzyα-idealare a new algebraicstructure of BP-algebra and to use necessity and possibility operator.The purpose ofthis study is to implement the fuzzy set theory and ideal theory of a BP-algebra. The relation between the operation on bipolar intuitionistic fuzzyα-ideal and bipolar intuitionistic anti fuzzyα-ideal are established.
Keywords: BP-algebra, fuzzy ideal, bipolar fuzzy ideal, bipolar intuitionistic fuzzyα-ideal, bipolar intuitionistic anti fuzzyα-ideal, necessity and possibility operator.

1. INTRODUCTION

The concept of fuzzy sets was initiated by I.A.Zadeh [13] then it has become a vigorous area of research in engineering, medical science, graph theory. S.S.Ahn [2] gave the idea of BP-algebra. Bipolar valued fuzzy sets was introduced by K.J.Lee [6] are an extension of fuzzy sets whose positive membership degree range is enlarged from the interval [0, 1] to [-1, 1]. In a bipolar valued fuzzy set, the membership degree 0 means that the elements are irrelevant to the corresponding property, the positive membership degree (0, 1] indicates that elements somewhat satisfy the property and the negative membership degree [-1, 0) indicates that elements somewhat satisfy the implicit counter property. The author W.R.Zhang [14] commenced the concept of bipolar fuzzy sets as a generalization of fuzzy sets in 1998. K.Chakrabarthy and Biswas R.Nanda [3] investigated note on union and intersection of intuitionistic fuzzy sets. A.Rajeshkumar [12] was analyzed fuzzy groups and level subgroups.M.Palanivelrajan and S.Nandakumar [11] introduced the definition and some operations of intuitionistic fuzzy primary and semiprimary ideal. K.Gunasekaran, S.Nandakumar and S.Sivakaminathan [15] introduced the definition of bipolar intuitionistic fuzzy α-ideal of a BP-algebra.

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2. PRELIMINARIES

Definition: 1

Let A and B be any two bipolar intuitionistic fuzzy set A = $(\mu^P_{\alpha_A}, \mu^N_{\alpha_A}, \nu^P_{\alpha_A}, \nu^N_{\alpha_A})$ and B = $(\mu^P_{\alpha_B}, \mu^N_{\alpha_B}, \nu^P_{\alpha_B}, \nu^N_{\alpha_B})$ in X, we define

(i) A ∩ B ={(x, min$(\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x))$, max$(\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x))$, max$(\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x))$, min$(\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x))$) /x ∈ X}

(ii) A ∪ B ={(x, max$(\mu^P_{\alpha_A}(x), \mu^P_{\alpha_B}(x))$, min$(\mu^N_{\alpha_A}(x), \mu^N_{\alpha_B}(x))$, min($\nu^P_{\alpha_A}(x), \nu^P_{\alpha_B}(x)$), max($\nu^N_{\alpha_A}(x), \nu^N_{\alpha_B}(x)$)) /x ∈ X}

(iii) $\bar{A}$= {(x,$\nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x), \mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x)$) / x ∈ X}.

Definition: 2

A bipolar intuitionistic fuzzy set A = {(x, $\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$)/ x ∈ X}, of BP-algebra X is called a bipolar intuitionistic fuzzy α-ideal of X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \geq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \leq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \geq \min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \leq \max\{\mu^N_{\alpha_A}(x * z), \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \leq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \geq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \leq \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \geq \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

Definition: 3

A bipolar intuitionistic fuzzy set A = {(x, $\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$)/ x ∈ X}, of BP-algebra X is called a bipolar intuitionistic anti fuzzy α-idealof X if it satisfies the following conditions:

(i) $\mu^P_{\alpha_A}(0) \leq \mu^P_{\alpha_A}(x)$ and $\mu^N_{\alpha_A}(0) \geq \mu^N_{\alpha_A}(x)$

(ii) $\mu^P_{\alpha_A}(y * z) \leq \max\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}$

(iii) $\mu^N_{\alpha_A}(y * z) \geq \min\{\mu^N_{\alpha_A}(x * z), \mu^N_{\alpha_A}(x * y)\}$

(iv) $\nu^P_{\alpha_A}(0) \geq \nu^P_{\alpha_A}(x)$ and $\nu^N_{\alpha_A}(0) \leq \nu^N_{\alpha_A}(x)$

(v) $\nu^P_{\alpha_A}(y * z) \geq \min\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$

(vi) $\nu^N_{\alpha_A}(y * z) \leq \max\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$, for all x, y, z ∈ X.

Definition: 4

Let A is a bipolar intuitionistic fuzzy set of X, then the necessity operator □ is defined

by then □A = { (x,$\mu^P_{\alpha_A}(x), \mu^N_{\alpha_A}(x), 1 - \mu^P_{\alpha_A}(x), -1 - \mu^N_{\alpha_A}(x)$) / x ∈ X }.

Definition: 5

Let A is a bipolar intuitionistic fuzzy set of X, then the possibility operator ◊ is defined

bythen ◊A = { (x, $1 - \nu^P_{\alpha_A}(x), -1 - \nu^N_{\alpha_A}(x), \nu^P_{\alpha_A}(x), \nu^N_{\alpha_A}(x)$) / x ∈ X}.

3. OPERATIONS ON BIPOLAR INTUITIONISTIC FUZZY $\boldsymbol{\alpha}$-IDEAL

Theorem: 1

If A is a bipolar intuitionistic fuzzy α-ideal of X, then □A is a bipolar intuitionistic

fuzzyα-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now$\mu^P_{\alpha_{\square A}}(0) = \mu^P_{\alpha_A}(0)$

$\geq \mu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{\square A}}(x)$

Therefore $\mu^P_{\alpha_{\square A}}(0) \geq \mu^P_{\alpha_{\square A}}(x)$

Now$\mu^N_{\alpha_{\square A}}(0) = \mu^N_{\alpha_A}(0)$

$\leq \mu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{\square A}}(x)$

Therefore $\mu^N_{\alpha_{\square A}}(0) \le \mu^N_{\alpha_{\square A}}(x)$

(ii) Now$\mu^P_{\alpha_{\square A}}(y*z) = \mu^P_{\alpha_A}(y*z)$

$\ge \min\{\mu^P_{\alpha_A}(x*z), \quad \mu^P_{\alpha_A}(x*y)\}$

$= \min\{\mu^P_{\alpha_{\square A}}(x*z),\ \mu^P_{\alpha_{\square A}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\square A}}(y*z) \ge \min\{\mu^P_{\alpha_{\square A}}(x*z),\ \mu^P_{\alpha_{\square A}}(x*y)\}$

(iii) Now$\mu^N_{\alpha_{\square A}}(y*z) = \mu^N_{\alpha_A}(y*z)$

$\le \max\{\mu^N_{\alpha_A}(x*z), \quad \mu^N_{\alpha_A}(x*y)\}$

$= \max\{\mu^N_{\alpha_{\square A}}(x*z),\ \mu^N_{\alpha_{\square A}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\square A}}(y*z) \le \max\{\mu^N_{\alpha_{\square A}}(x*z),\ \mu^N_{\alpha_{\square A}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\square A}}(0) = 1 - \mu^P_{\alpha_A}(0)$

