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A STUDY ON OPERATIONS OF BIPOLAR NEUTROSOPHIC CUBIC FUZZY GRAPHS

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Abstract. In this paper we introduce the idea of bipolar neutrosophic cubic fuzzy graphs. We discuss fundamental binary operations like Cartesian product, composition of bipolar neutrosophic cubic fuzzy graphs. We provide some results related with bipolar neutrosophic cubic fuzzy graphs.

1. Introduction

In 1975 Rosenfeld [10] introduced fuzzy graphs based on fuzzy set.Fuzzy graph theory plays essential roles in various discipines including information theory, neural networks,clustering problems and control theory,etc.Fuzzy models is more compatiable to the systemin compare with classical mode.Bhattacharya [5] gave some remarks on fuzzy graphs.Some operations on fuzzy graphs were introduced by mordeson and peng [7]. Akram et al. has introduced several new concepts including bipolar fuzzy graphs,regular bipolar fuzzy graphs, irregular bipolar fuzzy graphs etc. In this paper, we present certain operations on bipolar fuzzy graphs structures and investigate some of their properties.

2. Basic Definitions

Definition 2.1. Let X be a space of points with generic elements in X denoted by x. A neutrosophic fuzzy set A is characterized by truth-membership function $\mu_{AT}(x)$, an indeterminacy-membership function $\lambda_{AI}(x)$ a falsity – membership function $\gamma_{AF}(x)$.

For each point x in X $\mu_{AT}(x), \lambda_{AI}(x), \gamma_{AF}(x) \in [0,1]$. A neutrosophic fuzzy set A can be written as

$$A = \{< x : \mu_{AT}(x), \lambda_{AI}(x), \gamma_{AF}(x) >, x \in X\}$$

Definition 2.2. Let X be a space of points with generic elements in X denoted by x. A neutrosophic cubic fuzzy set in X is a pair $G = (M, N)$ where $M = \{< x : \mu_{MT}(x), \lambda_{MI}(x), \gamma_{MF}(x) >, x \in X\}$is an interval neutrosophic fuzzy set in X and $N = \{< x : \mu_{NT}(x), \lambda_{NI}(x), \gamma_{NF}(x) >, x \in X\}$ is a neutrosophic fuzzy set in X.

Definition 2.3. Let $G^* = (V, E)$ be a fuzzy graph. By neutrosophic cubic fuzzy graph of G^*, we mean a pair $G = (M, N)$ where

$$M = (A, B) = ((\mu_{AT}, \mu_{BT}), (\lambda_{AI}, \lambda_{BI}), (\gamma_{AF}, \gamma_{BF}))$$

is the neutrosophic cubic fuzzy set representation of vertex set V and $N = (C, D) = ((\mu_{CT}, \mu_{DT}), (\lambda_{CI}, \lambda_{DI}), (\gamma_{CF}, \gamma_{DF}))$ is the neutrosophic cubic fuzzy set representation of edge set E such that

(i) $(\mu_{TC}(x_i y_i) \leq r\min\{\mu_{AT}(x_i), \mu_{AT}(y_i)\}, \mu_{DT}(x_i y_i) \leq \max\{\mu_{BT}(x_i), \mu_{BT}(y_i)\})$
(ii) $(\lambda_{IC}(x_i y_i) \leq r\min\{\lambda_{AI}(x_i), \lambda_{AI}(y_i)\}, \lambda_{DI}(x_i y_i) \leq \max\{\lambda_{BI}(x_i), \lambda_{BI}(y_i)\})$
(iii) $(\gamma_{FC}(x_i y_i) \leq r\min\{\gamma_{AF}(x_i), \gamma_{AF}(y_i)\}, \gamma_{DF}(x_i y_i) \leq \max\{\gamma_{BF}(x_i), \gamma_{BF}(y_i)\})$.

3. Bipolar Neutrosophic Cubic Fuzzy Graphs (BNCFG)

Definition 3.1. Let X be a space of points with generic elements in X denoted by x. A Bipolar neutrosophic cubic fuzzy set in X is a pair $G = \left(\left(M^P, N^P\right), \left(M^N, N^N\right)\right)$ is defined as

$$M^P = \{< x^P : \mu^P_{MT}(x), \lambda^P_{MI}(x), \gamma^P_{MF}(x) > /x \in X\}$$
$$M^N = \{< x^N : \mu^N_{MT}(x), \lambda^N_{MI}(x), \gamma^N_{MF}(x) > /x \in X\}$$

is an interval neutrosophic fuzzy set in X and

$$N^P = \{< x^P : \mu^P_{NT}(x), \lambda^P_{NI}(x), \gamma^P_{NF}(x) > /x \in X\}$$
$$N^N = \{< x^N : \mu^N_{NT}(x), \lambda^N_{NI}(x), \gamma^N_{NF}(x) > /x \in X\}$$

is a neutrosophic fuzzy set in X, where $\mu^P_{MT}(x), \lambda^P_{MI}(x), \gamma^P_{MF}(x) \to [0, 1]$ and $\mu^N_{MT}(x), \lambda^N_{MI}(x), \gamma^N_{MF}(x) \to [-1, 0]$.

Definition 3.2. Let $G^* = (V, E)$ be a fuzzy graph. By a Bipolar neutrosophic cubic fuzzy graph of G^*. We mean a pair $G = \left(\left(M^P, N^P\right), \left(M^N, N^N\right)\right)$ where

$$M^P = (A, B) = \left(\left(\mu^P_{AT}, \mu^P_{BT}\right), \left(\lambda^P_{AI}, \lambda^P_{BI}\right), \left(\gamma^P_{AF}, \gamma^P_{BF}\right)\right)$$
$$M^N = (A, B) = \left(\left(\mu^N_{AT}, \mu^N_{BT}\right), \left(\lambda^N_{AI}, \lambda^N_{BI}\right), \left(\gamma^N_{AF}, \gamma^N_{BF}\right)\right)$$

is the neutrosophic cubic fuzzy set representation of vertex set V and

$$N^P = (C, D) = \left(\left(\mu^P_{CT}, \mu^P_{DT}\right), \left(\lambda^P_{CI}, \lambda^P_{DI}\right), \left(\gamma^P_{CF}, \gamma^P_{DF}\right)\right)$$
$$N^N = (C, D) = \left(\left(\mu^N_{CT}, \mu^N_{DT}\right), \left(\lambda^N_{CI}, \lambda^N_{DI}\right), \left(\gamma^N_{CF}, \gamma^N_{DF}\right)\right)$$

is the neutrosophic cubic fuzzy set representation of edge set Esuch that

(i) $\left(\mu^P_{TC}(x_i y_i) \leq r\min\left\{\mu^P_{AT}(x_i), \mu^P_{AT}(y_i)\right\}, \mu^P_{DT}(x_i y_i) \leq \max\left\{\mu^P_{BT}(x_i), \mu^P_{BT}(y_i)\right\}\right)$

$\left(\mu^N_{TC}(x_i y_i) \geq r\max\left\{\mu^N_{AT}(x_i), \mu^N_{AT}(y_i)\right\}, \mu^N_{DT}(x_i y_i) \geq \min\left\{\mu^N_{BT}(x_i), \mu^N_{BT}(y_i)\right\}\right)$

(ii) $\left(\lambda^P_{IC}(x_i y_i) \leq r\max\left\{\lambda^P_{AI}(x_i), \lambda^P_{AI}(y_i)\right\}, \lambda^P_{DI}(x_i y_i) \leq \min\left\{\lambda^P_{BI}(x_i), \lambda^P_{BI}(y_i)\right\}\right)$

$\left(\lambda^N_{IC}(x_i y_i) \geq r\max\left\{\lambda^N_{AI}(x_i), \lambda^N_{AI}(y_i)\right\}, \lambda^N_{DI}(x_i y_i) \geq \min\left\{\lambda^N_{BI}(x_i), \lambda^N_{BI}(y_i)\right\}\right)$

(iii) $\left(\gamma^P_{FC}(x_i y_i) \leq r\max\left\{\gamma^P_{AF}(x_i), \gamma^P_{AF}(y_i)\right\}, \gamma^P_{DF}(x_i y_i) \leq \min\left\{\gamma^P_{BF}(x_i), \gamma^P_{BF}(y_i)\right\}\right)$

$\left(\gamma^N_{FC}(x_i y_i) \geq r\max\left\{\gamma^N_{AF}(x_i), \gamma^N_{AF}(y_i)\right\}, \gamma^N_{DF}(x_i y_i) \geq \min\left\{\gamma^N_{BF}(x_i), \gamma^N_{BF}(y_i)\right\}\right)$

4. Operations of Two Bipolar Neutrosophic Cubic Fuzzy Graphs

Definition 4.1. Let $G_1 = \left(\left({M_1}^P, {N_1}^P\right), \left({M_1}^N, {N_1}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = \left(\left(V_1^P, E_1^P\right), \left(V_1^N, E_1^N\right)\right)$ and $G_2 = \left(\left({M_2}^P, {N_2}^P\right), \left({M_2}^N, {N_2}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^* = \left(\left(V_2^P, E_2^P\right), \left(V_2^N, E_2^N\right)\right)$. The Cartesian product of G_1 and G_2 is denoted by

$$G_1 \times G_2 = \left(\left(\left({M_1}^P \times {M_2}^P\right), \left({M_1}^N \times {M_2}^N\right)\right), \left(\left({N_1}^P \times {N_2}^P\right), \left({N_1}^N \times {N_2}^N\right)\right)\right)$$
$$= \begin{pmatrix} \left(\left(A_1^P, B_1^P\right), \left(A_1^N, B_1^N\right)\right) \times \left(\left(A_2^P, B_2^P\right), \left(A_2^N, B_2^N\right)\right), \\ \left(\left(C_1^P, D_1^P\right), \left(C_1^N, D_1^N\right)\right) \times \left(\left(C_2^P, D_2^P\right), \left(C_2^N, D_2^N\right)\right) \end{pmatrix}$$
$$\left\{ \begin{pmatrix} \left(\left(\widetilde{\mu}^P_{TA_1 \times TA_2}, \widetilde{\mu}^N_{TA_1 \times TA_2}\right), \left(\widetilde{\mu}^P_{TB_1 \times TB_2}, \widetilde{\mu}^N_{TB_1 \times TB_2}\right)\right), \\ \left(\left(\widetilde{\lambda}^P_{IA_1 \times IA_2}, \widetilde{\lambda}^N_{IA_1 \times IA_2}\right), \left(\widetilde{\lambda}^P_{IB_1 \times IB_2}, \widetilde{\lambda}^N_{IB_1 \times IB_2}\right)\right), \\ \left(\left(\widetilde{\gamma}^P_{FA_1 \times FA_2}, \widetilde{\gamma}^N_{FA_1 \times FA_2}\right), \left(\widetilde{\gamma}^P_{FB_1 \times FB_2}, \widetilde{\gamma}^N_{FB_1 \times FB_2}\right)\right) \end{pmatrix} \begin{pmatrix} \left(\left(\widetilde{\mu}^P_{TC_1 \times TC_2}, \widetilde{\mu}^N_{TC_1 \times TC_2}\right), \left(\widetilde{\mu}^P_{TD_1 \times TD_2}, \widetilde{\mu}^N_{TD_1 \times TD_2}\right)\right), \\ \left(\left(\widetilde{\lambda}^P_{IC_1 \times IC_2}, \widetilde{\lambda}^N_{IC_1 \times IC_2}\right), \left(\widetilde{\lambda}^P_{ID_1 \times ID_2}, \widetilde{\lambda}^N_{ID_1 \times ID_2}\right)\right), \\ \left(\left(\widetilde{\gamma}^P_{FC_1 \times FC_2}, \widetilde{\gamma}^N_{FC_1 \times FC_2}\right), \left(\widetilde{\gamma}^P_{FD_1 \times FD_2}, \widetilde{\gamma}^N_{FD_1 \times FD_2}\right)\right) \end{pmatrix} \right\}$$

