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SIX SIGMA BASED ROBUST CONTROL CHART UNDER MODERATENESS

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Abstract. Statistical process control methods extend the use of descriptive statistics to monitor the quality of the product and process. There are common and assignable causes of variation in the production of every product. Using statistical process control we want to determine the amount of variation that is common or normal. Then we monitor the production process to make sure that the production stays within this normal range. That is, it has to be making sure the process is in a state of control. The most commonly used tools for monitoring the production is a process control chart. This research paper introduces a new six sigma based control chart under moderate distribution for mean using standard deviation with varying sample size.

1. Introduction

A control chart is a graph that shows whether a sample of the data falls within the common or normal range of variation. A control chart has upper and lower control limits that separate common from assignable causes of variation (Duncan, 1958). The common range of variation is defined by the use of control chart limits. The process is out of control when a plot of data reveals that one or more samples fall outside the control limits.

3−Sigma control limits: Consider the statistic $t = t(x_1, x_2, ..., x_n)$, a function of the sample observations $x_1, x_2, ..., x_n$

$$E(t) = \mu_t \qquad and \qquad Var(t) = \sigma_t^2.$$

If the Statistic t is normally distributed, then from the fundamental area property of the normal distribution, we have

$$\begin{aligned} P[\mu_t - 3\sigma_t < t < \mu_t + 3\sigma_t] &= 0.9973 \\ \Rightarrow \qquad p[|t - \mu_t| < 3\sigma_t] &= 0.9973 \\ \Rightarrow \qquad p[|t - \mu_t| > 3\sigma_t] &= 0.0027 \end{aligned}$$

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Then the center line, the UCL and the LCL are

$$\begin{aligned} \text{UCL} &= \mu_t + k\sigma_t \\ \text{Center Line} &= \mu_t \\ \text{LCL} &= \mu_t - k\sigma_t \end{aligned}$$

where k is the distance of the control limits from the center line, expressed in terms of standard deviation units. When k is set to 3, we speak of $3-$sigma control charts.

The different characteristics that can be measured by control charts can be divided into two groups: variables and attributes.

2. Process capability

Capability indices have been developed to graphically portray that measure. Capability indices let to place the distribution of our process in relation to the product specification limits. Capability indices should be used to determine whether the process, given its natural variation, is capable of meeting established specifications. It is also a measure of the manufacturability of the product with the given processes.

Capability indices can be used to compare the product/process matches and identify the poorest match (lowest capability). The poorest matches then can be targeted on a priority basis for improvement (Montgomery, 2011).

If the process is in statistical control, via "normal" SPC charts, and the process mean is centered on the target, then Cp can be calculated as follows:

$$C_p = (\text{USL} - \text{LSL}) \ / \ 6 \text{ sigma}.$$

- $C_p < 1$ means the process variation exceeds specification, and a significant number of defects are being made.
- $C_p = 1$ means that the process is just meeting specifications. A minimum of .3% defects will be made and more if the process is not centered.
- $C_p > 1$ means that the process variation is less than the specification, however, defects might be made if the process is not centered on the target value.

While C_p relates the spread of the process relative to the specification width, it does not address how well the process average, X, is centered to the target value. C_p is often referred to as process "potential".

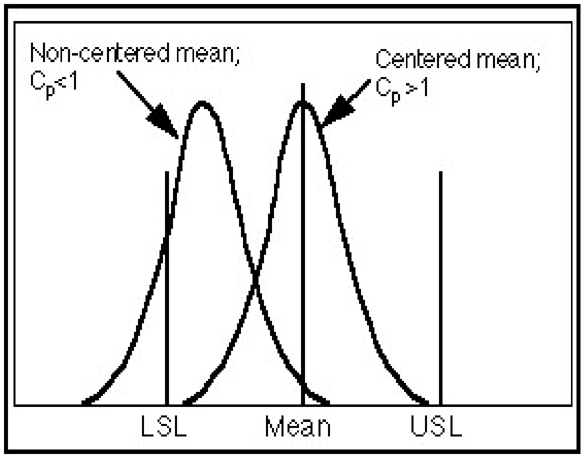


FIGURE 1

3. Conditions for Application

- Robust six sigma based control limits under moderateness will be used if the data is found to be non-normal
- Companies ready to utilize the concept of Interquartile range (IQR) in its processes.

4. Interquartile Range

The interquartile range ($IQR = Q_3 - Q_1$) is the difference between the first quartile and third quartile. Before determining the interquartile range, we first need to know the values of the first quartile and third quartile. There are many measurements of the variability of a set of data. Both the range and standard deviation tell us how spread out our data is. The problem with these descriptive statistics is that they are quite sensitive to outliers. A measurement of the spread of a dataset that is more resistant to the presence of outliers is the interquartile range.