$\le 1 - \mu^P_{\alpha_A}(x)$

$=\nu^P_{\alpha_{\square A}}(x)$

Therefore $\nu^P_{\alpha_{\square A}}(0) \le \nu^P_{\alpha_{\square A}}(x)$

Now$\nu^N_{\alpha_{\square A}}(0) = -1 - \mu^N_{\alpha_A}(0)$

$\ge -1 - \mu^N_{\alpha_A}(x)$

$=\nu^N_{\alpha_{\square A}}(x)$

Therefore$\nu^N_{\alpha_{\square A}}(0) \ge \nu^N_{\alpha_{\square A}}(x)$

(v) Now $\nu^P_{\alpha_{\square A}}(y*z) = 1 - \mu^P_{\alpha_A}(y*z)$

$\le \max\{1 - \mu^P_{\alpha_A}(x*z), \quad 1 - \mu^P_{\alpha_A}(x*y)\}$

$= \max\{\nu^P_{\alpha_{\square A}}(x*z),\ \nu^P_{\alpha_{\square A}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\square A}}(y*z) \le \max\{\nu^P_{\alpha_{\square A}}(x*z),\ \nu^P_{\alpha_{\square A}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\square A}}(y*z) = -1 - \mu^N_{\alpha_A}(y*z)$

$\ge \min\{-1 - \mu^N_{\alpha_A}(x*z), -1 - \mu^N_{\alpha_A}(x*y)\}$

$= \min\{\nu^N_{\alpha_{\square A}}(x*z),\ \nu^N_{\alpha_{\square A}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\square A}}(y*z) \ge \min\{\nu^N_{\alpha_{\square A}}(x*z),\ \nu^N_{\alpha_{\square A}}(x*y)\}$

Therefore□A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 2

If A and B are bipolar intuitionistic fuzzy α-ideal of X, then □ (A ∩ B) = □A∩□B
is also a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A ∩ B then 0, x, y, z∈A and 0, x, y, z∈B.

(i) Now $\mu^P_{\alpha_{\square(A\cap B)}}(0) = \mu^P_{\alpha_{A\cap B}}(0)$

$=\min\{\mu^P_{\alpha_A}(0),\ \mu^P_{\alpha_B}(0)\}$

$\ge\min\{\mu^P_{\alpha_A}(x),\ \mu^P_{\alpha_B}(x)\}$

$= \min\{\mu^P_{\alpha_{\square A}}(x),\ \mu^P_{\alpha_{\square B}}(x)\}$

$= \mu^P_{\alpha_{\square A\cap\square B}}(x)$

Therefore $\mu^P_{\alpha_{\square(A\cap B)}}(0) \ \ge \mu^P_{\alpha_{\square A\cap\square B}}(x)$

Now $\mu^N_{\alpha_{\square(A\cap B)}}(0) = \mu^N_{\alpha_{A\cap B}}(0)$

$= \max\{\mu^N_{\alpha_A}(0),\ \mu^N_{\alpha_B}(0)\}$

$\le\max\{\mu^N_{\alpha_A}(x),\ \mu^N_{\alpha_B}(x)\}$

$= \max\{\mu^N_{\alpha_{\square A}}(x),\ \mu^N_{\alpha_{\square B}}(x)\}$

$= \mu^N_{\alpha_{\square A\cap\square B}}(x)$

Therefore $\mu^N_{\alpha_{\square(A\cap B)}}(0) \le \mu^N_{\alpha_{\square A\cap\square B}}(x)$

(ii) Now $\mu^P_{\alpha_{\square(A\cap B)}}(y*z) = \mu^P_{\alpha_{A\cap B}}(y*z)$

$=\min\{\mu^P_{\alpha_A}(y*z),\ \mu^P_{\alpha_B}(y*z)\}$

$\ge\min \quad \{ \quad \min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_A}(x*y)\},\ \min\{\mu^P_{\alpha_B}(x*$
$z), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\min\{\mu^P_{\alpha_A}(x*z), \mu^P_{\alpha_B}(x*z)\}, \min\{\mu^P_{\alpha_A}(x*y), \mu^P_{\alpha_B}(x*y)\}\}$

$= \min\{\min\{\mu^P_{\alpha_{\Box A}}(x*z), \mu^P_{\alpha_{\Box B}}(x*z)\}, \min\{\mu^P_{\alpha_{\Box A}}(x*y), \mu^P_{\alpha_{\Box B}}(x*y)\}\}$

$= \min\{\mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\Box(A\cap B)}}(y*z) \geq \min\{\mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\Box(A\cap B)}}(y*z) = \mu^N_{\alpha_{A\cap B}}(y*z)$

$= \max\{\mu^N_{\alpha_A}(y*z), \mu^N_{\alpha_B}(y*z)\}$

$\leq \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_A}(x*y)\}, \max\{\mu^N_{\alpha_B}(x*z), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\mu^N_{\alpha_A}(x*z), \mu^N_{\alpha_B}(x*z)\}, \max\{\mu^N_{\alpha_A}(x*y), \mu^N_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\mu^N_{\alpha_{\Box A}}(x*z), \mu^N_{\alpha_{\Box B}}(x*z)\}, \max\{\mu^N_{\alpha_{\Box A}}(x*y), \mu^N_{\alpha_{\Box B}}(x*y)\}\}$

$= \max\{\mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\Box(A\cap B)}}(y*z) \leq \max\{\mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\Box(A\cap B)}}(0) = 1 - \mu^P_{\alpha_{A\cap B}}(0)$

$= 1 - \mu^P_{\alpha_{\Box(A\cap B)}}(0)$

$\leq 1 - \mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

$= \nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

Therefore $\nu^P_{\alpha_{\Box(A\cap B)}}(0) \leq \nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

Now $\nu^N_{\alpha_{\Box(A\cap B)}}(0) = -1 - \mu^N_{\alpha_{A\cap B}}(0)$

$= -1 - \mu^N_{\alpha_{\Box(A\cap B)}}(0)$

$\geq -1 - \mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

$= \nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

Therefore $\nu^N_{\alpha_{\Box(A\cap B)}}(0) \geq \nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x)$

(v) Now $\nu^P_{\alpha_{\Box(A\cap B)}}(y*z) = 1 - \mu^P_{\alpha_{A\cap B}}(y*z)$

$= 1 - \mu^P_{\alpha_{\Box(A\cap B)}}(y*z)$

$\leq \max\{1 - \mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), 1 - \mu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

$= \max\{\nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\Box(A\cap B)}}(y*z) \leq \max\{\nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \nu^P_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\Box(A\cap B)}}(y*z) = -1 - \mu^N_{\alpha_{A\cap B}}(y*z)$

$= -1 - \mu^N_{\alpha_{\Box(A\cap B)}}(y*z)$

$\geq \min\{-1 - \mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), -1 - \mu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

$= \min\{\nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\Box(A\cap B)}}(y*z) \geq \min\{\nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*z), \nu^N_{\alpha_{\Box A\,\cap\,\Box B}}(x*y)\}$

Therefore $\Box(A \cap B) = \Box A \cap \Box B$ is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 3

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\lozenge$A is a bipolar intuitionistic fuzzyα-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now$\mu^P_{\alpha_{\Diamond A}}(0) = 1 - \nu^P_{\alpha_A}(0)$

$\geq 1 - \nu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{\Diamond A}}(x)$

Therefore $\mu^P_{\alpha_{\Diamond A}}(0) \geq \mu^P_{\alpha_{\Diamond A}}(x)$

Now$\mu^N_{\alpha_{\Diamond A}}(0) = -1 - \nu^N_{\alpha_A}(0)$

$\leq -1 - \nu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{\Diamond A}}(x)$

Therefore $\mu^N_{\alpha_{\Diamond A}}(0) \leq \mu^N_{\alpha_{\Diamond A}}(x)$

(ii) Now$\mu^P_{\alpha_{\Diamond A}}(y * z) = 1 - \nu^P_{\alpha_A}(y * z)$

$\geq$min $\{1 - \nu^P_{\alpha_A}(x * z), 1 - \nu^P_{\alpha_A}(x * y)\}$

= min $\{\mu^P_{\alpha_{\Diamond A}}(x * z), \mu^P_{\alpha_{\Diamond A}}(x * y)\}$

