Stochastic Modeling and Applications
Vol. 24 No. 2 (July-December, 2020)

Received: 27th May 2020 *Revised: 13th July 2020* *Accepted: 02nd August 2020*

ON GENERALIZED CONFORMALLY AND SPECIAL WEAKLY PROJECTIVE SYMMETRIC MANIFOLDS

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Abstract. In this article, we have studied the nature of Ricci tensor R of type (1,1) in a special weakly projective Riemannian manifold and investigating some interesting results on it. Also, we have studied the two 1-forms η and ξ appear in the definition of the generalized recurrency of a Riemannian manifold and establish some results on a generalized conformally recurrent Riemannian manifold.

1. Introduction and Preliminaries

Let M_n be the n-dimensional Riemannian manifold and $\chi(M)$ denote the set of differential vector fieds on M_n. Let K(Q,S,T) be the Riemannian curvature tensor of type (1,3). The vector fields P, Q, S $\in \chi(M)$. The notion of a weakly symmetric and weakly projective symmetric Riemannian manifolds have been introduced by Tamassy and Binh ([9], [10]).

A non-flat Riemannian manifold (M_n,g), ($n \geq 2$) is called a special weakly symmetric Riemannian manifold if the curvature tensor K of type (1,3) satisfies the condition ([6], [12]).

$$\begin{aligned}(\nabla_Q K)(S,T,V) &= 2\eta(Q)K(S,T,V) + \eta(S)K(Q,T,V) + \eta(T)K(S,T,V) \\ &\quad +\eta(V)K(S,T,Q), \qquad (1.1)\end{aligned}$$

where η is a 1-form and ρ is associated vector field such that

$$\eta(Q) = g(Q,\rho), \qquad (1.2)$$

for every vector field Q and ∇ denotes the operator of covariant differentiation over metric g. Such a manifold is denoted by $(SWS)_n$. If we replace K by P in (1.1), then it reduces to

$$\begin{aligned}(\nabla_Q P)(S,T,V) &= 2\eta(X)P(S,T,V) + \eta(S)P(Q,T,V) + \eta(T)P(S,Q,V) \\ &\quad +\eta(V)P(S,T,Q), \qquad (1.3)\end{aligned}$$

where P is the projective curvature tensor defined by

$$P(S,T,V) = K(S,T,V) - \frac{1}{n-1}[Ric(T,V)S - Ric(S,V)T]. \qquad (1.4)$$

2000 *Mathematics Subject Classification.* 53B21, 53C25; Secondary 58A10.

Key words and phrases. Riemannian manifolds, Curvature tensors, Recurrent spaces, Projective curvature tensor, Ricci tensor, Einstein manifold, special weakly projective symmetric Riemannian manifold.

Here Ric is the Rcci tensor of type (0,2). Such an n-dimensional Riemannian manifold shall be called a special weakly projective symmetric Riemannian manifold ([1],[9]) and such a manifold is denoted by $(SWPS)_n$.
A Riemannian manifold is an Einstein manifold, if

$$Ric(Q,S) = \lambda g(Q,S) \tag{1.5}$$

where λ is constant. from the above equation we get

$$R(Q) = \lambda Q, \tag{1.6}$$

where R is the Ricci tensor of type (1,1) and is defined by

$$g(R(Q),S) = Ric(Q,S). \tag{1.7}$$

Contracting (1.6), we get

$$r = n\lambda \tag{1.8}$$

In a recent paper [13] De and Guha introduced and studied a type of non-flat Riemannian space whose curvature tensor K(Q,S,T) of type (1, 3) satisfies the condition:

$$(\nabla_U K)(Q,S,T) = \eta(U)K(Q,S,T) + \xi(U)[g(S,T)Q - g(Q,T)S] \tag{1.9}$$

where η and ξ are two 1-forms, ξ is non zero and ∇ denotes the operator of covariant differentiation with respect to the metric tensor g. Such a space has been called generalized recurrent space. Here ξ is called its associated 1-form. These spaces are related to the pseudo symmetric Riemannian spaces of M.C. Chaki [11] and are special cases of the weakly symmetric Riemannian spaces of L. Tamassay and T.Q. Binh ([9], [10]). If the 1-form ξ(U) becomes zero in (1.1), then the sapce reduces to a recurrent space according to Ruse and Walker [2].
Contracting (1.1) over 'Q', then