The range gives us a measurement of how spread out the entirety of our data set is. The interquartile range, which tells us how far apart the first and third quartile are, indicates how spread out the middle 50% of our set of data is. Due to its resistance to outliers, the interquartile range is useful in identifying when a value is an outlier.

5. Methods and Materials

The quality control constant $\sigma_{6\sigma.MD:\bar{X}-S.IQR}$ is coined by the six sigma based control limits using Interquartile range (IQR) under moderate distribution in the course of "z-score" that corresponds to the areas under the moderate curve of 0.25 and 0.75 respectively. Thus we have $Q_3 = 0.8453 + \mu$ and $Q_1 = -0.8453 + \mu$

(Desai, 2011) implies that $IQR_{Moderate} = 1.6906\ \delta_{6\ \sigma}$ because of the central limit theorem motivates the use of the normal distribution

$$f(x) = \frac{1}{\pi\delta}\ e^{-\frac{1}{\pi}\left(\frac{X-\mu}{\delta}\right)^2},$$

$-\infty < X < \infty$ and $\delta > 0$. A method is to build a six sigma based control chart using interquartile range (IQR) under moderate distribution for mean. Fix the tolerance level (TL) and process capability (C_p) to find out the process standard deviation $\sigma_{6\sigma.MD:\bar{X}-S.IQR^*}$. Apply the value of $\sigma_{6\sigma.MD:\bar{X}-S.IQR}$ in the control limits (Radhakrishnan and Balamurugan, 2016)

$$\overline{\overline{X}} \pm \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right)\ \sigma_{6\sigma.MD:\bar{X}-S.IQR},$$

to get the six sigma based control limits for mean. Where $\sigma_{6\sigma.MD:\bar{X}-S.IQR}$ is replaced instead of σ in the Shewhart 3−Sigma control chart and $\delta_{MD.6\sigma}$ is obtained using

$$p(z \leq z_{6\sigma}) = 1 - \alpha_1,$$

$\alpha_1 = 3.4 \times 10^{-6}$ and z is a standard moderate variate.

6. An Example

The example provided by R.C. Gupta (2001, Page No. 213) is considered here. The Specimens of concrete, collected daily from the work of a sub-contractor, yielded the results for compressive strength (in 100's p.s.i) give below.

Table 1: Measurements from compressive specification (in 100's p.s.i)

Sample No.	Observations					Mean	SD	IQR_{Z_S}
1	46.000	43.500	43.100	47.300	45.100	45.000	1.744	1.853
2	45.000	45.700	40.900	44.000	—	43.900	2.118	1.446
3	42.000	47.300	44.500	—	—	44.600	2.651	1.964
4	49.100	45.700	46.400	46.000	47.100	46.860	1.358	0.815
5	45.100	44.200	49.600	45.100	—	46.000	2.437	1.001
6	43.100	44.400	—	—	—	43.750	0.919	0.482
7	41.200	45.500	45.900	46.500	43.800	44.580	2.139	1.557
8	41.000	46.700	43.300	—	—	43.667	2.868	2.113
9	45.400	43.700	47.300	—	—	45.467	1.801	1.334
10	40.000	43.100	48.300	44.500	—	43.975	3.442	2.317
11	45.200	46.700	—	—	45.950	1.061	0.556	
12	44.200	43.500	45.000	44.000	44.700	44.280	0.589	0.519

a. Shewhart control chart for mean using standard deviation with varying sample size

The 3σ control limits suggested by Shewhart (1931) are $\overline{\overline{X}} \pm \left(\dfrac{3\overline{S}}{c_4\sqrt{n}}\right)$

$$\overline{\overline{X}} = \frac{\sum_{i=1}^{12} n_i\, \overline{X}_i}{\sum_{i=1}^{12} n_i} = \frac{5(45) + 4(43.9) + ... + 5(44.28)}{5 + 4 + ... + 5} = \frac{2019.7}{45} = 44.882$$

$$\overline{S} = \left[\frac{\sum_{i=1}^{12} (n_i - 1)S_i^2}{\sum_{i=1}^{12} (n_i - 12)}\right]^{\frac{1}{2}}$$

$$\overline{S} = \left[\frac{4(1.744)^2 + 3(2.118)^2 + ... + 4(0.589)^2}{5 + 4 + ... + 5 - 12}\right]^{\frac{1}{2}} = \left[\frac{3.223}{33}\right]^{\frac{1}{2}}$$