and is defined as follows

$$(1) \begin{pmatrix} \begin{pmatrix} \widetilde{\mu}^P_{TA_1 \times TA_2}(u, v) = r^P \min\left(\widetilde{\mu}^P_{TA_1}(u), \widetilde{\mu}^P_{TA_2}(v)\right), \\ \widetilde{\mu}^P_{TB_1 \times TB_2}(u, v) = \max\left(\widetilde{\mu}^P_{TB_1}(u), \widetilde{\mu}^P_{TB_2}(v)\right) \end{pmatrix}, \\ \begin{pmatrix} \widetilde{\mu}^N_{TA_1 \times TA_2}(u, v) = r^N \max\left(\widetilde{\mu}^N_{TA_1}(u), \widetilde{\mu}^N_{TA_2}(v)\right), \\ \widetilde{\mu}^N_{TB_1 \times TB_2}(u, v) = \min\left(\widetilde{\mu}^N_{TB_1}(u), \widetilde{\mu}^N_{TB_2}(v)\right) \end{pmatrix} \end{pmatrix}$$

$$(2) \begin{pmatrix} \begin{pmatrix} \widetilde{\lambda}^P_{IA_1 \times IA_2}(u, v) = r^P \min\left(\widetilde{\lambda}^P_{IA_1}(u), \widetilde{\lambda}^P_{IA_2}(v)\right), \\ \widetilde{\lambda}^P_{IB_1 \times IB_2}(u, v) = \max\left(\widetilde{\lambda}^P_{IB_1}(u), \widetilde{\lambda}^P_{IB_2}(v)\right) \end{pmatrix}, \\ \begin{pmatrix} \widetilde{\lambda}^N_{IA_1 \times IA_2}(u, v) = r^N \max\left(\widetilde{\lambda}^N_{IA_1}(u), \widetilde{\lambda}^N_{IA_2}(v)\right), \\ \widetilde{\lambda}^N_{IB_1 \times IB_2}(u, v) = \min\left(\widetilde{\lambda}^N_{IB_1}(u), \widetilde{\lambda}^N_{IB_2}(v)\right) \end{pmatrix} \end{pmatrix}$$

$$(3) \begin{pmatrix} \begin{pmatrix} \widetilde{\gamma}^P_{FA_1 \times FA_2}(u, v) = r^P \max\left(\widetilde{\gamma}^P_{FA_1}(u), \widetilde{\gamma}^P_{FA_2}(v)\right), \\ \widetilde{\gamma}^P_{FB_1 \times FB_2}(u, v) = \min\left(\widetilde{\gamma}^P_{FB_1}(u), \widetilde{\gamma}^P_{FB_2}(v)\right) \end{pmatrix}, \\ \begin{pmatrix} \widetilde{\gamma}^N_{FA_1 \times FA_2}(u, v) = r^N \min\left(\widetilde{\gamma}^N_{FA_1}(u), \widetilde{\gamma}^N_{FA_2}(v)\right), \\ \widetilde{\gamma}^N_{FB_1 \times FB_2}(u, v) = \max\left(\widetilde{\gamma}^N_{FB_1}(u), \widetilde{\gamma}^N_{FB_2}(v)\right) \end{pmatrix} \end{pmatrix}$$

$$(4) \begin{pmatrix} \begin{pmatrix} \left(\widetilde{\mu}^P_{TC_1 \times TC_2}((u, v_1)(u, v_2)) = r^P min\left(\widetilde{\mu}^P_{TA_1}(u), \widetilde{\mu}^P_{TC_2}(v_1 v_2)\right)\right), \\ \left(\widetilde{\mu}^N_{TC_1 \times TC_2}((u, v_1)(u, v_2)) = r^N max\left(\widetilde{\mu}^N_{TA_1}(u), \widetilde{\mu}^N_{TC_2}(v_1 v_2)\right)\right) \end{pmatrix}, \\ \begin{pmatrix} \left(\widetilde{\mu}^P_{TD_1 \times TD_2}((u, v_1)(u, v_2)) = max\left(\widetilde{\mu}^P_{TB_1}(u), \widetilde{\mu}^P_{TD_2}(v_1 v_2)\right)\right), \\ \left(\widetilde{\mu}^N_{TD_1 \times TD_2}((u, v_1)(u, v_2)) = min\left(\widetilde{\mu}^N_{TB_1}(u), \widetilde{\mu}^N_{TD_2}(v_1 v_2)\right)\right) \end{pmatrix} \end{pmatrix}$$

$\forall v \in V_2$and $u_1 u_2 \in E_1$

$$(5) \begin{pmatrix} \begin{pmatrix} \left(\widetilde{\lambda}^P_{IC_1 \times IC_2}((u, v_1)(u, v_2)) = r^P min\left(\widetilde{\lambda}^P_{IA_1}(u), \widetilde{\lambda}^P_{IC_2}(v_1 v_2)\right)\right), \\ \left(\widetilde{\lambda}^N_{IC_1 \times IC_2}((u, v_1)(u, v_2)) = r^N max\left(\widetilde{\lambda}^N_{IA_1}(u), \widetilde{\lambda}^N_{IC_2}(v_1 v_2)\right)\right) \end{pmatrix}, \\ \left(\widetilde{\lambda}^N_{ID_1 \times ID_2}((u, v_1)(u, v_2)) = \min\left(\widetilde{\lambda}^N_{IB_1}(u), \widetilde{\lambda}^N_{ID_2}(v_1 v_2)\right)\right), \\ \begin{pmatrix} \left(\widetilde{\lambda}^P_{ID_1 \times ID_2}((u, v_1)(u, v_2)) = max\left(\widetilde{\lambda}^P_{IB_1}(u), \widetilde{\lambda}^P_{ID_2}(v_1 v_2)\right)\right), \\ \left(\widetilde{\lambda}^N_{ID_1 \times ID_2}((u, v_1)(u, v_2)) = min\left(\widetilde{\lambda}^N_{IB_1}(u), \widetilde{\lambda}^N_{ID_2}(v_1 v_2)\right)\right) \end{pmatrix} \end{pmatrix}$$

$\forall v \in V_2$and $u_1 u_2 \in E_1$

$$(6)\left(\begin{array}{c}\left(\begin{array}{c}\left(\widetilde{\gamma}^{P}_{FC_1\times FC_2}\left((u,v_1)(u,v_2)\right)=r^P\max\left(\widetilde{\gamma}^{P}_{FA_1}(u),\widetilde{\gamma}^{P}_{FC_2}(v_1v_2)\right)\right),\\ \left(\widetilde{\gamma}^{N}_{FC_1\times FC_2}\left((u,v_1)(u,v_2)\right)=r^N\min\left(\widetilde{\gamma}^{N}_{FA_1}(u),\widetilde{\gamma}^{N}_{FC_2}(v_1v_2)\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\left(\widetilde{\gamma}^{P}_{FD_1\times FD_2}\left((u,v_1)(u,v_2)\right)=\min\left(\widetilde{\gamma}^{P}_{FB_1}(u),\widetilde{\gamma}^{P}_{FD_2}(v_1v_2)\right)\right),\\ \left(\widetilde{\gamma}^{N}_{FD_1\times FD_2}\left((u,v_1)(u,v_2)\right)=\max\left(\widetilde{\gamma}^{N}_{FB_1}(u),\widetilde{\gamma}^{N}_{FD_2}(v_1v_2)\right)\right)\end{array}\right)\end{array}\right)$$

$\forall v\in V_2$ and $u_1u_2\in E_1$

$$(7)\left(\begin{array}{c}\left(\begin{array}{c}\left(\widetilde{\mu}^{P}_{TC_1\times TC_2}\left((u_1,v)(u_2,v)\right)=r^P\min\left(\widetilde{\mu}^{P}_{TC_1}(u_1u_2),\widetilde{\mu}^{P}_{TA_2}(v)\right)\right),\\ \left(\widetilde{\mu}^{N}_{TC_1\times TC_2}\left((u_1,v)(u_2,v)\right)=r^N\max\left(\widetilde{\mu}^{N}_{TC_1}(u_1u_2),\widetilde{\mu}^{N}_{TA_2}(v)\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\left(\widetilde{\mu}^{P}_{TD_1\times TD_2}\left((u_1,v)(u_2,v)\right)=\max\left(\widetilde{\mu}^{P}_{TD_1}(u_1u_2),\widetilde{\mu}^{P}_{TB}(v)\right)\right),\\ \left(\widetilde{\mu}^{N}_{TD_1\times TD_2}\left((u_1,v)(u_2,v)\right)=\min\left(\widetilde{\mu}^{N}_{TD_1}(u_1u_2),\widetilde{\mu}^{N}_{TB_2}(v)\right)\right)\end{array}\right)\end{array}\right)$$

$$(8)\left(\begin{array}{c}\left(\begin{array}{c}\left(\widetilde{\lambda}^{P}_{IC_1\times IC_2}\left((u_1,v)(u_2,v)\right)=r^P\min\left(\widetilde{\lambda}^{P}_{IC_1}(u_1u_2),\widetilde{\lambda}^{P}_{IA_2}(v)\right)\right),\\ \left(\widetilde{\lambda}^{N}_{IC_1\times IC_2}\left((u_1,v)(u_2,v)\right)=r^N\max\left(\widetilde{\lambda}^{N}_{IC_1}(u_1u_2),\widetilde{\lambda}^{N}_{IA_2}(v)\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\left(\widetilde{\lambda}^{P}_{ID_1\times ID_2}\left((u_1,v)(u_2,v)\right)=\max\left(\widetilde{\lambda}^{P}_{ID_1}(u_1u_2),\widetilde{\lambda}^{P}_{IB_2}(v)\right)\right),\\ \left(\widetilde{\lambda}^{N}_{ID_1\times ID_2}\left((u_1,v)(u_2,v)\right)=\min\left(\widetilde{\lambda}^{N}_{ID_1}(u_1u_2),\widetilde{\lambda}^{N}_{IB_2}(v)\right)\right)\end{array}\right)\end{array}\right)$$

$$(9)\left(\begin{array}{c}\left(\begin{array}{c}\left(\widetilde{\gamma}^{P}_{FC_1\times FC_2}\left((u_1,v)(u_2,v)\right)=r^P\max\left(\widetilde{\gamma}^{P}_{FC_1}(u_1u_2),\widetilde{\gamma}^{P}_{FA_2}(v)\right)\right),\\ \left(\widetilde{\gamma}^{N}_{FC_1\times FC_2}\left((u_1,v)(u_2,v)\right)=r^N\min\left(\widetilde{\gamma}^{N}_{FC_1}(u_1u_2),\widetilde{\gamma}^{N}_{FA_2}(v)\right)\right)\end{array}\right),\\ \left(\begin{array}{c}\left(\widetilde{\gamma}^{P}_{FD_1\times FD_2}\left((u_1,v)(u_2,v)\right)=\min\left(\widetilde{\gamma}^{P}_{FD_1}(u_1u_2),\widetilde{\gamma}^{P}_{FB_2}(v)\right)\right),\\ \left(\widetilde{\gamma}^{N}_{FD_1\times FD_2}\left((u_1,v)(u_2,v)\right)=\max\left(\widetilde{\gamma}^{N}_{FD_1}(u_1u_2),\widetilde{\gamma}^{N}_{FB_2}(v)\right)\right)\end{array}\right)\end{array}\right)$$