Therefore $\mu^P_{\alpha_{\Diamond A}}(y * z) \geq$min $\{\mu^P_{\alpha_{\Diamond A}}(x * z), \mu^P_{\alpha_{\Diamond A}}(x * y)\}$

(iii) Now $\mu^N_{\alpha_{\Diamond A}}(y * z) = -1 - \nu^N_{\alpha_A}(y * z)$

$\leq$max $\{-1 - \nu^N_{\alpha_A}(x * z), -1 - \nu^N_{\alpha_A}(x * y)\}$

= max $\{\mu^N_{\alpha_{\Diamond A}}(x * z), \mu^N_{\alpha_{\Diamond A}}(x * y)\}$

Therefore $\mu^N_{\alpha_{\Diamond A}}(y * z) \leq$max $\{\mu^N_{\alpha_{\Diamond A}}(x * z), \mu^N_{\alpha_{\Diamond A}}(x * y)\}$

(iv) Now$\nu^P_{\alpha_{\Diamond A}}(0) = \nu^P_{\alpha_A}(0)$

$\leq \nu^P_{\alpha_A}(x)$

$= \nu^P_{\alpha_{\Diamond A}}(x)$

Therefore $\nu^P_{\alpha_{\Diamond A}}(0) \leq \nu^P_{\alpha_{\Diamond A}}(x)$

Now$\nu^N_{\alpha_{\Diamond A}}(0) = \nu^N_{\alpha_A}(0)$

$\geq \nu^N_{\alpha_A}(x)$

$= \nu^N_{\alpha_{\Diamond A}}(x)$

Therefore $\nu^N_{\alpha_{\Diamond A}}(0) \geq \nu^N_{\alpha_{\Diamond A}}(x)$

(v) Now$\nu^P_{\alpha_{\Diamond A}}(y * z) = \nu^P_{\alpha_A}(y * z)$

$\leq \max\{\nu^P_{\alpha_A}(x * z), \quad \nu^P_{\alpha_A}(x * y)\}$

$= \max\{\nu^P_{\alpha_{\Diamond A}}(x * z), \ \nu^P_{\alpha_{\Diamond A}}(x * y)\}$

Therefore $\nu^P_{\alpha_{\Diamond A}}(y * z) \leq \max\{\nu^P_{\alpha_{\Diamond A}}(x * z), \ \nu^P_{\alpha_{\Diamond A}}(x * y)\}$

(vi) Now$\nu^N_{\alpha_{\Diamond A}}(y * z) = \nu^N_{\alpha_A}(y * z)$

$\geq \min\{\nu^N_{\alpha_A}(x * z), \quad \nu^N_{\alpha_A}(x * y)\}$

$= \min\{\nu^N_{\alpha_{\Diamond A}}(x * z), \ \nu^N_{\alpha_{\Diamond A}}(x * y)\}$

Therefore $\nu^N_{\alpha_{\Diamond A}}(y * z) \geq \min\{\nu^N_{\alpha_{\Diamond A}}(x * z), \ \nu^N_{\alpha_{\Diamond A}}(x * y)\}$

Therefore◊A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 4

If A and B are bipolar intuitionistic fuzzy α-ideal of X, then ◊ (A ∩ B) = ◊A∩◊B
is also a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Let A and B are bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A ∩ B then 0, x, y, z∈A and 0, x, y, z∈B.

(i) Now $\mu^P_{\alpha_{\Diamond(A\cap B)}}(0) = 1 - \nu^P_{\alpha_{A\cap B}}(0)$

= 1- max $\{\nu^P_{\alpha_A}(0), \ \nu^P_{\alpha_B}(0)\}$

$\geq$ 1- max $\{\nu^P_{\alpha_A}(x), \ \nu^P_{\alpha_B}(x)\}$

= 1- max $\{\nu^P_{\alpha_{\Diamond A}}(x), \ \nu^P_{\alpha_{\Diamond B}}(x)\}$

= 1- $\nu^P_{\alpha_{\Diamond A \cap \Diamond B}}(x)$

$= \mu^P_{\alpha_{\Diamond A \cap \Diamond B}}(x)$

Therefore $\mu^P_{\alpha_{\Diamond(A\cap B)}}(0) \geq \mu^P_{\alpha_{\Diamond A \cap \Diamond B}}(x)$

Now $\mu^N_{\alpha_{\Diamond(A\cap B)}}(0) = -1 - \nu^N_{\alpha_{A\cap B}}(0)$

$= -1- \min\{\nu^N_{\alpha_A}(0),\ \nu^N_{\alpha_B}(0)\}$

$\leq -1- \min\{\nu^N_{\alpha_A}(x),\ \nu^N_{\alpha_B}(x)\}$

$= -1- \min\{\nu^N_{\alpha_{\Diamond A}}(x),\ \nu^N_{\alpha_{\Diamond B}}(x)\}$

$= -1- \nu^N_{\alpha_{\Diamond A\ \cap \Diamond B}}(x)$

$= \mu^N_{\alpha_{\Diamond A\ \cap \Diamond B}}(x)$

Therefore $\mu^N_{\alpha_{\Diamond(A\cap B)}}(0) \leq \mu^N_{\alpha_{\Diamond A\ \cap \Diamond B}}(x)$

(ii) Now$\mu^P_{\alpha_{\Diamond(A\cap B)}}(y*z) = 1-\nu^P_{\alpha_{A\cap B}}(y*z)$

$= 1- \max\{\square_{\square_A}(\square*\square),\ \square_{\square_B}(\square*\square)\}$

$\geq 1- \max\{\min\{\square_{\square_A}(\square*\square),\ \square_{\square_A}(\square*\square)\}, \min\{\square_{\square_B}(\square*\square),\ \square_{\square_B}(\square*\square)\}\}$

$= 1- \min\{\max\{\square_{\square_A}(\square*\square),\ \square_{\square_B}(\square*\square)\}, \max\{\square_{\square_A}(\square*\square),\ \square_{\square_B}(\square*\square)\}\}$

$= 1- \min\{\max\{\square_{\square_{\Diamond A}}(\square*\square),\ \square_{\square_{\Diamond B}}(\square*\square)\}, \max\{\square_{\square_{\Diamond A}}(\square*\square),\ \square_{\square_{\Diamond B}}(\square*\square)\}\}$

$= 1- \min\{\square_{\square_{\Diamond A\ \cap\Diamond B}}(\square*\square), \square_{\square_{\Diamond A\ \cap\Diamond B}}(\square*\square)\}$

$= \min\{1-\square_{\square_{\Diamond A\ \cap\Diamond B}}(\square*\square), 1-\square_{\square_{\Diamond A\ \cap\Diamond B}}(\square*\square)\}$

$= \min\{\square_{\square_{\Diamond A\ \cap\ \Diamond B}}(\square*\square), \square_{\square_{\Diamond A\ \cap\ \Diamond B}}(\square*\square)\}$

Therefore $\mu^P_{\alpha_{\Diamond(A\cap B)}}(y*z) \geq \min\{\mu^P_{\alpha_{\Diamond A\ \cap\ \Diamond B}}(x*z), \mu^P_{\alpha_{\Diamond A\ \cap\ \Diamond B}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\Diamond(A\cap B)}}(y*z) = -1-\nu^N_{\alpha_{A\cap B}}(y*z)$

$= -1- \min\{\nu^N_{\alpha_A}(y*z),\ \nu^N_{\alpha_B}(y*z)\}$

$\leq -1- \min\{\max\{\nu^N_{\alpha_A}(x*z),\ \nu^N_{\alpha_A}(x*y)\}, \max\{\nu^N_{\alpha_B}(x*z), \nu^N_{\alpha_B}(x*y)\}\}$