$$(\nabla_U Ric)(S,T) = \eta(U)Ric(S,T) + (n-1)\xi(U)g(S,T) \tag{1.10}$$

In this case, the riemannian manifold M is called a generalized Ricci recurrent space, where η and ξ are as stated earlier. If the 1-form ξ(U) becomes zero in (1.9), then the space reduces to a Ricci-recurrent space. In this article we have considered a non-flat n-dimensional Riemannian manifold in which the conformal curvature tensor C satisfies the condition:

$$(\nabla_U C)(Q,S,T) = \eta(U)C(Q,S,T) + \xi(U)[g(S,T)Q - g(Q,T)S]. \tag{1.11}$$

where η and ξ are two 1-forms, ξ is non zero and the conformal curvature tensor C is defined by ([3])

$$\begin{aligned} C(Q,S,T) &= K(Q,S,T) - \frac{1}{n-2}[Ric(S,T)Q - Ric(Q,T)S \\ &\quad + g(S,T)R(Q) - g(Q,T)R(S)] \\ &\quad + \frac{r}{(n-1)(n-2)}[g(S,T)Q - g(Q,T)S]. \end{aligned} \tag{1.12}$$

Here K is the curvature tensor of type (1, 3), Ric is the Ricci tensor of type (0, 2), r is the scalar curvature and R is the Ricci tensor of type (1, 1), defined by

$$Ric(Q,S) = g(R(Q),S). \tag{1.13}$$

Such an n-dimensional Riemannian manifold shall be called a generalized conformally recurrent Riemannian manifold. If the 1-form ξ is zero, then the manifold reduces to a conformally recurrent manifold [2].

The conharmonic curvature tensor N and the concircular curvature tensor W are given by [3]

$$\begin{aligned} N(Q,S,T) &= K(Q,S,T) - \frac{1}{n-2}[Ric(S,T)Q - Ric(Q,T)S \\ &\quad + g(S,T)R(Q) - g(Q,T)R(S)] \end{aligned} \tag{1.14}$$

and

$$W(Q,S,T) = K(Q,S,T) - \frac{r}{n(n-1)}[g(S,T)Q - g(Q,T)S], \tag{1.15}$$

respectively.
If the conharmonic curvature tensor N and concircular curvature tensor W satisfies the condition:

$$(\nabla_U N)(Q,S,T) = \eta(U)N(Q,S,T) + \xi(U)[g(S,T)Q - g(S,T)Q] \tag{1.16}$$

and

$$(\nabla_U W)(Q,S,T) = \eta(U)W(Q,S,T) + \xi(U)[g(S,T)Q - g(Q,T)S], \tag{1.17}$$

respectively, where η and ξ are two 1- forms, then the Riemannian manifold is known as a generalized conharmonically recurrent manifolds and generalized concircularly recurrent manifolds ([3],[4]).

2. Generalized conformally recurrent Riemannian manifolds

Let M be a generalized recurrent smooth Riemannian manifold of dimension n. Taking covariant derivative of (1.12) over 'U', we get

$$\begin{aligned} (\nabla_U C)(Q,S,T) &= (\nabla_U K)(Q,S,T) - \frac{1}{n-2}[(\nabla_U Ric)(S,T)Q \\ &\quad -(\nabla_U Ric)(Q,T)S + g(S,T)(\nabla_U R)(Q) - g(Q,T)(\nabla_U R)(S)] \\ &\quad + \frac{Ur}{(n-1)(n-2)}[g(S,T)Q - g(Q,T)S] \end{aligned} \tag{2.1}$$