$$\forall(u,v)\in(V_1,V_2)$$

Example 4.2. Let $G_1=\left(\left({M_1}^P,{N_1}^P\right),\left({M_1}^N,{N_1}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^*=(V_1,E_1)$ where $v_1=\{u,v,w\},E=\{uv,vw,uw\}$

$$ {M_1}^P=\left\langle\begin{array}{c}\{u,([0.1,0.1],0.4),([0.3,0.4],0.2),([0.5,0.6],0.1)\}\\ \{v,([0.1,0.3],0.1),([0.4,0.5],0.3),([0.1,0.1],0.2)\}\\ \{w,([0.2,0.3],0.1),([0.1,0.2],0.6),([0.3,0.5],0.2)\}\end{array}\right\rangle $$

$$ {N_1}^P=\left\langle\begin{array}{c}\{uv,([0.1,0.1],0.4),([0.3,0.4],0.2),([0.5,0.6],0.1)\}\\ \{vw,([0.1,0.3],0.1),([0.1,0.2],0.6),([0.3,0.5],0.2)\}\\ \{uw,([0.2,0.3],0.1),([0.1,0.2],0.6),([0.5,0.6],0.1)\}\end{array}\right\rangle $$

$$ {M_1}^N=\left\langle\begin{array}{c}\{u,([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.5,-0.6],-0.1)\}\\ \{v,([-0.1,-0.3],-0.1),([-0.4,-0.5],-0.3),([-0.1,-0.1],-0.2)\}\\ \{w,([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\}\end{array}\right\rangle $$

$$ {N_1}^N=\left\langle\begin{array}{c}\{uv,([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.5,-0.6],-0.1)\}\\ \{vw,([-0.1,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\}\\ \{uw,([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.1)\}\end{array}\right\rangle $$

and $G_2=\left(\left({M_2}^P,{N_2}^P\right),\left({M_2}^N,{N_2}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^*=(V_2,E_2)$ where $V_1=\{a,v,c\}$ and $E_2=\{ab,bc,ac\}$

$$ {M_2}^P=\left\langle\begin{array}{c}\{a,([0.6,0.7],0.5),([0.1,0.3],0.4),([0.2,0.3],0.6)\}\\ \{b,([0.1,0.2],0.3),([0.5,0.6],0.2),([0.8,0.9],0.4)\}\\ \{c,([0.3,0.4],0.1),([0.2,0.3],0.1),([0.5,0.6],0.3)\}\end{array}\right\rangle $$

$$ {N_2}^P=\left\langle\begin{array}{c}\{ab,([0.1,0.2],0.5),([0.1,0.3],0.4),([0.8,0.9],0.4)\}\\ \{bc,([0.1,0.2],0.3),([0.2,0.3],0.2),([0.8,0.9],0.3)\}\\ \{ac,([0.3,0.4],0.5),([0.1,0.3],0.4),([0.5,0.6],0.3)\}\end{array}\right\rangle $$

$$M_2{}^N = \left\langle \begin{array}{l} \{a, ([-0.6, -0.7], -0.5), ([-0.1, -0.3], -0.4), ([-0.2, -0.3], -0.6)\} \\ \{b, ([-0.1, -0.2], -0.3), ([-0.5, -0.6], -0.2), ([-0.8, -0.9], -0.4)\} \\ \{c, ([-0.3, -0.4], -0.1), ([-0.2, -0.3], -0.1), ([-0.5, -0.6], -0.3)\} \end{array} \right\rangle$$

$$N_2{}^N = \left\langle \begin{array}{l} \{ab, ([-0.1, -0.2], -0.5), ([-0.1, -0.3], -0.4), ([-0.8, -0.9], -0.4)\} \\ \{bc, ([-0.1, -0.2], -0.3), ([-0.2, -0.3], -0.2), ([-0.8, -0.9], -0.3)\} \\ \{ac, ([-0.3, -0.4], -0.5), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.3)\} \end{array} \right\rangle$$

then $G_1 \times G_2$ is a bipolar neutrosophic cubic fuzzy graph of $G_1^* \times G_2^*$, where $V_1 \times V_2 = \{(u,a), (u,b), (u,c), (v,a), (v,b), (v,c), (w,a), (w,b), (w,c)\}$ and

$$M_1^P \times M_2^P = \left\langle \begin{array}{l} \{(u,a), ([0.1, 0.1], 0.5), ([0.1, 0.3], 0.4), ([0.5, 0.6], 0.1)\} \\ \{(u,b), ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.8, 0.9], 0.1)\} \\ \{(u,c), ([0.1, 0.1], 0.6), ([0.2, 0.3], 0.2), ([0.5, 0.6], 0.1)\} \\ \{(v,a), ([0.1, 0.3], 0.5), ([0.1, 0.3], 0.4), ([0.2, 0.3], 0.2)\} \\ \{(v,b), ([0.1, 0.2], 0.3), ([0.4, 0.5], 0.3), ([0.8, 0.9], 0.2)\} \\ \{(v,c), ([0.1, 0.3], 0.1), ([0.2, 0.3], 0.3), ([0.5, 0.6], 0.2)\} \\ \{(w,a), ([0.2, 0.3], 0.5), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\} \\ \{(w,b), ([0.1, 0.2], 0.3), ([0.1, 0.2], 0.6), ([0.8, 0.9], 0.2)\} \\ \{(w,c), ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.5, 0.6], 0.2)\} \end{array} \right\rangle$$

$$M_1^N \times M_2^N = \left\langle \begin{array}{l} \{(u,a), ([-0.1, -0.1], -0.5), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.1)\} \\ \{(u,b), ([-0.1, -0.1], -0.4), ([-0.3, -0.4], -0.2), ([-0.8, -0.9], -0.1)\} \\ \{(u,c), ([-0.1, -0.1], -0.6), ([-0.2, -0.3], -0.2), ([-0.5, -0.6], -0.1)\} \\ \{(v,a), ([-0.1, -0.3], -0.5), ([-0.1, -0.3], -0.4), ([-0.2, -0.3], -0.2)\} \\ \{(v,b), ([-0.1, -0.2], -0.3), ([-0.4, -0.5], -0.3), ([-0.8, -0.9], -0.2)\} \\ \{(v,c), ([-0.1, -0.3], -0.1), ([-0.2, -0.3], -0.3), ([-0.5, -0.6], -0.2)\} \\ \{(w,a), ([-0.2, -0.3], -0.5), ([-0.1, -0.2], -0.6), ([-0.3, -0.5], -0.2)\} \\ \{(w,b), ([-0.1, -0.2], -0.3), ([-0.1, -0.2], -0.6), ([-0.8, -0.9], -0.2)\} \\ \{(w,c), ([-0.2, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.5, -0.6], -0.2)\} \end{array} \right\rangle$$

$$N_1^P \times N_2^P = \left\langle \begin{array}{l} \{((u,a),(u,b)), ([0.1, 0.1], 0.5), ([0.1, 0.3], 0.4), ([0.8, 0.9], 0.1)\} \\ \{((u,b),(u,c)), ([0.1, 0.1], 0.4), ([0.2, 0.3], 0.2), ([0.8, 0.9], 0.1)\} \\ \{((u,a),(v,c)), ([0.1, 0.1], 0.4), ([0.1, 0.3], 0.4), ([0.5, 0.6], 0.2)\} \\ \{((v,a),(v,c)), ([0.1, 0.3], 0.5), ([0.1, 0.3], 0.4), ([0.5, 0.6], 0.2)\} \\ \{((v,a),(v,b)), ([0.1, 0.2], 0.5), ([0.1, 0.3], 0.4), ([0.8, 0.9], 0.2)\} \\ \{((v,b),(w,b)), ([0.1, 0.2], 0.3), ([0.1, 0.2], 0.6), ([0.8, 0.9], 0.2)\} \\ \{((w,b),(w,c)), ([0.1, 0.2], 0.3), ([0.1, 0.2], 0.6), ([0.8, 0.9], 0.2)\} \\ \{((w,a),(w,c)), ([0.2, 0.3], 0.5), ([0.1, 0.2], 0.6), ([0.5, 0.6], 0.2)\} \\ \{((u,ab),(w,a)), ([0.1, 0.1], 0.5), ([0.1, 0.2], 0.6), ([0.5, 0.6], 0.1)\} \end{array} \right\rangle$$

$$N_1^N \times N_2^N = \left\langle \begin{array}{l} \{((u,a),(u,b)), ([-0.1, -0.1], -0.5), ([-0.1, -0.3], -0.4), ([-0.8, -0.9], -0.1)\} \\ \{((u,b),(u,c)), ([-0.1, -0.1], -0.4), ([-0.2, -0.3], -0.2), ([-0.8, -0.9], -0.1)\} \\ \{((u,a),(v,c)), ([-0.1, -0.1], -0.4), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.2)\} \\ \{((v,a),(v,c)), ([-0.1, -0.3], -0.5), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.2)\} \\ \{((v,a),(v,b)), ([-0.1, -0.2], -0.5), ([-0.1, -0.3], -0.4), ([-0.8, -0.9], -0.2)\} \\ \{((v,b),(w,b)), ([-0.1, -0.2], -0.3), ([-0.1, -0.2], -0.6), ([-0.8, -0.9], -0.2)\} \\ \{((w,b),(w,c)), ([-0.1, -0.2], -0.3), ([-0.1, -0.2], -0.6), ([-0.8, -0.9], -0.2)\} \\ \{((w,a),(w,c)), ([-0.2, -0.3], -0.5), ([-0.1, -0.2], -0.6), ([-0.5, -0.6], -0.2)\} \\ \{((u,ab),(w,a)), ([-0.1, -0.1], -0.5), ([-0.1, -0.2], -0.6), ([-0.5, -0.6], -0.1)\} \end{array} \right\rangle$$