$= -1- \max\{\min\{\nu^N_{\alpha_A}(x*z), \nu^N_{\alpha_B}(x*z)\}, \min\{\nu^N_{\alpha_A}(x*y), \nu^N_{\alpha_B}(x*y)\}\}$

$= -1- \max\{\min\{\nu^N_{\alpha_{\Diamond A}}(x*z), \nu^N_{\alpha_{\Diamond B}}(x*z)\}, \min\{\nu^N_{\alpha_{\Diamond A}}(x*y),\ \nu^N_{\alpha_{\Diamond B}}(x*y)\}\}$

$= -1- \max\{\nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), \nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

$= \max\{-1-\nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), -1-\nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

$= \max\{\mu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), \mu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\Diamond(A\cap B)}}(y*z) \leq \max\{\mu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), \mu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\Diamond(A\cap B)}}(0) = \nu^P_{\alpha_{A\cap B}}(0)$

$= \max\{\nu^P_{\alpha_A}(0),\ \nu^P_{\alpha_B}(0)\}$

$\leq \max\{\nu^P_{\alpha_A}(x),\ \nu^P_{\alpha_B}(x)\}$

$= \max\{\nu^P_{\alpha_{\Diamond A}}(x),\ \nu^P_{\alpha_{\Diamond B}}(x)\}$

$= \nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x)$

Therefore $\nu^P_{\alpha_{\Diamond(A\cap B)}}(0) \leq \nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x)$

Now $\nu^N_{\alpha_{\Diamond(A\cap B)}}(0) = \nu^N_{\alpha_{A\cap B}}(0)$

$= \min\{\nu^N_{\alpha_A}(0),\ \nu^N_{\alpha_B}(0)\}$

$\geq \min\{\nu^N_{\alpha_A}(x),\ \nu^N_{\alpha_B}(x)\}$

$= \min\{\nu^N_{\alpha_{\Diamond A}}(x),\ \nu^N_{\alpha_{\Diamond B}}(x)\}$

$= \nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x)$

Therefore $\nu^N_{\alpha_{\Diamond(A\cap B)}}(0) \geq \nu^N_{\alpha_{\Diamond A\ \cap\Diamond B}}(x)$

(v) Now $\nu^P_{\alpha_{\Diamond(A\cap B)}}(y*z) = \nu^P_{\alpha_{A\cap B}}(y*z)$

$= \max\{\nu^P_{\alpha_A}(y*z),\ \nu^P_{\alpha_B}(y*z)\}$

$\leq \max\{\max\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_A}(x*y)\}, \max\{\nu^P_{\alpha_B}(x*z), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\nu^P_{\alpha_A}(x*z),\ \nu^P_{\alpha_B}(x*z)\}, \max\{\nu^P_{\alpha_A}(x*y), \nu^P_{\alpha_B}(x*y)\}\}$

$= \max\{\max\{\nu^P_{\alpha_{\Diamond A}}(x*z),\ \nu^P_{\alpha_{\Diamond B}}(x*z)\}, \max\{\nu^P_{\alpha_{\Diamond A}}(x*y),\ \nu^P_{\alpha_{\Diamond B}}(x*y)\}\}$

$= \max\{\nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), \nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\Diamond(A\cap B)}}(y*z) \leq \max\{\nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*z), \nu^P_{\alpha_{\Diamond A\ \cap\Diamond B}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\lozenge(\mathrm{A}\cap\mathrm{B})}}(y*z) = \nu^N_{\alpha_{\mathrm{A}\cap\mathrm{B}}}(y*z)$

$=\min\{\nu^N_{\alpha_\mathrm{A}}(y*z),\ \nu^N_{\alpha_\mathrm{B}}(y*z)\}$

$\geq \min\{\min\{\nu^N_{\alpha_\mathrm{A}}(x*z),\ \nu^N_{\alpha_\mathrm{A}}(x*y)\}, \min\{\nu^N_{\alpha_\mathrm{B}}(x*z),\ \nu^N_{\alpha_\mathrm{B}}(x*y)\}\}$

$= \min\{\min\{\nu^N_{\alpha_\mathrm{A}}(x*z),\ \nu^N_{\alpha_\mathrm{B}}(x*z)\}, \min\{\nu^N_{\alpha_\mathrm{A}}(x*y),\ \nu^N_{\alpha_\mathrm{B}}(x*y)\}\}$

$= \min\{\min\{\nu^N_{\alpha_{\lozenge\mathrm{A}}}(x*z),\ \nu^N_{\alpha_{\lozenge\mathrm{B}}}(x*z)\}, \min\{\nu^N_{\alpha_{\lozenge\mathrm{A}}}(x*y),\ \nu^N_{\alpha_{\lozenge\mathrm{B}}}(x*y)\}\}$

$= \min\{\nu^N_{\alpha_{\lozenge\mathrm{A}\cap\lozenge\mathrm{B}}}(x*z),\ \nu^N_{\alpha_{\lozenge\mathrm{A}\cap\lozenge\mathrm{B}}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\lozenge(\mathrm{A}\cap\mathrm{B})}}(y*z) \geq \min\{\nu^N_{\alpha_{\lozenge\mathrm{A}\cap\lozenge\mathrm{B}}}(x*z),\ \nu^N_{\alpha_{\lozenge\mathrm{A}\cap\lozenge\mathrm{B}}}(x*y)\}$

Therefore $\lozenge(\mathrm{A}\cap\mathrm{B}) = \lozenge\mathrm{A}\cap\lozenge\mathrm{B}$ is also a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 5

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\Box\Box\mathrm{A} = \Box\mathrm{A}$ is a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now $\mu^P_{\alpha_{\Box\Box\mathrm{A}}}(0) = \mu^P_{\alpha_{\Box\mathrm{A}}}(0)$

$=\mu^P_{\alpha_\mathrm{A}}(0)$

$\geq \mu^P_{\alpha_\mathrm{A}}(x)$

$= \mu^P_{\alpha_{\Box\mathrm{A}}}(x)$

Therefore $\mu^P_{\alpha_{\Box\Box\mathrm{A}}}(0) \geq \mu^P_{\alpha_{\Box\mathrm{A}}}(x)$

Now $\mu^N_{\alpha_{\Box\Box\mathrm{A}}}(0) = \mu^N_{\alpha_{\Box\mathrm{A}}}(0)$

$= \mu^N_{\alpha_\mathrm{A}}(0)$

$\leq \mu^N_{\alpha_\mathrm{A}}(x)$

$= \mu^N_{\alpha_{\Box\mathrm{A}}}(x)$

Therefore $\mu^N_{\alpha_{\Box\Box\mathrm{A}}}(0) \leq \mu^N_{\alpha_{\Box\mathrm{A}}}(x)$

(ii) Now $\mu^P_{\alpha_{\Box\Box\mathrm{A}}}(y*z) = \mu^P_{\alpha_{\Box\mathrm{A}}}(y*z)$

$=\mu^P_{\alpha_\mathrm{A}}(y*z)$

$\geq \min\{\mu^P_{\alpha_\mathrm{A}}(x*z), \mu^P_{\alpha_\mathrm{A}}(x*y)\}$

$= \min\{\mu^P_{\alpha_{\Box\mathrm{A}}}(x*z), \mu^P_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\Box\Box\mathrm{A}}}(y*z) \geq \min\{\mu^P_{\alpha_{\Box\mathrm{A}}}(x*z), \mu^P_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\Box\Box\mathrm{A}}}(y*z) = \mu^N_{\alpha_{\Box\mathrm{A}}}(y*z)$

$= \mu^N_{\alpha_\mathrm{A}}(y*z)$

$\leq \max\{\mu^N_{\alpha_\mathrm{A}}(x*z), \mu^N_{\alpha_\mathrm{A}}(x*y)\}$

$= \max\{\mu^N_{\alpha_{\Box\mathrm{A}}}(x*z), \mu^N_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\Box\Box\mathrm{A}}}(y*z) \leq \max\{\mu^N_{\alpha_{\Box\mathrm{A}}}(x*z), \mu^N_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\Box\Box\mathrm{A}}}(0) = 1 - \mu^P_{\alpha_{\Box\mathrm{A}}}(0)$