$$\overline{S} = 0.313$$

Table 2: UCL and LCL values for 3Sigma control limits

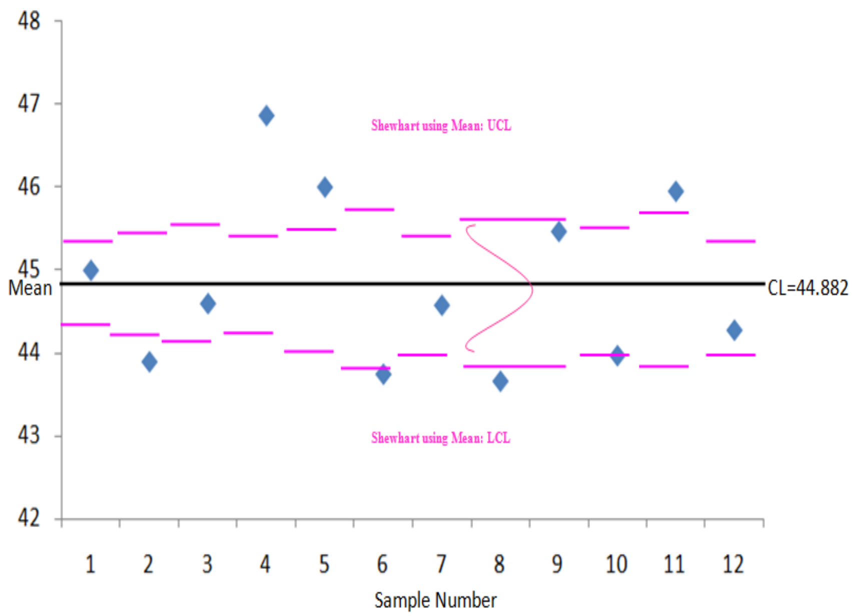
Sample Number	Sample Size n	LCL	UCL
1	5	44.353	45.247
2	4	44.290	45.310
3	3	44.188	45.412
4	5	44.353	45.247
5	4	44.290	45.310
6	2	43.968	45.632
7	5	44.353	45.247
8	3	44.188	45.412
9	3	44.188	45.412
10	4	44.290	45.310
11	2	43.968	45.632
12	5	44.353	45.247

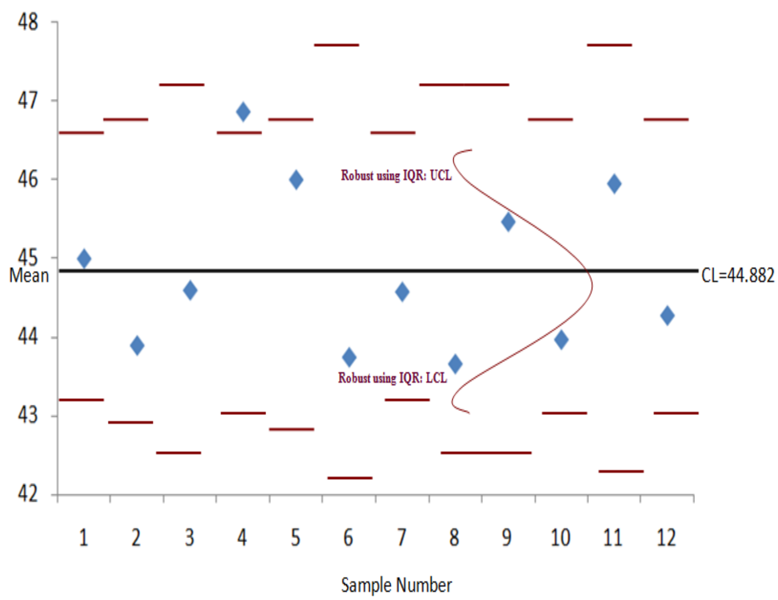
(Quality control constants $c_4 = 0.7979$ for $n = 2$, $c_4 = 0.8862$ for $n = 3$, $c_4 = 0.9213$ for $n = 4$, $c_4 = 0.9400$ for $n = 5$)

From the resulting Figure 1, it is clear that the process is out of control, since the sample numbers 4, 5 and 11 goes above the upper control limit and the sample numbers 2, 6 and 8 goes below the lower control limit.

b. Inter quartile range (IQR) control chart for mean using standard deviation with varying sample size

The 3σ control limits based on IQR are $\overline{\overline{X}} \pm \left(\dfrac{3\overline{IQR}_{z_s}}{c_4\sqrt{n}}\right)$