Definition 4.3. Let $G_1 = \left(\left(M_1{}^P, N_1{}^P\right), \left(M_1{}^N, N_1{}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = (V_1, E_1)$ and $G_2 = \left(\left(M_2{}^P, N_2{}^P\right), \left(M_2{}^N, N_2{}^N\right)\right)$ be a Bipolar neutrosophic cubic fuzzy graph of $G_2^* = (V_2, E_2)$. Then composition of G_1

and G_2 is denoted by $G_1[G_2]$ and defined as follows

$$\begin{aligned}
G_1\left[G_2\right] &= \left(\left({M_1}^P, {N_1}^P\right), \left({M_1}^N, {N_1}^N\right)\right) \ \left[\left({M_2}^P, {N_2}^P\right), \left({M_2}^N, {N_2}^N\right)\right] \\
&= \left\{\left(M_1^P, M_1^N\right)\left[M_2^P, M_2^N\right], \left(N_1^P, N_1^N\right)\left[N_2^P, N_2^N\right]\right\} \\
&= \left\{ \begin{array}{l} \left(\left(A_1^P, A_1^N\right), \left(B_1^P, B_1^N\right)\right)\left[\left(\left(A_2^P, A_2^N\right), \left(B_2^P, B_2^N\right)\right)\right] \\ \left(\left(C_1^P, D_1^N\right), \left(C_1^P, D_1^N\right)\right)\left[\left(\left(C_2^P, D_2^N\right), \left(C_2^P, D_2^N\right)\right)\right] \end{array} \right\} \\
&= \left\{ \begin{array}{l} \left(A_1^P, A_1^N\right), \left[A_2^P, A_2^N\right], \left(B_1^P, B_1^N\right), \left[B_2^P, B_2^N\right] \\ \left(C_1^P, C_1^N\right), \left[C_2^P, C_2^N\right], \left(D_1^P, D_1^N\right), \left[D_2^P, D_2^N\right] \end{array} \right\} \\
&= \left\{ \begin{array}{l} \left\langle \begin{array}{l} \left(\left(\widetilde{\mu}_{TA_1}^P, \widetilde{\mu}_{TA_1}^N\right) \circ \left(\widetilde{\mu}_{TA_2}^P, \widetilde{\mu}_{TA_2}^N\right)\right), \left(\left(\widetilde{\mu}_{TB_1}^P, \widetilde{\mu}_{TB_1}^N\right) \circ \left(\widetilde{\mu}_{TB_2}^P, \widetilde{\mu}_{TB_2}^N\right)\right), \\ \left(\left(\widetilde{\lambda}_{IA_1}^P, \widetilde{\lambda}_{IA_1}^N\right) \circ \left(\widetilde{\lambda}_{IA_2}^P, \widetilde{\lambda}_{IA_2}^N\right)\right), \left(\left(\widetilde{\lambda}_{IB_1}^P, \widetilde{\lambda}_{IB_1}^N\right) \circ \left(\widetilde{\lambda}_{IB_2}^P, \widetilde{\lambda}_{IB_2}^N\right)\right), \\ \left(\left(\widetilde{\gamma}_{FA_1}^P, \widetilde{\gamma}_{FA_1}^N\right) \circ \left(\widetilde{\gamma}_{FA_2}^P, \widetilde{\gamma}_{FA_2}^N\right)\right), \left(\left(\widetilde{\gamma}_{FB_1}^P, \widetilde{\gamma}_{FB_1}^N\right) \circ \left(\widetilde{\gamma}_{FB_2}^P, \widetilde{\gamma}_{FB_2}^N\right)\right) \end{array} \right\rangle \\ \left\langle \begin{array}{l} \left(\left(\widetilde{\mu}_{TC_1}^P, \widetilde{\mu}_{TC_1}^N\right) \circ \left(\widetilde{\mu}_{TC_2}^P, \widetilde{\mu}_{TC_2}^N\right)\right), \left(\left(\widetilde{\mu}_{TD_1}^P, \widetilde{\mu}_{TD_1}^N\right) \circ \left(\widetilde{\mu}_{TD_2}^P, \widetilde{\mu}_{TD_2}^N\right)\right), \\ \left(\left(\widetilde{\lambda}_{IC_1}^P, \widetilde{\lambda}_{IC_1}^N\right) \circ \left(\widetilde{\lambda}_{IC_2}^P, \widetilde{\lambda}_{IC_2}^N\right)\right), \left(\left(\widetilde{\lambda}_{ID_1}^P, \widetilde{\lambda}_{ID_1}^N\right) \circ \left(\widetilde{\lambda}_{ID_2}^P, \widetilde{\lambda}_{ID_2}^N\right)\right), \\ \left(\left(\widetilde{\gamma}_{FC_1}^P, \widetilde{\gamma}_{FC_1}^N\right) \circ \left(\widetilde{\gamma}_{FC_2}^P, \widetilde{\gamma}_{FC_2}^N\right)\right), \left(\left(\widetilde{\gamma}_{FD_1}^P, \widetilde{\gamma}_{FD_1}^N\right) \circ \left(\widetilde{\gamma}_{FD_2}^P, \widetilde{\gamma}_{FD_2}^N\right)\right) \end{array} \right\rangle \end{array} \right\}
\end{aligned}$$

where

(i) $\forall\left(\left(u^P, u^N\right)\left(v^P, v^N\right)\right) \in \left(V_1, V_2\right)$

$$\left\{ \begin{array}{l} \left(\widetilde{\mu}_{TA_1}^P \circ \widetilde{\mu}_{TA_2}^P\right)\left(u^P, v^P\right) = r^P \min\left(\widetilde{\mu}_{TA_1}^P\left(u^P\right), \widetilde{\mu}_{TA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\mu}_{TB_1}^P \circ \widetilde{\mu}_{TB_2}^P\right)\left(u^P, v^P\right) = \max\left(\widetilde{\mu}_{TB_1}^P\left(u^P\right), \widetilde{\mu}_{TB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\mu}_{TA_1}^N \circ \widetilde{\mu}_{TA_2}^N\right)\left(u^N, v^N\right) = r^N \max\left(\widetilde{\mu}_{TA_1}^N\left(u^N\right), \widetilde{\mu}_{TA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\mu}_{TB_1}^N \circ \widetilde{\mu}_{TB_2}^N\right)\left(u^N, v^N\right) = \min\left(\widetilde{\mu}_{TB_1}^N\left(u^N\right), \widetilde{\mu}_{TB_2}^N\left(v^N\right)\right) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \left(\widetilde{\lambda}_{IA_1}^P \circ \widetilde{\lambda}_{IA_2}^P\right)\left(u^P, v^P\right) = r^P \min\left(\widetilde{\lambda}_{IA_1}^P\left(u^P\right), \widetilde{\lambda}_{IA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\lambda}_{IB_1}^P \circ \widetilde{\lambda}_{IB_2}^P\right)\left(u^P, v^P\right) = \max\left(\widetilde{\lambda}_{IB_1}^P\left(u^P\right), \widetilde{\lambda}_{IB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\lambda}_{IA_1}^N \circ \widetilde{\lambda}_{IA_2}^N\right)\left(u^N, v^N\right) = r^N \max\left(\widetilde{\lambda}_{IA_1}^N\left(u^N\right), \widetilde{\lambda}_{IA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\lambda}_{IB_1}^N \circ \widetilde{\lambda}_{IB_2}^N\right)\left(u^N, v^N\right) = \min\left(\widetilde{\lambda}_{IB_1}^N\left(u^N\right), \widetilde{\lambda}_{IB_2}^N\left(v^N\right)\right) \end{array} \right\}$$

$$\left\{ \begin{array}{l} \left(\widetilde{\gamma}_{FA_1}^P \circ \widetilde{\gamma}_{FA_2}^P\right)\left(u^P, v^P\right) = r^P \max\left(\widetilde{\gamma}_{FA_1}^P\left(u^P\right), \widetilde{\gamma}_{FA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\gamma}_{FB_1}^P \circ \widetilde{\gamma}_{FB_2}^P\right)\left(u^P, v^P\right) = \min\left(\widetilde{\gamma}_{FB_1}^P\left(u^P\right), \widetilde{\gamma}_{FB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\gamma}_{FA_1}^N \circ \widetilde{\gamma}_{FA_2}^N\right)\left(u^N, v^N\right) = r^N \min\left(\widetilde{\gamma}_{FA_1}^N\left(u^N\right), \widetilde{\gamma}_{FA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\gamma}_{FB_1}^N \circ \widetilde{\gamma}_{FB_2}^N\right)\left(u^N, v^N\right) = \max\left(\widetilde{\gamma}_{FB_1}^N\left(u^N\right), \widetilde{\gamma}_{FB_2}^N\left(v^N\right)\right) \end{array} \right\}$$

(ii) $\forall\left(u^P, u^N\right) \in V_1$ and $(v_1^P v_2^P)(v_1^N v_2^N) \in E$

$$\left\{ \begin{array}{l} \left(\widetilde{\mu}_{TC_1}^P \circ \widetilde{\mu}_{TC_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = r^P \min\left(\widetilde{\mu}_{TC_1}^P\left(u^P\right), \widetilde{\mu}_{TC_2}^P\left(v_1^P v_2^P\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^P \circ \widetilde{\mu}_{TD_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = \max\left(\widetilde{\mu}_{TD_1}^P\left(u^P\right), \widetilde{\mu}_{TD_2}^P\left(v_1^P v_2^P\right)\right) \\ \left(\widetilde{\mu}_{TC_1}^N \circ \widetilde{\mu}_{TC_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = r^N \max\left(\widetilde{\mu}_{TC_1}^N\left(u^N\right), \widetilde{\mu}_{TC_2}^N\left(v_1^N v_2^N\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^N \circ \widetilde{\mu}_{TD_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = \min\left(\widetilde{\mu}_{TD_1}^N\left(u^N\right), \widetilde{\mu}_{TD_2}^N\left(v_1^N v_2^N\right)\right) \end{array} \right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\lambda}_{IC_1}^P \circ \widetilde{\lambda}_{IC_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = r^P \min\left(\widetilde{\lambda}_{IC_1}^P\left(u^P\right), \widetilde{\lambda}_{IC_2}^P\left(v_1^P v_2^P\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^P \circ \widetilde{\lambda}_{ID_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = \max\left(\widetilde{\lambda}_{ID_1}^P\left(u^P\right), \widetilde{\lambda}_{ID_2}^P\left(v_1^P v_2^P\right)\right) \\ \left(\widetilde{\lambda}_{IC_1}^N \circ \widetilde{\lambda}_{IC_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = r^N \max\left(\widetilde{\lambda}_{IC_1}^N\left(u^N\right), \widetilde{\lambda}_{IC_2}^N\left(v_1^N v_2^N\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^N \circ \widetilde{\lambda}_{ID_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = \min\left(\widetilde{\lambda}_{ID_1}^N\left(u^N\right), \widetilde{\lambda}_{ID_2}^N\left(v_1^N v_2^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\gamma}_{FC_1}^P \circ \widetilde{\gamma}_{FC_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = r^P \max\left(\widetilde{\gamma}_{FC_1}^P\left(u^P\right), \widetilde{\gamma}_{FC_2}^P\left(v_1^P v_2^P\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^P \circ \widetilde{\gamma}_{FD_2}^P\right)\left(\left(u^P, {v_1}^P\right)\left(u^P, {v_2}^P\right)\right) = \min\left(\widetilde{\gamma}_{FD_1}^P\left(u^P\right), \widetilde{\gamma}_{FD_2}^P\left(v_1^P v_2^P\right)\right) \\ \left(\widetilde{\gamma}_{FC_1}^N \circ \widetilde{\gamma}_{FC_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = r^N \min\left(\widetilde{\gamma}_{FC_1}^N\left(u^N\right), \widetilde{\gamma}_{FC_2}^N\left(v_1^N v_2^N\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^N \circ \widetilde{\gamma}_{FD_2}^N\right)\left(\left(u^N, {v_1}^N\right)\left(u^N, {v_2}^N\right)\right) = \max\left(\widetilde{\gamma}_{FD_1}^N\left(u^N\right), \widetilde{\gamma}_{FD_2}^N\left(v_1^N v_2^N\right)\right) \end{array}\right\}$$