$= 1 - \mu^P_{\alpha_\mathrm{A}}(0)$

$\leq 1 - \mu^P_{\alpha_\mathrm{A}}(x)$

$=\nu^P_{\alpha_{\Box\mathrm{A}}}(x)$

Therefore $\nu^P_{\alpha_{\Box\Box\mathrm{A}}}(0) \leq \nu^P_{\alpha_{\Box\mathrm{A}}}(x)$

Now $\nu^N_{\alpha_{\Box\Box\mathrm{A}}}(0) = -1 - \mu^N_{\alpha_{\Box\mathrm{A}}}(0)$

$= -1 - \mu^N_{\alpha_\mathrm{A}}(0)$

$\geq -1 - \mu^N_{\alpha_\mathrm{A}}(x)$

$=\nu^N_{\alpha_{\Box\mathrm{A}}}(x)$

Therefore $\nu^N_{\alpha_{\Box\Box\mathrm{A}}}(0) \geq \nu^N_{\alpha_{\Box\mathrm{A}}}(x)$

(v) Now $\nu^P_{\alpha_{\Box\Box\mathrm{A}}}(y*z) = 1 - \mu^P_{\alpha_{\Box\mathrm{A}}}(y*z)$

$= 1 - \mu^P_{\alpha_\mathrm{A}}(y*z)$

$\leq$max$\{1-\mu^P_{\alpha_{\mathrm{A}}}(x*z), 1-\mu^P_{\alpha_{\mathrm{A}}}(x*y)\}$

$= \max\{\nu^P_{\alpha_{\Box\mathrm{A}}}(x*z),\ \nu^P_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\Box\Box\mathrm{A}}}(y*z) \leq$max$\{\nu^P_{\alpha_{\Box\mathrm{A}}}(x*z),\ \nu^P_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\Box\Box\mathrm{A}}}(y*z) = -1-\mu^N_{\alpha_{\Box\mathrm{A}}}(y*z)$

$= -1-\mu^N_{\alpha_{\mathrm{A}}}(y*z)$

$\geq$min$\{-1-\mu^N_{\alpha_{\mathrm{A}}}(x*z), -1-\mu^N_{\alpha_{\mathrm{A}}}(x*y)\}$

$= \min\{\nu^N_{\alpha_{\Box\mathrm{A}}}(x*z), \nu^N_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\Box\Box\mathrm{A}}}(y*z) \geq$min$\{\nu^N_{\alpha_{\Box\mathrm{A}}}(x*z), \nu^N_{\alpha_{\Box\mathrm{A}}}(x*y)\}$

Therefore □□A = □A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 6

If A is a bipolar intuitionistic fuzzy α-ideal of X, then □◊A = ◊A is a bipolar
intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now$\mu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(0) = \mu^P_{\alpha_{\Diamond\mathrm{A}}}(0)$

$= 1-\nu^P_{\alpha_{\mathrm{A}}}(0)$

$\geq 1-\nu^P_{\alpha_{\mathrm{A}}}(x)$

$= \mu^P_{\alpha_{\Diamond\mathrm{A}}}(x)$

Therefore $\mu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(0) \geq \mu^P_{\alpha_{\Diamond\mathrm{A}}}(x)$

Now$\mu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(0) = \mu^N_{\alpha_{\Diamond\mathrm{A}}}(0)$

$= -1-\nu^N_{\alpha_{\mathrm{A}}}(0)$

$\leq -1-\nu^N_{\alpha_{\mathrm{A}}}(x)$

$= \mu^N_{\alpha_{\Diamond\mathrm{A}}}(x)$

Therefore $\mu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(0) \leq \mu^N_{\alpha_{\Diamond\mathrm{A}}}(x)$

(ii) Now$\mu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(y*z) = \mu^P_{\alpha_{\Diamond\mathrm{A}}}(y*z)$

$= 1-\nu^P_{\alpha_{\mathrm{A}}}(y*z)$

$\geq$min$\{1-\nu^P_{\alpha_{\mathrm{A}}}(x*z), 1-\nu^P_{\alpha_{\mathrm{A}}}(x*y)\}$

$= \min\{\mu^P_{\alpha_{\Diamond\mathrm{A}}}(x*z), \mu^P_{\alpha_{\Diamond\mathrm{A}}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(y*z) \geq$min$\{\mu^P_{\alpha_{\Diamond\mathrm{A}}}(x*z), \mu^P_{\alpha_{\Diamond\mathrm{A}}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(y*z) = \mu^N_{\alpha_{\Diamond\mathrm{A}}}(y*z)$

$= -1-\nu^N_{\alpha_{\mathrm{A}}}(y*z)$

$\leq$max$\{-1-\nu^N_{\alpha_{\mathrm{A}}}(x*z), -1-\nu^N_{\alpha_{\mathrm{A}}}(x*y)\}$

$= \max\{\mu^N_{\alpha_{\Diamond\mathrm{A}}}(x*z), \mu^N_{\alpha_{\Diamond\mathrm{A}}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(y*z) \leq$max$\{\mu^N_{\alpha_{\Diamond\mathrm{A}}}(x*z), \mu^N_{\alpha_{\Diamond\mathrm{A}}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(0) = 1-\mu^P_{\alpha_{\Diamond\mathrm{A}}}(0)$

$= 1-[1-\nu^P_{\alpha_{\mathrm{A}}}(0)]$

$= \nu^P_{\alpha_{\mathrm{A}}}(0)$

$\leq \nu^P_{\alpha_{\mathrm{A}}}(x)$

$= \nu^P_{\alpha_{\Diamond\mathrm{A}}}(x)$

Therefore $\nu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(0) \leq \nu^P_{\alpha_{\Diamond\mathrm{A}}}(x)$

Now$\nu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(0) = -1-\mu^N_{\alpha_{\Diamond\mathrm{A}}}(0)$

$= -1-[-1-\nu^N_{\alpha_{\mathrm{A}}}(0)]$

$= \nu^N_{\alpha_{\mathrm{A}}}(0)$

$\geq \nu^N_{\alpha_{\mathrm{A}}}(x)$

$= \nu^N_{\alpha_{\Diamond\mathrm{A}}}(x)$

Therefore $\nu^N_{\alpha_{\Box\Diamond\mathrm{A}}}(0) \geq \nu^N_{\alpha_{\Diamond\mathrm{A}}}(x)$

(v) Now$\nu^P_{\alpha_{\Box\Diamond\mathrm{A}}}(y*z) = 1-\mu^P_{\alpha_{\Diamond\mathrm{A}}}(y*z)$

$= 1 - [1 - \nu^P_{\alpha_A}(y * z)]$
$= \nu^P_{\alpha_A}(y * z)$
$\leq \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$
$= \max\{\nu^P_{\alpha_{\Diamond A}}(x * z), \nu^P_{\alpha_{\Diamond A}}(x * y)\}$
Therefore $\nu^P_{\alpha_{\Box\Diamond A}}(y * z) \leq \max\{\nu^P_{\alpha_{\Diamond A}}(x * z), \nu^P_{\alpha_{\Diamond A}}(x * y)\}$
(vi) Now $\nu^N_{\alpha_{\Box\Diamond A}}(y * z) = -1 - \mu^N_{\alpha_{\Diamond A}}(y * z)$
$= -1 - [-1 - \nu^N_{\alpha_A}(y * z)]$
$= \nu^N_{\alpha_A}(y * z)$
$\geq \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$
$= \min\{\nu^N_{\alpha_{\Diamond A}}(x * z), \nu^N_{\alpha_{\Diamond A}}(x * y)\}$
Therefore $\nu^N_{\alpha_{\Box\Diamond A}}(y * z) \geq \min\{\nu^N_{\alpha_{\Diamond A}}(x * z), \nu^N_{\alpha_{\Diamond A}}(x * y)\}$
Therefore □◊A = ◊A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 7