Using (1.9), (1.10) and (2.1), we get

$$\begin{aligned}(\nabla_U C)(Q,S,T) \quad = \quad & \eta(U)[K(Q,S,T) - \frac{1}{n-2}\{Ric(S,T)Q - Ric(Q,T)S \\ & -g(S,T)R(Q) - g(Q,T)R(S)\} + \frac{r}{(n-1)(n-2)}\{g(S,T)Q \\ & -g(Q,T)S\}] + \xi(U)[g(S,T)Q - g(Q,T)S \\ & -\frac{2(n-1)}{(n-2)}\{g(S,T)Q - g(Q,T)S\} \\ & +\frac{n(n-1)}{(n-1)(n-2)}\{g(S,T)Q - g(Q,T)S\}] \qquad (2.2)\end{aligned}$$

Using (1.12) in (2.2), we have

$$(\nabla_U C)(Q,S,T) = \eta(U)C(Q,S,T) \qquad (2.3)$$

which shows the condition of a conformally recurrent Riemannian manifold. Thus we can state the following:

Theorem 2.1. *A generalized recurrent Riemannian manifold is conformally recurrent for the same recurrence parameter.*

From (1.11) and (1.12) it follows that

$$\begin{aligned}& (\nabla_U K)(Q,S,T) - \eta(U)K(Q,S,T) - \xi(U)[g(S,T)Q - g(Q,T)S] \\ & = \frac{1}{n-2}[(\nabla_U Ric)(S,T)Q - (\nabla_U Ric)(Q,T)S \\ & +g(S,T)(\nabla_U R)(Q) - g(Q,T)(\nabla_U R)(S) \\ & -\eta(U)\{Ric(S,T)Q - Ric(Q,T)S + g(S,T)R(Q) - g(Q,T)R(S)\}] \\ & +\frac{r}{(n-1)(n-2)}[\eta(U)\{g(S,T)Q - g(Q,T)(S)\} \\ & -U\{g(S,T)R(Q) - g(Q,T)R(S)\}]. \qquad (2.4)\end{aligned}$$

Permutting equation (2.4) twice over U, Q, S; adding the three equations and using Bianchi's 2nd identity, we get

$$\begin{aligned}
&\eta(U)K(Q,S,T)+\eta(X)K(S,U,T)+\eta(S)K(U,Q,T)\\
&+\xi(U)[g(S,T)Q-g(Q,T)S]+\xi(Q)[g(U,T)S-g(S,T)U]\\
&+\xi(S)[g(Q,T)U-g(U,T)Q]+\frac{1}{n-2}[(\nabla_U Ric)(S,T)Q\\
&-(\nabla_U Ric)(Q,T)S+g(S,T)(\nabla_U R)(Q)\\
&-g(Q,T)(\nabla_U R)(S)+(\nabla_Q Ric)(U,T)S\\
&-(\nabla_Q Ric)(S,T)U+g(U,T)(\nabla_Q R)(S)\\
&-g(S,T)(\nabla_Q R)(U)+(\nabla_S Ric)(Q,T)U\\
&-(\nabla_Y Ric)(U,Z)X+g(X,Z)(\nabla_Y R)(U)\\
&-g(U,T)(\nabla_S R)(Q)-\eta(U)\{Ric(S,T)Q-Ric(Q,T)S\\
&+g(S,T)R(Q)-g(Q,T)R(S)\}-\eta(Q)\{Ric(U,T)S-Ric(S,T)U\\
&+g(U,T)R(S)-g(S,T)R(U)\}-\eta(S)\{Ric(Q,T)U-Ric(U,T)Q\\
&+g(Q,T)R(U)-g(U,T)R(Q)\}]\\
&+\frac{r}{(n-1)(n-2)}[\eta(U)\{g(S,T)Q-g(Q,T)S\}\\
&-U\{g(S,T)Q-g(Q,T)S\}]\\
&+\eta(Q)\{g(U,T)S-g(S,T)U\}-Q\{g(U,T)S-g(S,T)U\}\\
&+\eta(S)\{g(Q,T)U-g(U,T)Q\}-S\{g(Q,T)U-g(U,T)Q\}\\
&=0
\end{aligned}\tag{2.5}$$