(iii) $\forall\left(v^P, v^N\right) \in V_1$and $(u_1^P u_2^P)(u_1^N u_2^N) \in E_1$

$$\left\{\begin{array}{l} \left(\widetilde{\mu}_{TC_1}^P \circ \widetilde{\mu}_{TC_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = r^P \min\left(\widetilde{\mu}_{TC_1}^P\left(u_1^P u_2^P\right), \widetilde{\mu}_{TA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^P \circ \widetilde{\mu}_{TD_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = \max\left(\widetilde{\mu}_{TD_1}^P\left(u_1^P u_2^P\right), \widetilde{\mu}_{TB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\mu}_{TC_1}^N \circ \widetilde{\mu}_{TC_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = r^N \max\left(\widetilde{\mu}_{TC_1}^N\left(u_1^N u_2^N\right), \widetilde{\mu}_{TA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^N \circ \widetilde{\mu}_{TD_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = \min\left(\widetilde{\mu}_{TD_1}^N\left(u_1^N u_2^N\right), \widetilde{\mu}_{TB_2}^N\left(v^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\lambda}_{IC_1}^P \circ \widetilde{\lambda}_{IC_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = r^P \min\left(\widetilde{\lambda}_{IC_1}^P\left(u_1^P u_2^P\right), \widetilde{\lambda}_{IA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^P \circ \widetilde{\lambda}_{ID_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = \max\left(\widetilde{\lambda}_{ID_1}^P\left(u_1^P u_2^P\right), \widetilde{\lambda}_{IB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\lambda}_{IC_1}^N \circ \widetilde{\lambda}_{IC_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = r^N \max\left(\widetilde{\lambda}_{IC_1}^N\left(u_1^N u_2^N\right), \widetilde{\lambda}_{IA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^N \circ \widetilde{\lambda}_{ID_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = \min\left(\widetilde{\lambda}_{ID_1}^N\left(u_1^N u_2^N\right), \widetilde{\lambda}_{IB_2}^N\left(v^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\gamma}_{FC_1}^P \circ \widetilde{\gamma}_{FC_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = r^P \max\left(\widetilde{\gamma}_{FC_1}^P\left(u_1^P u_2^P\right), \widetilde{\gamma}_{FA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^P \circ \widetilde{\gamma}_{FD_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = \min\left(\widetilde{\gamma}_{FD_1}^P\left(u_1^P u_2^P\right), \widetilde{\gamma}_{FB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\gamma}_{FC_1}^N \circ \widetilde{\gamma}_{FC_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = r^N \min\left(\widetilde{\gamma}_{FC_1}^N\left(u_1^N u_2^N\right), \widetilde{\gamma}_{FA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^N \circ \widetilde{\gamma}_{FD_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = \max\left(\widetilde{\gamma}_{FD_1}^N\left(u_1^N u_2^N\right), \widetilde{\gamma}_{FB_2}^N\left(v^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\lambda}_{IC_1}^P \circ \widetilde{\lambda}_{IC_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = r^P \min\left(\widetilde{\lambda}_{IC_1}^P\left(u_1^P u_2^P\right), \widetilde{\lambda}_{IA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^P \circ \widetilde{\lambda}_{ID_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = \max\left(\widetilde{\lambda}_{ID_1}^P\left(u_1^P u_2^P\right), \widetilde{\lambda}_{IB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\lambda}_{IC_1}^N \circ \widetilde{\lambda}_{IC_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = r^N \max\left(\widetilde{\lambda}_{IC_1}^N\left(u_1^N u_2^N\right), \widetilde{\lambda}_{IA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\lambda}_{ID_1}^N \circ \widetilde{\lambda}_{ID_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = \min\left(\widetilde{\lambda}_{ID_1}^N\left(u_1^N u_2^N\right), \widetilde{\lambda}_{IB_2}^N\left(v^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l} \left(\widetilde{\gamma}_{FC_1}^P \circ \widetilde{\gamma}_{FC_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = r^P \max\left(\widetilde{\gamma}_{FC_1}^P\left(u_1^P u_2^P\right), \widetilde{\gamma}_{FA_2}^P\left(v^P\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^P \circ \widetilde{\gamma}_{FD_2}^P\right)\left(\left({u_1}^P, v^P\right)\left({u_2}^P, v^P\right)\right) = \min\left(\widetilde{\gamma}_{FD_1}^P\left(u_1^P u_2^P\right), \widetilde{\gamma}_{FB_2}^P\left(v^P\right)\right) \\ \left(\widetilde{\gamma}_{FC_1}^N \circ \widetilde{\gamma}_{FC_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = r^N \min\left(\widetilde{\gamma}_{FC_1}^N\left(u_1^N u_2^N\right), \widetilde{\gamma}_{FA_2}^N\left(v^N\right)\right), \\ \left(\widetilde{\gamma}_{FD_1}^N \circ \widetilde{\gamma}_{FD_2}^N\right)\left(\left({u_1}^N, v^N\right)\left({u_2}^N, v^N\right)\right) = \max\left(\widetilde{\gamma}_{FD_1}^N\left(u_1^N u_2^N\right), \widetilde{\gamma}_{FB_2}^N\left(v^N\right)\right) \end{array}\right\}$$

(iv) $\forall\left(\left(u_1^P, v_1^P\right)\left(u_2^P, v_2^P\right)\right), \left(\left(u_1^N, v_1^N\right)\left(u_2^N, v_2^N\right)\right) \in E^\circ - E$

$$\left\{\begin{array}{l} \left(\widetilde{\mu}_{TC_1}^P \circ \widetilde{\mu}_{TC_2}^P\right)\left(\left(u_1^P, v_1^P\right)\left(u_2^P, v_2^P\right)\right) = r^P \min\left(\widetilde{\mu}_{TA_2}^P\left({v_1}^P\right), \widetilde{\mu}_{TA_2}^P\left({v_2}^P\right), \widetilde{\mu}_{TC_1}^P\left(u_1^P u_2^P\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^P \circ \widetilde{\mu}_{TD_2}^P\right)\left(\left(u_1^P, v_1^P\right)\left(u_2^P, v_2^P\right)\right) = \max\left(\widetilde{\mu}_{TB_2}^P\left({v_1}^P\right), \widetilde{\mu}_{TB_2}^P\left({v_2}^P\right), \widetilde{\mu}_{TD_1}^P\left(u_1^P u_2^P\right)\right) \\ \left(\widetilde{\mu}_{TC_1}^N \circ \widetilde{\mu}_{TC_2}^N\right)\left(\left(u_1^N, v_1^N\right)\left(u_2^N, v_2^N\right)\right) = r^N \max\left(\widetilde{\mu}_{TA_2}^N\left({v_1}^N\right), \widetilde{\mu}_{TA_2}^N\left({v_2}^N\right), \widetilde{\mu}_{TC_1}^N\left(u_1^N u_2^N\right)\right), \\ \left(\widetilde{\mu}_{TD_1}^N \circ \widetilde{\mu}_{TD_2}^N\right)\left(\left(u_1^N, v_1^N\right)\left(u_2^N, v_2^N\right)\right) = \min\left(\widetilde{\mu}_{TB_2}^N\left({v_1}^N\right), \widetilde{\mu}_{TB_2}^N\left({v_2}^N\right), \widetilde{\mu}_{TD_1}^N\left(u_1^N u_2^N\right)\right) \end{array}\right\}$$

$$\left\{\begin{array}{l}\left(\widetilde{\lambda}^P_{IC_1}\circ\widetilde{\lambda}^P_{IC_2}\right)\left(\left(u_1^P,v_1^P\right)\left(u_2^P,v_2^P\right)\right)=r^P\min\left(\widetilde{\lambda}^P_{IA_2}\left(v_1{}^P\right),\widetilde{\lambda}^P_{IA_2}\left(v_2{}^P\right),\widetilde{\lambda}^P_{IC_1}\left(u_1^Pu_2^P\right)\right),\\ \left(\widetilde{\lambda}^P_{ID_1}\circ\widetilde{\lambda}^P_{ID_2}\right)\left(\left(u_1^P,v_1^P\right)\left(u_2^P,v_2^P\right)\right)=\max\left(\widetilde{\lambda}^P_{IB_2}\left(v_1{}^P\right),\widetilde{\lambda}^P_{IB_2}\left(v_2{}^P\right),\widetilde{\lambda}^P_{ID_1}\left(u_1^Pu_2^P\right)\right)\\ \left(\widetilde{\lambda}^N_{IC_1}\circ\widetilde{\lambda}^N_{IC_2}\right)\left(\left(u_1^N,v_1^N\right)\left(u_2^N,v_2^N\right)\right)=r^N\max\left(\widetilde{\lambda}^N_{IA_2}\left(v_1{}^N\right),\widetilde{\lambda}^N_{IA_2}\left(v_2{}^N\right),\widetilde{\lambda}^N_{IC_1}\left(u_1^Nu_2^N\right)\right),\\ \left(\widetilde{\lambda}^N_{ID_1}\circ\widetilde{\lambda}^N_{ID_2}\right)\left(\left(u_1^N,v_1^N\right)\left(u_2^N,v_2^N\right)\right)=\min\left(\widetilde{\lambda}^N_{IB_2}\left(v_1{}^N\right),\widetilde{\lambda}^N_{IB_2}\left(v_2{}^N\right),\widetilde{\lambda}^N_{IB_1}\left(u_1^Nu_2^N\right)\right)\end{array}\right\}$$

$$\left\{\begin{array}{l}\left(\widetilde{\gamma}^P_{FC_1}\circ\widetilde{\gamma}^P_{FC_2}\right)\left(\left(u_1^P,v_1^P\right)\left(u_2^P,v_2^P\right)\right)=r^P\max\left(\widetilde{\gamma}^P_{FA_2}\left(v_1{}^P\right),\widetilde{\gamma}^P_{FA_2}\left(v_2{}^P\right),\widetilde{\gamma}^P_{FC_1}\left(u_1^Pu_2^P\right)\right),\\ \left(\widetilde{\gamma}^P_{FD_1}\circ\widetilde{\gamma}^P_{FD_2}\right)\left(\left(u_1^P,v_1^P\right)\left(u_2^P,v_2^P\right)\right)=\min\left(\widetilde{\gamma}^P_{FB_2}\left(v_1{}^P\right),\widetilde{\gamma}^P_{FB_2}\left(v_2{}^P\right),\widetilde{\gamma}^P_{FD_1}\left(u_1^Pu_2^P\right)\right)\\ \left(\widetilde{\gamma}^N_{FC_1}\circ\widetilde{\gamma}^N_{FC_2}\right)\left(\left(u_1^N,v_1^N\right)\left(u_2^N,v_2^N\right)\right)=r^N\min\left(\widetilde{\gamma}^N_{FA_2}\left(v_1{}^N\right),\widetilde{\gamma}^N_{FA_2}\left(v_2{}^N\right),\widetilde{\gamma}^N_{FC_1}\left(u_1^Nu_2^N\right)\right),\\ \left(\widetilde{\gamma}^N_{FD_1}\circ\widetilde{\gamma}^N_{FD_2}\right)\left(\left(u_1^N,v_1^N\right)\left(u_2^N,v_2^N\right)\right)=\max\left(\widetilde{\gamma}^N_{FB_2}\left(v_1{}^N\right),\widetilde{\gamma}^N_{FB_2}\left(v_2{}^N\right),\widetilde{\gamma}^N_{FD_1}\left(u_1^Nu_2^N\right)\right)\end{array}\right\}$$