If A is a bipolar intuitionistic fuzzy α-ideal of X, then ◊□A = □A is a bipolar
intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now $\mu^P_{\alpha_{\Diamond\Box A}}(0) = 1 - \nu^P_{\alpha_{\Box A}}(0)$
$= 1 - [1 - \mu^P_{\alpha_A}(0)]$
$= \mu^P_{\alpha_A}(0)$
$\geq \mu^P_{\alpha_A}(x)$
$= \mu^P_{\alpha_{\Box A}}(x)$
Therefore $\mu^P_{\alpha_{\Diamond\Box A}}(0) \geq \mu^P_{\alpha_{\Box A}}(x)$
Now $\mu^N_{\alpha_{\Diamond\Box A}}(0) = -1 - \nu^N_{\alpha_{\Box A}}(0)$
$= -1 - [-1 - \mu^N_{\alpha_A}(0)]$
$= \mu^N_{\alpha_A}(0)$
$\leq \mu^N_{\alpha_A}(x)$
$= \mu^N_{\alpha_{\Box A}}(x)$
Therefore $\mu^N_{\alpha_{\Diamond\Box A}}(0) \leq \mu^N_{\alpha_{\Box A}}(x)$
(ii) Now $\mu^P_{\alpha_{\Diamond\Box A}}(y * z) = 1 - \nu^P_{\alpha_{\Box A}}(y * z)$
$= 1 - [1 - \mu^P_{\alpha_A}(y * z)]$
$= \mu^P_{\alpha_A}(y * z)$
$\geq \min\{\mu^P_{\alpha_A}(x * z), \mu^P_{\alpha_A}(x * y)\}$
$= \min\{\mu^P_{\alpha_{\Box A}}(x * z), \mu^P_{\alpha_{\Box A}}(x * y)\}$
Therefore $\mu^P_{\alpha_{\Diamond\Box A}}(y * z) \geq \min\{\mu^P_{\alpha_{\Box A}}(x * z), \mu^P_{\alpha_{\Box A}}(x * y)\}$
(iii) Now $\mu^N_{\alpha_{\Diamond\Box A}}(y * z) = -1 - \nu^N_{\alpha_{\Box A}}(y * z)$
$= -1 - [-1 - \mu^N_{\alpha_A}(y * z)]$
$= \mu^N_{\alpha_A}(y * z)$
$\leq \max\{\mu^N_{\alpha_A}(x * z), \mu^N_{\alpha_A}(x * y)\}$
$= \max\{\mu^N_{\alpha_{\Box A}}(x * z), \mu^N_{\alpha_{\Box A}}(x * y)\}$
Therefore $\mu^N_{\alpha_{\Diamond\Box A}}(y * z) \leq \max\{\mu^N_{\alpha_{\Box A}}(x * z), \mu^N_{\alpha_{\Box A}}(x * y)\}$
(iv) Now $\nu^P_{\alpha_{\Diamond\Box A}}(0) = \nu^P_{\alpha_{\Box A}}(0)$
$= 1 - \mu^P_{\alpha_A}(0)$
$\leq 1 - \mu^P_{\alpha_A}(x)$
$= \nu^P_{\alpha_{\Box A}}(x)$
Therefore $\nu^P_{\alpha_{\Diamond\Box A}}(0) \leq \nu^P_{\alpha_{\Box A}}(x)$
Now $\nu^N_{\alpha_{\Diamond\Box A}}(0) = \nu^N_{\alpha_{\Box A}}(0)$

$= -1 - \mu^N_{\alpha_A}(0)$

$\geq -1 - \mu^N_{\alpha_A}(x)$

$= \nu^N_{\alpha_{\Box A}}(x)$

Therefore $\nu^N_{\alpha_{\Diamond\Box A}}(0) \geq \nu^N_{\alpha_{\Box A}}(x)$

(v) Now $\nu^P_{\alpha_{\Diamond\Box A}}(y*z) = \nu^P_{\alpha_{\Box A}}(y*z)$

$= 1 - \mu^P_{\alpha_A}(y*z)$

$\leq \max\{1 - \mu^P_{\alpha_A}(x*z), 1 - \mu^P_{\alpha_A}(x*y)\}$

$= \max\{\nu^P_{\alpha_{\Box A}}(x*z), \nu^P_{\alpha_{\Box A}}(x*y)\}$

Therefore $\nu^P_{\alpha_{\Diamond\Box A}}(y*z) \leq \max\{\nu^P_{\alpha_{\Box A}}(x*z), \nu^P_{\alpha_{\Box A}}(x*y)\}$

(vi) Now $\nu^N_{\alpha_{\Diamond\Box A}}(y*z) = \nu^N_{\alpha_{\Box A}}(y*z)$

$= -1 - \mu^N_{\alpha_A}(y*z)$

$\geq \min\{-1 - \mu^N_{\alpha_A}(x*z), -1 - \mu^N_{\alpha_A}(x*y)\}$

$= \min\{\nu^N_{\alpha_{\Box A}}(x*z), \nu^N_{\alpha_{\Box A}}(x*y)\}$

Therefore $\nu^N_{\alpha_{\Diamond\Box A}}(y*z) \geq \min\{\nu^N_{\alpha_{\Box A}}(x*z), \nu^N_{\alpha_{\Box A}}(x*y)\}$

Therefore ◊□A = □A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 8

If A is a bipolar intuitionistic fuzzy α-ideal of X, then ◊◊A = ◊A is a bipolar intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now $\mu^P_{\alpha_{\Diamond\Diamond A}}(0) = 1 - \nu^P_{\alpha_{\Diamond A}}(0)$

$= 1 - \nu^P_{\alpha_A}(0)$

$\geq 1 - \nu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{\Diamond A}}(x)$

Therefore $\mu^P_{\alpha_{\Diamond\Diamond A}}(0) \geq \mu^P_{\alpha_{\Diamond A}}(x)$

Now $\mu^N_{\alpha_{\Diamond\Diamond A}}(0) = -1 - \nu^N_{\alpha_{\Diamond A}}(0)$

$= -1 - \nu^N_{\alpha_A}(0)$

$\leq -1 - \nu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{\Diamond A}}(x)$

Therefore $\mu^N_{\alpha_{\Diamond\Diamond A}}(0) \leq \mu^N_{\alpha_{\Diamond A}}(x)$

(ii) Now $\mu^P_{\alpha_{\Diamond\Diamond A}}(y*z) = 1 - \nu^P_{\alpha_{\Diamond A}}(y*z)$

$= 1 - \nu^P_{\alpha_A}(y*z)$

$\geq \min\{1 - \nu^P_{\alpha_A}(x*z), 1 - \nu^P_{\alpha_A}(x*y)\}$

$= \min\{\mu^P_{\alpha_{\Diamond A}}(x*z), \mu^P_{\alpha_{\Diamond A}}(x*y)\}$

Therefore $\mu^P_{\alpha_{\Diamond\Diamond A}}(y*z) \geq \min\{\mu^P_{\alpha_{\Diamond A}}(x*z), \mu^P_{\alpha_{\Diamond A}}(x*y)\}$

(iii) Now $\mu^N_{\alpha_{\Diamond\Diamond A}}(y*z) = -1 - \nu^N_{\alpha_{\Diamond A}}(y*z)$