Contracting (2.5) over 'Q', we have

$$\begin{aligned}
&\eta(U)Ric(S,T)-\eta(S)Ric(U,T)+K(S,U,T,\rho)\\
&+(n-1)\xi(U)g(S,T)+\xi(S)g(U,T)-\xi(U)g(S,T)\\
&+(1-n)B(Y)g(U,Z)+\frac{1}{n-2}[(n-1)(\nabla_U Ric)(Y,Z)\\
&+g(S,T)(Ur)-g((\nabla_U R)(S),T)+(\nabla_S Ric)(U,T)\\
&-(\nabla_U Ric)(S,T)+\frac{1}{2}g(U,T)(Sr)-\frac{1}{2}g(S,T)(Ur)\\
&+(1-n)(\nabla_S Ric)(U,T)-g(U,T)(Sr)\\
&-(n-1)\eta(U)Ric(S,T)-\eta(U)g(S,T)r+\eta(U)Ric(Q,T)\\
&-\eta(S)Ric(U,T)-\eta(U)Ric(S,T)-\eta(R(S))g(U,T)\\
&+\eta(R(U))g(S,T)+(n-1)\eta(S)Ric(U,T)-\eta(S)Ric(U,T)\\
&+\eta(S)g(U,T)r]+\frac{r}{(n-1)(n-2)}[(n-1)\eta(U)g(S,T)\\
&-(n-1)g(S,T)U+\eta(S)g(U,T)-\eta(U)g(S,T)-ng(U,T)S\\
&+ng(S,T)U+(1-n)\eta(S)g(U,T)+(n-1)g(U,T)S]=0
\end{aligned}\tag{2.6}$$

where ρ is vector field defined by

$$g(Q,\rho)=\eta(Q).\tag{2.7}$$

Using (1.13) in (2.6), we have

$$\begin{aligned}
&\eta(U)R(S)-\eta(S)R(U)-K(S,U,\rho)+(n-2)[\xi(U)Y-\xi(Y)U]\\
&+\frac{1}{n-2}[(n-1)(\nabla_U R)(S)+S(Ur)-(\nabla_U R)(S)+(\nabla_S R)(U)\\
&-(\nabla_U R)(S)+\frac{1}{2}U(Sr)-\frac{1}{2}S(Ur)+(1-n)(\nabla_S R)(U)+(\nabla_S R)(U)\\
&-U(Sr)-(n-1)\eta(U)R(S)-\eta(U)Sr+\eta(U)Sr+\eta(U)R(S)-\eta(S)R(U)\\
&+\eta(U)R(S)-\eta(R(S))U+\eta(R(U))S+(n-1)\eta(S)R(U)\\
&-\eta(S)R(U)+\eta(S)Ur]+\frac{r}{(n-1)(n-2)}[(n-1)\eta(U)S-(n-1)SU\\
&+\eta(S)U-\eta(U)S-nUS+nSU+(1-n)\eta(S)U+(n-1)US]\\
&=0
\end{aligned}$$

or

$$\begin{aligned}
&K(S,U,\rho)-(n-2)[\xi(U)S-\xi(S)U]\\
&=\frac{1}{n-2}[(n-3)(\nabla_U R)(S)-(n-3)(\nabla_S R)(U)\\
&+\eta(U)R(S)-\eta(S)R(U)+\eta(R(U))S-\eta(R(S))U]\\
&+\frac{r}{(n-1)(n-2)}[\eta(S)U-\eta(U)S]. \qquad (2.8)
\end{aligned}$$

Contracting (2.9) with respect to ‘S’, we get

$$\begin{aligned}
&Ric(U,\rho)-(n-1)(n-2)\xi(U)=\frac{1}{n-2}[(n-3)(Ur)\\
&-\frac{1}{2}(n-3)(Ur)+\eta(U)r-\eta(R(U))+(n-1)\eta(R(U))]\\
&+\frac{r}{(n-1)(n-2)}[(n-1)\eta(U)]
\end{aligned}$$

or

$$2\eta(U)r+2(n-1)(n-2)^2\xi(U)=(n-3)(Ur).$$

Thus we can state the following:

Theorem 2.2. *The necessary and sufficient condition that the scalar curvature r of a generalized conformally recurrent Riemannian manifold be constant is that*

$$\eta(U)r+(n-1)(n-2)^2\xi(U)=0. \qquad (2.9)$$

From (1.12),(1.14) and (1.15), we have

$$C(Q,S,T)=N(Q,S,T)+\frac{n}{n-2}[K(Q,S,T)-W(Q,S,T)]. \qquad (2.10)$$

Taking covariant derivative of (2.13) over ‘U’, we get

$$\begin{aligned}
(\nabla_U C)(Q,S,T) &= (\nabla_U N)(Q,S,T)\\
&+\frac{n}{n-2}[(\nabla_U C)(Q,S,T)-(\nabla_U W)(Q,S,T)]. \qquad (2.11)
\end{aligned}$$

Using (1.9) in (2.11), we get

$$(\nabla_U C)(Q,S,T) - (\nabla_U N)(Q,S,T) + \frac{n}{n-2}(\nabla_U W)(Q,S,T)$$
$$= \frac{n}{n-2}[\eta(U)K(Q,S,T) + \xi(U)\{g(S,T)Q - g(Q,T)S\}] \tag{2.12}$$

From (2.12) it is conclude that if any two of the equations (1.11), (1.16) and (1.17) holds then the third also hold.

Thus we can state the following:

Theorem 2.3. *let M be the generalized recurrent Riemannian manifold of dimension n. If any two of the following hold then the third also holds:*
(i) It is generalized conformally recurrent manifold.
(ii) It is generalized conharmonically recurrent manifold.
(iii) It is generalized concircularlly recurrent manifold.
For the same recurrence parameters.

3. Special weakly projective symmetric Riemannian manifold

Let (M_n, g) be a $(SWPS)_n$. Taking covariant derivative of (1.4) with respect to Q and then using (1.3), we get

$$\begin{aligned}2\eta(Q)P(S,T,V) \quad & +\eta(S)P(Q,T,V) + \eta(T)P(S,Q,V) + \eta(V)P(S,T,Q) \\ & = (\nabla_Q K)(S,T,V) - \frac{1}{n-1}[(\nabla_Q Ric)(T,V)S \\ & -(\nabla_Q Ric)(S,V)T]\end{aligned} \tag{3.1}$$

By virtue of (1.4), the equation (3.1) reduces to

$$\begin{aligned}&(\nabla_Q K)(S,T,V) - 2\eta(Q)K(S,T,V) - \eta(S)K(Q,T,V) - \eta(T)K(S,Q,V) \\ &-\eta(V)K(S,T,Q) - \frac{1}{n-1}[(\nabla_Q Ric)(T,V)S - (\nabla_Q Ric)(S,V)T \\ &-2\eta(X)\{Ric(Z,V)Y - Ric(Y,V)Z\} - \eta(Y)\{Ric(Z,V)X - Ric(X,V)Z\} \\ &-\eta(T)\{Ric(Q,V)S - Ric(S,V)Q\} - \eta(V)\{Ric(T,Q)V - Ric(S,Q)T] = 0\end{aligned} \tag{3.2}$$

Permuting equation (3.2) twice over Q, S, T; adding the three obtained equations and using Bianchi's 1st and 2nd identities; and using symmetric property of Ricci tensor and the skew-symmetric properties of curvature tensor, we get

$$\begin{aligned}(\nabla_Q Ric)(T,V)S \quad + \quad & (\nabla_S Ric)(Q,V)T + (\nabla_T Ric)(S,V)Q - (\nabla_Q Ric)(S,V)T \\ & -(\nabla_S Ric)(T,V)Q - (\nabla_T Ric)(Q,V)S = 0\end{aligned} \tag{3.3}$$