Example 4.4. Let $G_1^* = (V_1, E_1)$ and $G_2^* = (V_2, E_2)$ be two fuzzy graphs, where $V_1 = (u, v)$ and $V_2 = (x, y)$. Suppose M_1 and M_2 be the bipolar neutrosophic fuzzy cubic set representations of V_1 and V_2. Also N_1 and N_2 be the bipolar neutrosophic fuzzy cubic set representations of E_1 and E_2 and defined as

$$M_1{}^P=\left\langle\begin{array}{l}\{u,([0.4,0.5],0.1),([0.1,0.1],0.4),([0.7,0.8],0.2)\}\\ \{v,([0.3,0.4],0.2),([0.1,0.2],0.1),([0.4,0.5],0.5)\}\end{array}\right\rangle$$

$$M_1{}^N=\left\langle\begin{array}{l}\{u,([-0.4,-0.5],-0.1),([-0.1,-0.1],-0.4),([-0.7,-0.8],-0.2)\}\\ \{v,([-0.3,-0.4],-0.2),([-0.1,-0.2],-0.1),([-0.4,-0.5],-0.5)\}\end{array}\right\rangle$$

$$N_1{}^P=\left\langle\ \{uv,([0.3,0.4],0.2),([0.1,0.1],0.4),([0.7,0.8],0.2)\}\ \right\rangle$$

$$N_1{}^N=\left\langle\ \{uv,([-0.3,-0.4],-0.2),([-0.1,-0.1],-0.4),([-0.7,-0.8],-0.2)\}\ \right\rangle$$

and

$$M_2{}^P=\left\langle\begin{array}{l}\{x,([0.5,0.6],0.3),([0.7,0.8],0.7),([0.1,0.1],0.5)\}\\ \{y,([0.2,0.3],0.6),([0.5,0.6],0.4),([0.8,0.9],0.8)\}\end{array}\right\rangle$$

$$M_2{}^N=\left\langle\begin{array}{l}\{x,([-0.5,-0.6],-0.3),([-0.7,-0.8],-0.7),([-0.7,-0.8],-0.2)\}\\ \{y,([-0.2,-0.3],-0.6),([-0.5,-0.6],-0.4),([-0.8,-0.9],-0.8)\}\end{array}\right\rangle$$

$$N_2{}^P=\left\langle\ \{xy,([0.2,0.3],0.6),([0.5,0.6],0.7),([0.8,0.9],0.5)\}\ \right\rangle$$

$$N_2{}^N=\left\langle\ \{xy,([-0.2,-0.3],-0.6),([-0.5,-0.6],-0.7),([-0.8,-0.9],-0.5)\}\ \right\rangle$$

Clearly $G_1 = \left(\left(M_1{}^P, N_1{}^P\right), \left(M_1{}^N, N_1{}^N\right)\right)$ and $G_2 = \left(\left(M_2{}^P, N_2{}^P\right), \left(M_2{}^N, N_2{}^N\right)\right)$ are bipolar neutrosophic cubic fuzzy graphs. So, the composition of two bipolar neutrosophic cubic fuzzy graphs G_1 and G_2 is again a bipolar neutrosophic cubic fuzzy graph, where

$$M_1^P\ \left[M_2^P\right]=\left\langle\begin{array}{l}\{(u,x),([0.4,0.5],0.3),([0.1,0.1],0.7),([0.7,0.8],0.2)\}\\ \{(u,y),([0.2,0.3],0.6),([0.1,0.1],0.4),([0.8,0.9],0.2)\}\\ \{(v,x),([0.3,0.4],0.3),([0.1,0.2],0.7),([0.4,0.5],0.5)\}\\ \{(v,y),([0.2,0.3],0.6),([0.1,0.2],0.4),([0.8,0.9],0.5)\}\end{array}\right\rangle$$

$$M_1^N\ \left[M_2^N\right]=\left\langle\begin{array}{l}\{(u,x),([-0.4,-0.5],-0.3),([-0.1,-0.1],-0.7),([-0.7,-0.8],-0.2)\}\\ \{(u,y),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.4),([-0.8,-0.9],-0.2)\}\\ \{(v,x),([-0.3,-0.4],-0.3),([-0.1,-0.2],-0.7),([-0.4,-0.5],-0.5)\}\\ \{(v,y),([-0.2,-0.3],-0.6),([-0.1,-0.2],-0.4),([-0.8,-0.9],-0.5)\}\end{array}\right\rangle$$

$$N_1^P \left[N_2^P\right] = \left\langle \begin{array}{l} \{((u,x),(u,y)),([0.2,0.3],0.6),([0.1,0.1],0.7),([0.8,0.9],0.2)\} \\ \{((u,y),(v,y)),([0.2,0.3],0.6),([0.1,0.1],0.4),([0.8,0.9],0.2)\} \\ \{((v,y),(v,x)),([0.2,0.3],0.6),([0.1,0.2],0.7),([0.8,0.9],0.5)\} \\ \{((v,x),(u,x)),([0.3,0.4],0.3),([0.1,0.1],0.7),([0.7,0.8],0.2)\} \\ \{((u,x),(v,y)),([0.2,0.3],0.6),([0.1,0.1],0.7),([0.8,0.9],0.2)\} \\ \{((u,y),(v,x)),([0.2,0.3],0.6),([0.1,0.1],0.7),([0.8,0.9],0.2)\} \end{array} \right\rangle$$

$$N_1^N \left[N_2^N\right] = \left\langle \begin{array}{l} \{((u,x),(u,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \\ \{((u,y),(v,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.4),([-0.8,-0.9],-0.2)\} \\ \{((v,y),(v,x)),([-0.2,-0.3],-0.6),([-0.1,-0.2],-0.7),([-0.8,-0.9],-0.5)\} \\ \{((v,x),(u,x)),([-0.3,-0.4],-0.3),([-0.1,-0.1],-0.7),([-0.7,-0.8],-0.2)\} \\ \{((u,x),(v,y)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \\ \{((u,y),(v,x)),([-0.2,-0.3],-0.6),([-0.1,-0.1],-0.7),([-0.8,-0.9],-0.2)\} \end{array} \right\rangle$$

Proposition 4.5. *Let* $G_1 = \left(\left({M_1}^P, {N_1}^P\right), \left({M_1}^N, {N_1}^N\right)\right)$ *and* $G_2 = \left(\left({M_2}^P, {N_2}^P\right), \left({M_2}^N, {N_2}^N\right)\right)$ *be two bipolar neutrosophic cubic fuzzy graphs, then the Cartesian product of two bipolar neutrosophic cubic fuzzy graphs is again a bipolar neutrosophic cubic fuzzy graph.*

Proof. Condition is oblivious for $\left({M_1}^P, {M_1}^N\right) \times \left({M_2}^P, {M_2}^N\right)$. Therefore we verify conditions only for $\left({N_1}^P, {N_1}^N\right) \times \left({N_2}^P, {N_2}^N\right)$, where

$$\left({N_1}^P, {N_1}^N\right) \times \left({N_2}^P, {N_2}^N\right) = \left\{ \begin{array}{l} \left(\left(\widetilde{\mu}^P_{TC_1 \times TC_2}, \widetilde{\mu}^P_{TD_1 \times TD_2}\right), \left(\widetilde{\mu}^N_{TC_1 \times TC_2}, \widetilde{\mu}^N_{TD_1 \times TD_2}\right)\right), \\ \left(\left(\widetilde{\lambda}^P_{IC_1 \times IC_2, ID_1 \times ID_2}\right), \left(\widetilde{\lambda}^P_{IC_1 \times IC_2, ID_1 \times ID_2}\right)\right), \\ \left(\left(\widetilde{\gamma}^P_{FC_1 \times FC_2, FD_1 \times FD_2}\right), \left(\widetilde{\gamma}^N_{FC_1 \times FC_2, FD_1 \times FD_2}\right)\right) \end{array} \right\}$$

Let $\left(u^P, u^N\right) \in V_1$ and $u_2 v_2 \in E_2$. Then

$$\begin{array}{l}
\left(\widetilde{\mu}^P_{TC_1 \times TC_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right)\right), \widetilde{\mu}^N_{TC_1 \times TC_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right)\right)\right) \\
= \left\{ r^P \min\left\{\left(\widetilde{\mu}^P_{TA_1}\left(u^P\right), \widetilde{\mu}^P_{TC_2}\left({u_2}^P, v_2^P\right)\right)\right\}, r^N \max\left\{\left(\widetilde{\mu}^N_{TA_1}\left(u^P\right), \widetilde{\mu}^N_{TC_2}\left({u_2}^P, v_2^P\right)\right)\right\} \right\} \\
\leq \left\{ \begin{array}{l} r^P \min\left\{\left(\widetilde{\mu}^P_{TA_1}\left(u^P\right),\ r^P \min\left(\widetilde{\mu}^P_{TA_2}\left({u_2}^P\right), \widetilde{\mu}^P_{TA_2}\left({v_2}^P\right)\right)\right)\right\}, \\ r^N \max\left\{\left(\widetilde{\mu}^N_{TA_1}\left(u^N\right),\ r^N \max\left(\widetilde{\mu}^N_{TA_2}\left({u_2}^N\right), \widetilde{\mu}^N_{TA_2}\left({v_2}^N\right)\right)\right)\right\} \end{array} \right\} \\
= \left\{ \begin{array}{l} r^P \min\left\{ r^P \min\left(\left(\widetilde{\mu}^P_{TA_1}\left(u^P\right), \widetilde{\mu}^P_{TA_2}\left({u_2}^P\right)\right), r^P \min\left(\widetilde{\mu}^P_{TA_1}\left(u^P\right), \widetilde{\mu}^P_{TA_2}\left({v_2}^P\right)\right)\right)\right\}, \\ r^N \max\left\{ r^N \max\left(\left(\widetilde{\mu}^N_{TA_1}\left(u^N\right), \widetilde{\mu}^N_{TA_2}\left({u_2}^N\right)\right), r^N \max\left(\widetilde{\mu}^N_{TA_1}\left(u^N\right), \widetilde{\mu}^N_{TA_2}\left({v_2}^N\right)\right)\right)\right\} \end{array} \right\} \\
= \left\{ \begin{array}{l} r^P \min\left\{\left(\widetilde{\mu}^P_{TA_1} \times \widetilde{\mu}^P_{TA_2}\right)\left(u^P, u_2^P\right), \left(\widetilde{\mu}^P_{TA_1} \times \widetilde{\mu}^P_{TA_2}\right)\left(u^P, v_2^P\right)\right\}, \\ r^N \max\left\{\left(\widetilde{\mu}^N_{TA_1} \times \widetilde{\mu}^N_{TA_2}\right)\left(u^N, u_2^N\right), \left(\widetilde{\mu}^N_{TA_1} \times \widetilde{\mu}^N_{TA_2}\right)\left(u^N, v_2^N\right)\right\} \end{array} \right\}
\end{array}$$