$= -1 - \nu^N_{\alpha_A}(y*z)$

$\leq \max\{-1 - \nu^N_{\alpha_A}(x*z), -1 - \nu^N_{\alpha_A}(x*y)\}$

$= \max\{\mu^N_{\alpha_{\Diamond A}}(x*z), \mu^N_{\alpha_{\Diamond A}}(x*y)\}$

Therefore $\mu^N_{\alpha_{\Diamond\Diamond A}}(y*z) \leq \max\{\mu^N_{\alpha_{\Diamond A}}(x*z), \mu^N_{\alpha_{\Diamond A}}(x*y)\}$

(iv) Now $\nu^P_{\alpha_{\Diamond\Diamond A}}(0) = \nu^P_{\alpha_{\Diamond A}}(0)$

$= \nu^P_{\alpha_A}(0)$

$\leq \nu^P_{\alpha_A}(x)$

$= \nu^P_{\alpha_{\Diamond A}}(x)$

Therefore $\nu^P_{\alpha_{\Diamond\Diamond A}}(0) \leq \nu^P_{\alpha_{\Diamond A}}(x)$

Now $\nu^N_{\alpha_{\Diamond\Diamond A}}(0) = \nu^N_{\alpha_{\Diamond A}}(0)$

$= \nu^N_{\alpha_A}(0)$

$\geq \nu^N_{\alpha_A}(x)$

$= \nu^N_{\alpha_{\lozenge A}}(x)$

Therefore $\nu^N_{\alpha_{\lozenge\lozenge A}}(0) \geq \nu^N_{\alpha_{\lozenge A}}(x)$

(v) Now$\nu^P_{\alpha_{\lozenge\lozenge A}}(y * z) = \nu^P_{\alpha_{\lozenge A}}(y * z)$

$= \nu^P_{\alpha_A}(y * z)$

$\leq \max\{\nu^P_{\alpha_A}(x * z), \nu^P_{\alpha_A}(x * y)\}$

$= \max\{\nu^P_{\alpha_{\lozenge A}}(x * z), \nu^P_{\alpha_{\lozenge A}}(x * y)\}$

Therefore $\nu^P_{\alpha_{\lozenge\lozenge A}}(y * z) \leq \max\{\nu^P_{\alpha_{\lozenge A}}(x * z), \nu^P_{\alpha_{\lozenge A}}(x * y)\}$

(vi) Now$\nu^N_{\alpha_{\lozenge\lozenge A}}(y * z) = \nu^N_{\alpha_{\lozenge A}}(y * z)$

$= \nu^N_{\alpha_A}(y * z)$

$\geq \min\{\nu^N_{\alpha_A}(x * z), \nu^N_{\alpha_A}(x * y)\}$

$= \min\{\nu^N_{\alpha_{\lozenge A}}(x * z), \nu^N_{\alpha_{\lozenge A}}(x * y)\}$

Therefore $\nu^N_{\alpha_{\lozenge\lozenge A}}(y * z) \geq \min\{\nu^N_{\alpha_{\lozenge A}}(x * z), \nu^N_{\alpha_{\lozenge A}}(x * y)\}$

Therefore ◊◊A = ◊A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 9

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\Box\bar{\bar{A}}$ = ◊A is a bipolar
bipolar
intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now$\mu^P_{\alpha_{\Box\bar{A}}}(0) = \nu^P_{\alpha_{\Box\bar{A}}}(0)$

$= 1 - \nu^P_{\alpha_A}(0)$

$\geq 1 - \nu^P_{\alpha_A}(x)$

$= \mu^P_{\alpha_{\lozenge A}}(x)$

Therefore $\mu^P_{\alpha_{\Box\bar{A}}}(0) \geq \mu^P_{\alpha_{\lozenge A}}(x)$

Now$\mu^N_{\alpha_{\Box\bar{A}}}(0) = \nu^N_{\alpha_{\Box\bar{A}}}(0)$

$= -1 - \nu^N_{\alpha_A}(0)$

$\leq -1 - \nu^N_{\alpha_A}(x)$

$= \mu^N_{\alpha_{\lozenge A}}(x)$

Therefore $\mu^N_{\alpha_{\Box\bar{A}}}(0) \leq \mu^N_{\alpha_{\lozenge A}}(x)$

(ii) Now$\mu^P_{\alpha_{\Box\bar{A}}}(y * z) = \nu^P_{\alpha_{\Box\bar{A}}}(y * z)$

$= 1 - \nu^P_{\alpha_A}(y * z)$

$\geq \min\{1 - \nu^P_{\alpha_A}(x * z), 1 - \nu^P_{\alpha_A}(x * y)\}$

$= \min\{\mu^P_{\alpha_{\lozenge A}}(x * z), \mu^P_{\alpha_{\lozenge A}}(x * y)\}$

Therefore $\mu^P_{\alpha_{\Box\bar{A}}}(y * z) \geq \min\{\mu^P_{\alpha_{\lozenge A}}(x * z), \mu^P_{\alpha_{\lozenge A}}(x * y)\}$

(iii) Now$\mu^N_{\alpha_{\Box\bar{A}}}(y * z) = \nu^N_{\alpha_{\Box\bar{A}}}(y * z)$

$= -1 - \nu^N_{\alpha_A}(y * z)$

$\leq \max\{-1 - \nu^N_{\alpha_A}(x * z), -1 - \nu^N_{\alpha_A}(x * y)\}$

$= \max\{\mu^N_{\alpha_{\lozenge A}}(x * z), \mu^N_{\alpha_{\lozenge A}}(x * y)\}$

Therefore $\mu^N_{\alpha_{\Box\bar{A}}}(y * z) \leq \max\{\mu^N_{\alpha_{\lozenge A}}(x * z), \mu^N_{\alpha_{\lozenge A}}(x * y)\}$

(iv) Now$\nu^P_{\alpha_{\Box\bar{A}}}(0) = \mu^P_{\alpha_{\Box\bar{A}}}(0)$

$= \nu^P_{\alpha_A}(0)$

$\leq \nu^P_{\alpha_A}(x)$

$= \nu^P_{\alpha_{\lozenge A}}(x)$

Therefore $\nu^P_{\alpha_{\Box\bar{A}}}(0) \leq \nu^P_{\alpha_{\lozenge A}}(x)$

Now$\nu^N_{\alpha_{\Box\bar{A}}}(0) = \mu^N_{\alpha_{\Box\bar{A}}}(0)$

$= \nu^N_{\alpha_A}(0)$

$\geq \nu_{\alpha_{\mathrm{A}}}^{N}(x)$
$= \nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x)$

Therefore $\nu_{\alpha_{\Box\bar{\bar{A}}}}^{N}(0) \geq \nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x)$

(v) Now$\nu_{\alpha_{\Box\bar{\bar{A}}}}^{P}(y*z) = \mu_{\alpha_{\Box\bar{\mathrm{A}}}}^{P}(y*z)$
$= \nu_{\alpha_{\mathrm{A}}}^{P}(y*z)$
$\leq$max $\{\nu_{\alpha_{\mathrm{A}}}^{P}(x*z), \nu_{\alpha_{\mathrm{A}}}^{P}(x*y)\}$
$=$max $\{\nu_{\alpha_{\Diamond\mathrm{A}}}^{P}(x*z), \nu_{\alpha_{\Diamond\mathrm{A}}}^{P}(x*y)\}$

Therefore $\nu_{\alpha_{\Box\bar{\bar{A}}}}^{P}(y*z) \leq$max $\{\nu_{\alpha_{\Diamond\mathrm{A}}}^{P}(x*z), \nu_{\alpha_{\Diamond\mathrm{A}}}^{P}(x*y)\}$

(vi) Now$\nu_{\alpha_{\Box\bar{\bar{A}}}}^{N}(y*z) = \mu_{\alpha_{\Box\bar{\mathrm{A}}}}^{N}(y*z)$
$= \nu_{\alpha_{\mathrm{A}}}^{N}(y*z)$
$\geq$min $\{\nu_{\alpha_{\mathrm{A}}}^{N}(x*z), \nu_{\alpha_{\mathrm{A}}}^{N}(x*y)\}$
$=$ min $\{\nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x*z), \nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x*y)\}$

Therefore $\nu_{\alpha_{\Box\bar{\bar{A}}}}^{N}(y*z) \geq$min $\{\nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x*z), \nu_{\alpha_{\Diamond\mathrm{A}}}^{N}(x*y)\}$

Therefore$\Box\bar{\bar{A}}$ = ◊A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 10

If A is a bipolar intuitionistic fuzzy α-ideal of X, then $\Diamond\bar{\bar{A}}$ = □A is a bipolar
intuitionistic fuzzy α-ideal of X.