From the resulting Figure 2, it is clear that the process is out of control, since the sample number 4 goes above the upper control limit.

c. Proposed six sigma based inter quartile range (IQR) control chart under moderate distribution for mean using standard deviation with

varying sample size

Difference between upper specification and lower specification limits is 1.835 ($USL-LSL = 2.317-0.482$), which termed as tolerance level (TL) and choose the process capability (C_p) is 2.0, the value of $\sigma_{6\sigma.MD:\bar{X}-S.IQR}$ is 0.122. The six sigma based control limits of inter quartile range (IQR) under moderate distribution for mean using standard deviation with varying sample size, a specified tolerance level with the control limits

$$\overline{\overline{X}} \pm \left(\frac{\delta_{6\sigma}}{\sqrt{n}}\right) \ \sigma_{6\sigma.MD:\bar{X}-S.IQR},$$

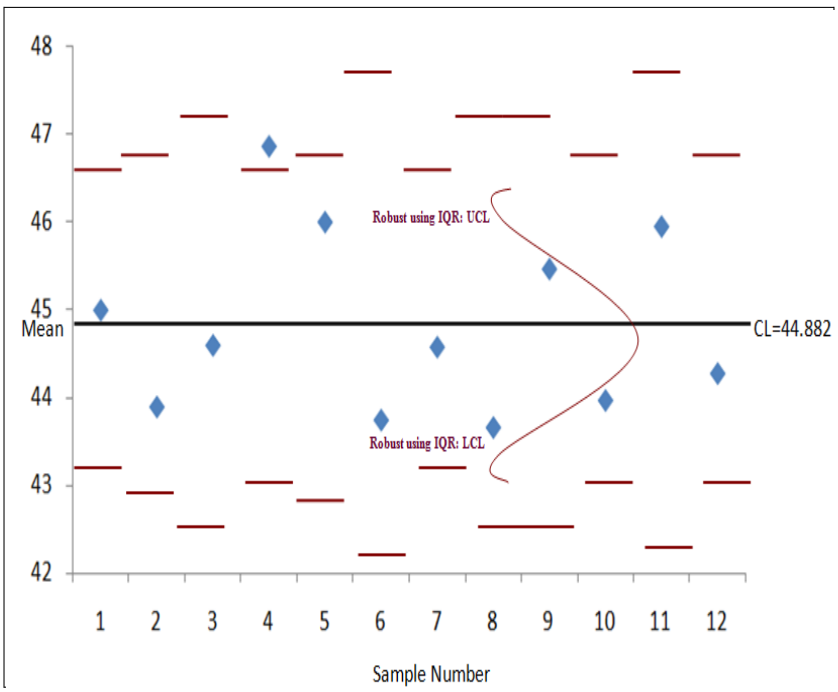


Table 4: UCL and LCL values for six sigma based IQR under moderate distribution

Sample Number	Sample Size (n)	UCL	LCL
1	5	45.128	44.636
2	4	45.157	44.608
3	3	45.199	44.565
4	5	45.128	44.636
5	4	45.157	44.608
6	2	45.270	44.494
7	5	45.128	44.636
8	3	45.199	44.565
9	3	45.199	44.565
10	4	45.157	44.608
11	2	45.270	44.494
12	5	45.128	44.636

From the above results, it is clear that the process is out of control, since the many sample numbers goes above the upper control limit and the below the lower control limit.

Table 5: Assessment of Shewhart, IQR and IQR using process capability control charts

Sample Size (n)	Control limits	Shewhart control chart	IQR	Six sigma based IQR under moderate distribution
2	LCL	43.968	42.061	44.494
	CL	44.882	44.882	44.882
	UCL	45.632	47.703	45.270
3	LCL	44.188	42.578	44.565
	CL	44.882	44.882	44.882
	UCL	45.412	47.186	45.199
4	LCL	44.290	42.887	44.608
	CL	44.882	44.882	44.882
	UCL	45.310	46.877	45.157
5	LCL	44.353	43.098	44.636
	CL	44.882	44.882	44.882
	UCL	45.247	46.666	45.128

7. Conclusion

It is found from the results that the process is out of statistical control when Shewhart 3−Sigma, IQR and six sigma based IQR under moderate distribution limits are adopted. In this case many points fall outside the control limits compared than the control limits of Shewhart 3−Sigma and IQR control limits but in this case the six sigma based control limits interval of IQR under moderate distribution is smaller than the control limits interval of Shewhart and IQR control charts. It is clear that the estimate can be valuable to practitioners when they are searching for the special cause that produced the signal on the control chart.

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