$$\begin{array}{l}
\left(\widetilde{\mu}^P_{TD_1 \times TD_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right)\right), \widetilde{\mu}^N_{TD_1 \times TD_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right)\right)\right) \\
= \left\{ \max\left\{\left(\widetilde{\mu}^P_{TB_1}\left(u^P\right), \widetilde{\mu}^P_{TD_2}\left({u_2}^P, v_2^P\right)\right)\right\}, \min\left\{\left(\widetilde{\mu}^N_{TB_1}\left(u^N\right), \widetilde{\mu}^N_{TD_2}\left({u_2}^N, v_2^N\right)\right)\right\} \right\} \\
\leq \left\{ \begin{array}{l} \max\left\{\left(\widetilde{\mu}^P_{TB_1}\left(u^P\right), max(\widetilde{\mu}^P_{TB_2}\left({u_2}^P\right)\right), \widetilde{\mu}^P_{TB_2}\left({v_2}^P\right)\right\}, \\ \min\left\{\left(\widetilde{\mu}^N_{TB_1}\left(u^N\right), min(\widetilde{\mu}^N_{TB_2}\left({u_2}^N\right)\right), \widetilde{\mu}^N_{TB_2}\left({v_2}^N\right)\right\} \end{array} \right\} \\
= \left\{ \begin{array}{l} \max\left\{ \max\left(\widetilde{\mu}^P_{TB_1}\left(u^P\right), \widetilde{\mu}^P_{TB_2}\left({u_2}^P\right)\right), \max\left(\widetilde{\mu}^P_{TB_1}\left(u^P\right), \widetilde{\mu}^P_{TB_2}\left(v_2^P\right)\right) \right\} \\ \min\left\{ \min\left(\widetilde{\mu}^N_{TB_1}\left(u^N\right), \widetilde{\mu}^N_{TB_2}\left({u_2}^N\right)\right), \min\left(\widetilde{\mu}^N_{TB_1}\left(u^N\right), \widetilde{\mu}^N_{TB_2}\left({v_2}^N\right)\right) \right\} \end{array} \right\} \\
= \left\{ \begin{array}{l} \max\left\{\left(\widetilde{\mu}^P_{TB_1} \times \widetilde{\mu}^P_{TB_2}\right)\left(u^P, u_2^P\right), \left(\left(\widetilde{\mu}^P_{TB_1} \times \widetilde{\mu}^P_{TB_2}\right)\left(u^P, v_2^P\right)\right)\right\}, \\ \min\left\{\left(\widetilde{\mu}^N_{TB_1} \times \widetilde{\mu}^N_{TB_2}\right)\left(u^N, u_2^N\right), \left(\left(\widetilde{\mu}^N_{TB_1} \times \widetilde{\mu}^N_{TB_2}\right)\left(u^N, v_2^N\right)\right)\right\} \end{array} \right\}
\end{array}$$

$$\left(\widetilde{\lambda}^P_{IC_1 \times IC_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right)\right), \widetilde{\lambda}^N_{IC_1 \times IC_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right)\right)\right)$$

$$= \left\{ r^P \min\left\{ \left(\widetilde{\lambda}^P_{IA_1}\left(u^P\right), \widetilde{\lambda}^P_{IC_2}\left(u_2{}^P, v_2^P\right) \right) \right\}, \quad \left\{ \left(\widetilde{\lambda}^N_{IA_1}\left(u^N\right), \widetilde{\lambda}^N_{IC_2}\left(u_2{}^N, v_2^N\right) \right) \right\} \right\}$$

$$\leq \left\{ \begin{array}{l} r^P \min\left\{ \left(\widetilde{\lambda}^P_{IA_1}\left(u^P\right),\ r^P \min\left(\widetilde{\lambda}^P_{IA_2}\left(u_2{}^P\right), \widetilde{\lambda}^P_{IA_2}\left(v_2{}^P\right) \right) \right) \right\}, \\ r^N \max\left\{ \left(\widetilde{\lambda}^N_{IA_1}\left(u^N\right),\ r^N \max\left(\widetilde{\lambda}^N_{IA_2}\left(u_2{}^N\right), \widetilde{\lambda}^N_{IA_2}\left(v_2{}^N\right) \right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} r^P \min\left\{ r^P \min\left(\left(\widetilde{\lambda}^P_{IA_1}\left(u^P\right), \widetilde{\lambda}^P_{IA_2}\left(u_2{}^P\right) \right), r^P \min\left(\widetilde{\lambda}^P_{IA_1}\left(u^P\right), \widetilde{\lambda}^P_{IA_2}\left(v_2{}^P\right) \right) \right) \right\}, \\ r^N \min\left\{ r^N \max\left(\left(\widetilde{\lambda}^N_{IA_1}\left(u^N\right), \widetilde{\lambda}^N_{IA_2}\left(u_2{}^N\right) \right), r^N \min\left(\widetilde{\lambda}^N_{IA_1}\left(u^N\right), \widetilde{\lambda}^N_{IA_2}\left(v_2{}^N\right) \right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} r^P \min\left\{ \left(\widetilde{\lambda}^P_{IA_1} \times \widetilde{\lambda}^P_{IA_2} \right)\left(u^P, u_2^P\right), \left(\widetilde{\lambda}^P_{IA_1} \times \widetilde{\lambda}^P_{IA_2} \right)\left(u^P, v_2^P\right) \right\}, \\ r^N \max\left\{ \left(\widetilde{\lambda}^N_{IA_1} \times \widetilde{\lambda}^N_{IA_2} \right)\left(u^N, u_2^N\right), \left(\widetilde{\lambda}^N_{IA_1} \times \widetilde{\lambda}^N_{IA_2} \right)\left(u^N, v_2^N\right) \right\} \end{array} \right\}$$

$$\left(\widetilde{\lambda}^P_{ID_1 \times ID_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right) \right), \widetilde{\lambda}^N_{ID_1 \times ID_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right) \right) \right)$$

$$= \{ \max\left\{ \left(\widetilde{\lambda}^P_{IB_1}\left(u^P\right), \widetilde{\lambda}^P_{ID_2}\left(u_2{}^P, v_2^P\right) \right) \right\}, \min\left\{ \left(\widetilde{\lambda}^N_{IB_1}\left(u^N\right), \widetilde{\lambda}^N_{ID_2}\left(u_2{}^N, v_2^N\right) \right) \right\}$$

$$\leq \left\{ \begin{array}{l} \max\left\{ \left(\widetilde{\lambda}^P_{IB_1}\left(u^P\right), max(\widetilde{\lambda}^P_{IB_2}\left(u_2{}^P\right) \right), \widetilde{\lambda}^P_{IB_2}\left(v_2{}^P\right) \right\}, \\ \min\left\{ \left(\widetilde{\lambda}^N_{IB_1}\left(u^N\right), min(\widetilde{\lambda}^N_{IB_2}\left(u_2{}^N\right) \right), \widetilde{\lambda}^N_{IB_2}\left(v_2{}^N\right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \max\left\{ \max\left(\widetilde{\lambda}^P_{IB_1}\left(u^P\right), \widetilde{\lambda}^P_{IB_2}\left(u_2{}^P\right) \right), \max\left(\widetilde{\lambda}^P_{IB_1}\left(u^P\right), \widetilde{\lambda}^P_{IB_2}\left(v_2^P\right) \right) \right\} \\ \min\left\{ \min\left(\widetilde{\lambda}^N_{IB_1}\left(u^N\right), \widetilde{\lambda}^N_{IB_2}\left(u_2{}^N\right) \right), \min\left(\widetilde{\lambda}^N_{IB_1}\left(u^N\right), \widetilde{\lambda}^N_{IB_2}\left(v_2{}^N\right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \max\left\{ \left(\widetilde{\lambda}^P_{IB_1} \times \widetilde{\lambda}^P_{IB_2} \right)\left(u^P, u_2^P\right), \left(\left(\widetilde{\lambda}^P_{IB_1} \times \widetilde{\lambda}^P_{IB_2} \right)\left(u^P, v_2^P\right) \right) \right\}, \\ \min\left\{ \left(\widetilde{\lambda}^N_{IB_1} \times \widetilde{\lambda}^N_{IB_2} \right)\left(u^N, u_2^N\right), \left(\left(\widetilde{\lambda}^N_{IB_1} \times \widetilde{\lambda}^N_{IB_2} \right)\left(u^N, v_2^N\right) \right) \right\} \end{array} \right\}$$

$$\left(\widetilde{\gamma}^P_{FC_1 \times FC_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right) \right), \widetilde{\gamma}^N_{FC_1 \times FC_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right) \right) \right)$$

$$= \left\{ r^P \max\left\{ \left(\widetilde{\gamma}^P_{FA_1}\left(u^P\right), \widetilde{\gamma}^P_{FC_2}\left(u_2{}^P, v_2^P\right) \right) \right\}, \mathrm{r^N}\ \min\ \left\{ \left(\widetilde{\gamma}^N_{FA_1}\left(u^N\right), \widetilde{\gamma}^N_{FC_2}\left(u_2{}^N, v_2^N\right) \right) \right\} \right\}$$

$$\leq \left\{ \begin{array}{l} r^P \max\left\{ \left(\widetilde{\gamma}^P_{FA_1}\left(u^P\right),\ r^P \max\left(\widetilde{\gamma}^P_{FA_2}\left(u_2{}^P\right), \widetilde{\gamma}^P_{FA_2}\left(v_2{}^P\right) \right) \right) \right\}, \\ r^N \min\left\{ \left(\widetilde{\gamma}^N_{FA_1}\left(u^N\right),\ r^N \max\left(\widetilde{\gamma}^N_{FA_2}\left(u_2{}^N\right), \widetilde{\gamma}^N_{FA_2}\left(v_2{}^N\right) \right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} r^P \max\left\{ r^P \max\left(\left(\widetilde{\gamma}^P_{FA_1}\left(u^P\right), \widetilde{\gamma}^P_{FA_2}\left(u_2{}^P\right) \right), r^P \max\left(\widetilde{\gamma}^P_{FA_1}\left(u^P\right), \widetilde{\gamma}^P_{FA_2}\left(v_2{}^P\right) \right) \right) \right\}, \\ r^N \min\left\{ r^N \min\left(\left(\widetilde{\gamma}^N_{FA_1}\left(u^N\right), \widetilde{\gamma}^N_{FA_2}\left(u_2{}^N\right) \right), r^N \min\left(\widetilde{\gamma}^N_{FA_1}\left(u^N\right), \widetilde{\gamma}^N_{FA_2}\left(v_2{}^N\right) \right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} r^P \max\left\{ \left(\widetilde{\gamma}^P_{FA_1} \times \widetilde{\gamma}^P_{FA_2} \right)\left(u^P, u_2^P\right), \left(\widetilde{\gamma}^P_{FA_1} \times \widetilde{\gamma}^P_{FA_2} \right)\left(u^P, v_2^P\right) \right\}, \\ r^N \min\left\{ \left(\widetilde{\gamma}^N_{FA_1} \times \widetilde{\gamma}^N_{FA_2} \right)\left(u^N, u_2^N\right), \left(\widetilde{\gamma}^N_{FA_1} \times \widetilde{\gamma}^N_{FA_2} \right)\left(u^N, v_2^N\right) \right\} \end{array} \right\}$$

$$\left(\widetilde{\gamma}^P_{FD_1 \times FD_2}\left(\left(u^P, u_2^P\right)\left(u^P, v_2^P\right) \right), \widetilde{\gamma}^N_{FD_1 \times FD_2}\left(\left(u^N, u_2^N\right)\left(u^N, v_2^N\right) \right) \right)$$

$$= \{ \min\left\{ \left(\widetilde{\gamma}^P_{FB_1}\left(u^P\right), \widetilde{\gamma}^P_{FD_2}\left(u_2{}^P, v_2^P\right) \right) \right\}, \max\left\{ \left(\widetilde{\gamma}^N_{FB_1}\left(u^N\right), \widetilde{\gamma}^N_{FD_2}\left(u_2{}^N, v_2^N\right) \right) \right\}$$