Proof: Given A is a bipolar intuitionistic fuzzy α-ideal of X.

Consider 0, x, y, z∈A.

(i) Now$\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(0) = \nu_{\alpha_{\Diamond\bar{A}}}^{P}(0)$
$= \mu_{\alpha_{\mathrm{A}}}^{P}(0)$
$\geq \mu_{\alpha_{\mathrm{A}}}^{P}(x)$
$= \mu_{\alpha_{\Box\mathrm{A}}}^{P}(x)$

Therefore $\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(0) \geq \mu_{\alpha_{\Box\mathrm{A}}}^{P}(x)$

Now$\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{N}(0) = \nu_{\alpha_{\Diamond\bar{A}}}^{N}(0)$
$= \mu_{\alpha_{\mathrm{A}}}^{N}(0)$
$\leq \mu_{\alpha_{\mathrm{A}}}^{N}(x)$
$= \mu_{\alpha_{\Box\mathrm{A}}}^{N}(x)$

Therefore $\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{N}(0) \leq \mu_{\alpha_{\Box\mathrm{A}}}^{N}(x)$

(ii) Now$\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(y*z) = \nu_{\alpha_{\Diamond\bar{A}}}^{P}(y*z)$
$= \mu_{\alpha_{\mathrm{A}}}^{P}(y*z)$
$\geq$min $\{\mu_{\alpha_{\mathrm{A}}}^{P}(x*z),\ \mu_{\alpha_{\mathrm{A}}}^{P}(x*y)\}$
$=$ min $\{\mu_{\alpha_{\Box\mathrm{A}}}^{P}(x*z),\ \mu_{\alpha_{\Box\mathrm{A}}}^{P}(x*y)\}$

Therefore $\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(y*z) \geq$min $\{\mu_{\alpha_{\Box\mathrm{A}}}^{P}(x*z),\ \mu_{\alpha_{\Box\mathrm{A}}}^{P}(x*y)\}$

(iii) Now$\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{N}(y*z) = \nu_{\alpha_{\Diamond\bar{A}}}^{N}(y*z)$
$= \mu_{\alpha_{\mathrm{A}}}^{N}(y*z)$
$\leq$max $\{\mu_{\alpha_{\mathrm{A}}}^{N}(x*z),\ \mu_{\alpha_{\mathrm{A}}}^{N}(x*y)\}$
$=$ max $\{\mu_{\alpha_{\Box\mathrm{A}}}^{N}(x*z),\ \mu_{\alpha_{\Box\mathrm{A}}}^{N}(x*y)\}$

Therefore $\mu_{\alpha_{\Diamond\bar{\bar{A}}}}^{N}(y*z) \leq$max $\{\mu_{\alpha_{\Box\mathrm{A}}}^{N}(x*z),\ \mu_{\alpha_{\Box\mathrm{A}}}^{N}(x*y)\}$

(iv) Now$\nu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(0) = \mu_{\alpha_{\Diamond\bar{A}}}^{P}(0)$
$=1 - \mu_{\alpha_{\mathrm{A}}}^{P}(0)$
$\leq 1 - \mu_{\alpha_{\mathrm{A}}}^{P}(x)$
$= \nu_{\alpha_{\Box\mathrm{A}}}^{P}(x)$

Therefore $\nu_{\alpha_{\Diamond\bar{\bar{A}}}}^{P}(0) \leq \nu_{\alpha_{\Box\mathrm{A}}}^{P}(x)$

Now$\nu_{\alpha_{\Diamond\bar{\bar{A}}}}^{N}(0) = \mu_{\alpha_{\Diamond\bar{A}}}^{N}(0)$

$= -1 - \mu^N_{\alpha_A}(0)$
$\geq -1 - \mu^N_{\alpha_A}(x)$
$= \nu^N_{\alpha_{\Box A}}(x)$

Therefore $\nu^N_{\alpha_{\Diamond\bar{\bar{A}}}}(0) \geq \nu^N_{\alpha_{\Box A}}(x)$

(v) Now$\nu^P_{\alpha_{\Diamond\bar{\bar{A}}}}(y * z) = \mu^P_{\alpha_{\Diamond\bar{A}}}(y * z)$
$= 1 - \mu^P_{\alpha_A}(y * z)$
$\leq \max\{1 - \mu^P_{\alpha_A}(x * z),\ 1 - \mu^P_{\alpha_A}(x * y)\}$
$= \max\{\nu^P_{\alpha_{\Box A}}(x * z),\ \nu^P_{\alpha_{\Box A}}(x * y)\}$
Therefore $\nu^P_{\alpha_{\Diamond\bar{\bar{A}}}}(y * z) \leq \max\{\nu^P_{\alpha_{\Box A}}(x * z),\ \nu^P_{\alpha_{\Box A}}(x * y)\}$

(vi) Now$\nu^N_{\alpha_{\Diamond\bar{\bar{A}}}}(y * z) = \mu^N_{\alpha_{\Diamond\bar{A}}}(y * z)$
$= -1 - \mu^N_{\alpha_A}(y * z)$
$\geq \min\{-1 - \mu^N_{\alpha_A}(x * z), -1 - \mu^N_{\alpha_A}(x * y)\}$
$= \min\{\nu^N_{\alpha_{\Box A}}(x * z), \nu^N_{\alpha_{\Box A}}(x * y)\}$

Therefore $\nu^N_{\alpha_{\Diamond\bar{\bar{A}}}}(y * z) \geq \min\{\nu^N_{\alpha_{\Box A}}(x * z), \nu^N_{\alpha_{\Box A}}(x * y)\}$

Therefore$\Diamond\bar{\bar{A}}$ = □A is a bipolar intuitionistic fuzzy α-ideal of X.

Theorem: 11

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then □A is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:12

If A and B are bipolar intuitionistic anti fuzzy α-idealof X, then □ (A ∩ B) = □A∩□B
is also a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:13

If A is a bipolar intuitionistic anti fuzzy α-idealof X, then ◊A is a bipolar intuitionisticanti fuzzy α-ideal of X.

Theorem:14

If A and B are bipolar intuitionistic anti fuzzy α-ideal of X, then ◊ (A ∩ B) = ◊A∩◊B
is also a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:15

If A is a bipolar intuitionistic anti fuzzy α-idealof X, then □□A = □A is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:16

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then □◊A = ◊A is a bipolar
intuitionistic anti fuzzy α-ideal of X.

Theorem:17

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then ◊□A = □A is a bipolar
intuitionistic anti fuzzy α-ideal of X.

Theorem:18

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then ◊◊A = ◊A is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem:19

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then □$\bar{\bar{A}}$ = ◊A is a bipolar intuitionistic anti fuzzy α-ideal of X.

Theorem: 20

If A is a bipolar intuitionistic anti fuzzy α-ideal of X, then ◊$\bar{\bar{A}}$ = □A is a bipolar intuitionistic anti fuzzy α-ideal of X.

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