Contracting (3.3) with respect to 'Q', we get

$$(\nabla_T Ric)(S,V) - (\nabla_S Ric)(T,V) = 0. \tag{3.4}$$

Consequently relation (3.4) gives

$$(\nabla_T R)(S) - (\nabla_S R)(T) = 0 \tag{3.5}$$

Thus we can state the following:

Theorem 3.1. *In a $(SWPS)_n$, the Ricci tensor of type (1,1) is closed.*

Contracting (3.5) with respect to 'S', we get

$$Tr = 0, \tag{3.6}$$

which has shows that the scalar curvature r is constant.
Thus we have the following result:

Theorem 3.2. *In a $(SWPS)_n$, the scalar curvature r is constant.*

By virtue of (1.5), the equation (1.4) reduces to the form

$$P(S,T,V) = K(S,T,V) - \frac{k}{n-1}[g(T,V)S - g(S,V)T]. \tag{3.7}$$

Taking covariant derivative of (3.7) over 'Q', we get

$$(\nabla_Q P)(S,T,V) = (\nabla_Q K)(S,T,V). \tag{3.8}$$

Using (1.3) in (3.8), we get

$$\begin{aligned}(\nabla_Q P)(S,T,V) &= 2\alpha(Q)P(S,T,V) + \alpha(S)P(Q,T,V) + \alpha(T)P(S,Q,V) \\ &\quad +\alpha(V)P(S,T,X)\end{aligned} \tag{3.9}$$

By virtue of (3.7), the equation (3.9) reduces to the form

$$\begin{aligned}(\nabla_Q K)(S,T,V) &= 2\eta(Q)[K(S,T,V) - \frac{k}{n-1}\{g(T,V)S - g(S,V)T\}] \\ &\quad +\eta(S)[K(Q,T,V) - \frac{k}{n-1}\{g(T,V)Q - g(Q,V)T] \\ &\quad +\eta(T)[K(S,Q,V) - \frac{k}{n-1}\{g(Q,V)S - g(S,V)Q\}] \\ &\quad +\eta(V)[K(S,T,Q) - \frac{k}{n-1}\{g(T,Q)S - g(S,Q)T]\end{aligned} \tag{3.10}$$

Thus, we can state the following:

Theorem 3.3. *The necessary and sufficient condition for an Einstein $(SWPS)_n$ to be a $(SWS)_n$ is that*

$$\begin{aligned}&[2\eta(Q)S + \eta(S)Q]g(T,V) - [2\eta(Q)T - \eta(T)Q]g(S,V) \\ &+[\eta(T)S - \eta(S)T]g(Q,V) + \eta(V)[g(T,Q)S - g(S,Q)T] \\ &= 0\end{aligned} \tag{3.11}$$

4. Manifold satisfying P(S,T,V)=0

Let (M_n, g) be a projectively flat, that is, P(S,T,V)=0, then the relation (1.4) reduces to

$$K(S,T,V) = \frac{1}{n-1}[Ric(T,V)S - Ric(S,V)T]. \tag{4.1}$$

Taking covariant derivative of (4.1) over 'Q', we have

$$(\nabla_Q K)(S,T,V) = \frac{1}{n-1}[(\nabla_Q Ric)(T,V)S - (\nabla_Q Ric)(S,V)T]. \tag{4.2}$$

Permuting equation (4.2) twice over Q, S, T; adding the three obtained equations and then using Bianchi's 2nd identity, we get

$$(\nabla_Q Ric)(T,V)S + (\nabla_S Ric)(Q,V)T + (\nabla_T Ric)(S,V)Q$$
$$-(\nabla_Q Ric)(S,V)T - (\nabla_S Ric)(T,V)Q - (\nabla_T Ric)(Q,V)S = 0. \quad (4.3)$$

Contracting (4.3) with respect to 'Q', we have

$$(\nabla_T Ric)(S,V) - (\nabla_S Ric)(T,V) = 0. \quad (4.4)$$

Consequently equation (4.4) gives

$$(\nabla_T R)(S) - (\nabla_S R)(Q) = 0. \quad (4.5)$$

Thus we can state the following:

Theorem 4.1. *it In a projectively flat Riemannian manifold, the Ricci tensor R is closed.*

An n-dimensional Riemannian manifold is called a special weakly Ricci symmetric manifold if the Ricci tensor Ric of type (0,2) satisfies the condition

$$(\nabla_Q Ric)(S,T) = 2\eta(Q)Ric(S,T) + \eta(S)Ric(Q,T) + \eta(T)Ric(S,Q), \quad (4.6)$$

where η is a 1-form. Such a manifolds were denoted by $(SWRS)_n$.
Now, using (4.6) in (4.3), we have

$$\eta(Q)Ric(T,V)S + \eta(S)Ric(Q,V)T + \eta(T)Ric(S,V)Q$$
$$-\eta(Q)Ric(S,V)T - \eta(S)Ric(T,V)Q - \eta(T)Ric(Q,V)S = 0. \quad (4.7)$$

Contracting (4.7) with respect to 'Q', we have

$$\eta(T)Ric(S,V) - \eta(S)Ric(T,V) = 0. \quad (4.8)$$

Consequently (4.8) gives

$$\eta(T)R(S) - \eta(S)R(T) = 0. \quad (4.9)$$

Hence we can state the following:

Theorem 4.2. *In a projectively flat $(SWRS)_n$, the 1-form η is collinear with the Ricci tensor R.*

Acknowledgment. The authors wish to express their sincere thanks and gratitude to the referee for his valuable suggestions towards the improvement of the paper.

References

1. Shaikh, A.A. and Jana, S. K.: On weakly symmetric Riemannian manifolds, *Publ. Math.Debrecen* **71** (2007) 27—41.
2. Walker, A.G.: On Ruse's space of recurrent curvature, *Proc. London Math. Soc.* **52** (1951) 36–64.
3. Sinha, B.B.: On Tachibana concircular curvature tensor, *The Academy for Progress of Mathematics Univ. of Allahabad, India* **4** (1970).
4. Singh, H. Khan, Q.: On generalized recurrent Riemannian manifolds, *Publ Math debr* **56** (2000) 87–95.

5. Singh, H., Khan, Q.: On symmetric Riemannian manifolds, *Novi Sad J. Math.* **293** (1999) 301–308.
6. Singh, H., Khan, Q.: On special weakly symmetric Riemannian manifolds, *publ. Math. Debrecen* **58** (2001) 523–536.
7. Ruse, H.S.: On simply harmonic space, *J. London Math. Soc.* **21** (1946).
8. Arslan, K., De, U.C., Murathan, C. and Yildiz, A.: On generalized recurrent Riemannian manifolds, *Acta Math Hung* **123** (2009) 27–39.
9. Tamassy, L., Binh, T.Q.: On weakly symmetric and weakly projective symmetric Riemannian manifolds, *Differential Geometry and its Application (Eger, Hungary, 1989), Colloquia Mathematica Societatis Janos Bolyani, North-Holland, Amsterdam* **56** (1992) 663–670.
10. Tamassy, L., Binh, T.Q.: On weakly symmetric and weakly projectively symmetric Riemannian manifolds, *Colloq. Math. Soc. J. Bolyai* **50** (1989) 663–670.
11. Chaki, M.C.: On pseudo symmetric maniffolds, *An. Sti. Univ. Al. J. Cuza Iasi* **33** (1987) 53–58.
12. De, U.C., Bandyopadhyay, S.: On weakly symmetric Riemannian spaces, *Publ. Math. Debrecen* **54** (1999) 377–381.
13. De, U.C., Guha, N.: On generalised recurrent manifold, *J. National Academy of Math. India* **9** (1991) 85–92.
14. Maralabhavi, Y.B., Rathnamma, M.: Generalized recurrent and concircular recurrent manifolds, *Indian J. Pure Appl Math* **30** (2001) 1167–1171.

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