$$\leq \left\{ \begin{array}{l} \min\left\{ min\left(\widetilde{\gamma}^P_{FB_1}\left(u^P\right), \widetilde{\gamma}^P_{FB_2}\left(u_2{}^P\right) \right), \min\left(\widetilde{\gamma}^P_{FB_1}\left(u^P\right), \widetilde{\gamma}^P_{FB_2}\left(v_2{}^P\right) \right) \right\}, \\ \max\left\{ \max\left(\widetilde{\gamma}^N_{FB_1}\left(u^N\right), \widetilde{\gamma}^N_{FB_2}\left(u_2{}^N\right) \right), \max\left(\widetilde{\gamma}^N_{FB_1}\left(u^N\right), \widetilde{\gamma}^N_{FB_2}\left(u_2{}^N\right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \min\left\{ \min\left(\widetilde{\gamma}^P_{FB_1}\left(u^P\right), \widetilde{\gamma}^P_{FB_2}\left(u_2{}^P\right) \right), \min\left(\widetilde{\gamma}^P_{FB_1}\left(u^P\right), \widetilde{\gamma}^P_{FB_2}\left(v_2^P\right) \right) \right\} \\ \max\left\{ \max\left(\widetilde{\gamma}^N_{FB_1}\left(u^N\right), \widetilde{\gamma}^N_{FB_2}\left(u_2{}^N\right) \right), \max\left(\widetilde{\gamma}^N_{FB_1}\left(u^N\right), \widetilde{\gamma}^N_{FB_2}\left(v_2{}^N\right) \right) \right\} \end{array} \right\}$$

$$= \left\{ \begin{array}{l} \min\left\{ \left(\widetilde{\gamma}^P_{FB_1} \times \widetilde{\gamma}^P_{FB_2} \right)\left(u^P, u_2^P\right), \left(\left(\widetilde{\gamma}^P_{FB_1} \times \widetilde{\gamma}^P_{FB_2} \right)\left(u^P, v_2^P\right) \right) \right\}, \\ \max\left\{ \left(\widetilde{\gamma}^N_{FB_1} \times \widetilde{\gamma}^N_{FB_2} \right)\left(u^N, u_2^N\right), \left(\left(\widetilde{\gamma}^N_{FB_1} \times \widetilde{\gamma}^N_{FB_2} \right)\left(u^N, v_2^N\right) \right) \right\} \end{array} \right\}$$

Similarly we can prove it for $w \in V_2$ and $u_1 v_1 \in E_1$.

Proposition 4.6. *Let* $G_1 = \left(\left({M_1}^P, {N_1}^P\right), \left({M_1}^N, {N_1}^N\right)\right)$ *and* $G_2 = \left({M_2}^P, {N_2}^P\right), \left({M_2}^N, {N_2}^N\right)$ *be two bipolar neutrosophic cubic fuzzy graphs, then the composition of two bipolar neutrosophic cubic fuzzy graphs is again a bipolar neutrosophic cubic fuzzy graph.*

Example 4.7. Let $G_1 = \left(\left({M_1}^P, {N_1}^P\right), \left({M_1}^N, {N_1}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_1^* = (V_1, E_1)$ where $v_1 = \{u, v, w\}, E = \{uv, vw, uw\}$.

$$
{M_1}^P = \left\langle \begin{array}{l} \{u, ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.5, 0.6], 0.1)\} \\ \{v, ([0.1, 0.3], 0.1), ([0.4, 0.5], 0.3), ([0.1, 0.1], 0.2)\} \\ \{w, ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\} \end{array} \right\rangle
$$

$$
{N_1}^P = \left\langle \begin{array}{l} \{uv, ([0.1, 0.1], 0.4), ([0.3, 0.4], 0.2), ([0.5, 0.6], 0.1)\} \\ \{vw, ([0.1, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.3, 0.5], 0.2)\} \\ \{uw, ([0.2, 0.3], 0.1), ([0.1, 0.2], 0.6), ([0.5, 0.6], 0.1)\} \end{array} \right\rangle
$$

$$
{M_1}^N = \left\langle \begin{array}{l} \{u, ([-0.1, -0.1], -0.4), ([-0.3, -0.4], -0.2), ([-0.5, -0.6], -0.1)\} \\ \{v, ([-0.1, -0.3], -0.1), ([-0.4, -0.5], -0.3), ([-0.1, -0.1], -0.2)\} \\ \{w, ([-0.2, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.3, -0.5], -0.2)\} \end{array} \right\rangle
$$

$$
{N_1}^N = \left\langle \begin{array}{l} \{uv, ([-0.1, -0.1], -0.4), ([-0.3, -0.4], -0.2), ([-0.5, -0.6], -0.1)\} \\ \{vw, ([-0.1, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.3, -0.5], -0.2)\} \\ \{uw, ([-0.2, -0.3], -0.1), ([-0.1, -0.2], -0.6), ([-0.5, -0.6], -0.1)\} \end{array} \right\rangle
$$

and $G_2 = \left(\left({M_2}^P, {N_2}^P\right), \left({M_2}^N, {N_2}^N\right)\right)$ be a bipolar neutrosophic cubic fuzzy graph of $G_2^* = (V_2, E_2)$ where $V_1 = \{a, v, c\}$ and $E_2 = \{ab, bc, ac\}$

$$
{M_2}^P = \left\langle \begin{array}{l} \{a, ([0.6, 0.7], 0.5), ([0.1, 0.3], 0.4), ([0.2, 0.3], 0.6)\} \\ \{b, ([0.1, 0.2], 0.3), ([0.5, 0.6], 0.2), ([0.8, 0.9], 0.4)\} \\ \{c, ([0.3, 0.4], 0.1), ([0.2, 0.3], 0.1), ([0.5, 0.6], 0.3)\} \end{array} \right\rangle
$$

$$
{N_2}^P = \left\langle \begin{array}{l} \{ab, ([0.1, 0.2], 0.5), ([0.1, 0.3], 0.4), ([0.8, 0.9], 0.4)\} \\ \{bc, ([0.1, 0.2], 0.3), ([0.2, 0.3], 0.2), ([0.8, 0.9], 0.3)\} \\ \{ac, ([0.3, 0.4], 0.5), ([0.1, 0.3], 0.4), ([0.5, 0.6], 0.3)\} \end{array} \right\rangle
$$

$$
{M_2}^N = \left\langle \begin{array}{l} \{a, ([-0.6, -0.7], -0.5), ([-0.1, -0.3], -0.4), ([-0.2, -0.3], -0.6)\} \\ \{b, ([-0.1, -0.2], -0.3), ([-0.5, -0.6], -0.2), ([-0.8, -0.9], -0.4)\} \\ \{c, ([-0.3, -0.4], -0.1), ([-0.2, -0.3], -0.1), ([-0.5, -0.6], -0.3)\} \end{array} \right\rangle
$$

$$
{N_2}^N = \left\langle \begin{array}{l} \{ab, ([-0.1, -0.2], -0.5), ([-0.1, -0.3], -0.4), ([-0.8, -0.9], -0.4)\} \\ \{bc, ([-0.1, -0.2], -0.3), ([-0.2, -0.3], -0.2), ([-0.8, -0.9], -0.3)\} \\ \{ac, ([-0.3, -0.4], -0.5), ([-0.1, -0.3], -0.4), ([-0.5, -0.6], -0.3)\} \end{array} \right\rangle
$$

then $G_1 \times G_2$ is a bipolar neutrosophic cubic fuzzy graph of $G_1^* \times G_2^*$, where $V_1 \times V_2 = \{(u,a),(u,b),(u,c),(v,a),(v,b),(v,c),(w,a),(w,b),(w,c)\}$ and

$$M_1^P \times M_2^P = \left\langle \begin{array}{l} \{(u,a),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.5,0.6],0.1)\} \\ \{(u,b),([0.1,0.1],0.4),([0.3,0.4],0.2),([0.8,0.9],0.1)\} \\ \{(u,c),([0.1,0.1],0.6),([0.2,0.3],0.2),([0.5,0.6],0.1)\} \\ \{(v,a),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.2,0.3],0.2)\} \\ \{(v,b),([0.1,0.2],0.3),([0.4,0.5],0.3),([0.8,0.9],0.2)\} \\ \{(v,c),([0.1,0.3],0.1),([0.2,0.3],0.3),([0.5,0.6],0.2)\} \\ \{(w,a),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.3,0.5],0.2)\} \\ \{(w,b),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{(w,c),([0.2,0.3],0.1),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \end{array} \right\rangle$$

$$M_1^N \times M_2^N = \left\langle \begin{array}{l} \{(u,a),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.1)\} \\ \{(u,b),([-0.1,-0.1],-0.4),([-0.3,-0.4],-0.2),([-0.8,-0.9],-0.1)\} \\ \{(u,c),([-0.1,-0.1],-0.6),([-0.2,-0.3],-0.2),([-0.5,-0.6],-0.1)\} \\ \{(v,a),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.2,-0.3],-0.2)\} \\ \{(v,b),([-0.1,-0.2],-0.3),([-0.4,-0.5],-0.3),([-0.8,-0.9],-0.2)\} \\ \{(v,c),([-0.1,-0.3],-0.1),([-0.2,-0.3],-0.3),([-0.5,-0.6],-0.2)\} \\ \{(w,a),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.3,-0.5],-0.2)\} \\ \{(w,b),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{(w,c),([-0.2,-0.3],-0.1),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \end{array} \right\rangle$$

$$N_1^P \times N_2^P = \left\langle \begin{array}{l} \{((u,a),(u,b)),([0.1,0.1],0.5),([0.1,0.3],0.4),([0.8,0.9],0.1)\} \\ \{((u,b),(u,c)),([0.1,0.1],0.4),([0.2,0.3],0.2),([0.8,0.9],0.1)\} \\ \{((u,a),(v,c)),([0.1,0.1],0.4),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,c)),([0.1,0.3],0.5),([0.1,0.3],0.4),([0.5,0.6],0.2)\} \\ \{((v,a),(v,b)),([0.1,0.2],0.5),([0.1,0.3],0.4),([0.8,0.9],0.2)\} \\ \{((v,b),(w,b)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,b),(w,c)),([0.1,0.2],0.3),([0.1,0.2],0.6),([0.8,0.9],0.2)\} \\ \{((w,a),(w,c)),([0.2,0.3],0.5),([0.1,0.2],0.6),([0.5,0.6],0.2)\} \\ \{((u,ab),(w,a)),([0.1,0.1],0.5),([0.1,0.2],0.6),([0.5,0.6],0.1)\} \end{array} \right\rangle$$

$$N_1^N \times N_2^N = \left\langle \begin{array}{l} \{((u,a),(u,b)),([-0.1,-0.1],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.1)\} \\ \{((u,b),(u,c)),([-0.1,-0.1],-0.4),([-0.2,-0.3],-0.2),([-0.8,-0.9],-0.1)\} \\ \{((u,a),(v,c)),([-0.1,-0.1],-0.4),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,c)),([-0.1,-0.3],-0.5),([-0.1,-0.3],-0.4),([-0.5,-0.6],-0.2)\} \\ \{((v,a),(v,b)),([-0.1,-0.2],-0.5),([-0.1,-0.3],-0.4),([-0.8,-0.9],-0.2)\} \\ \{((v,b),(w,b)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,b),(w,c)),([-0.1,-0.2],-0.3),([-0.1,-0.2],-0.6),([-0.8,-0.9],-0.2)\} \\ \{((w,a),(w,c)),([-0.2,-0.3],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.2)\} \\ \{((u,ab),(w,a)),([-0.1,-0.1],-0.5),([-0.1,-0.2],-0.6),([-0.5,-0.6],-0.1)\} \end{array} \right\rangle$$

Conclusion

In this paper, we introduced Cartesian product and composition of bipolar neutrosophic bipolar fuzzy graphs.we investigate some of their properties.

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