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SUPER ROOT CUBE OF CUBE DIFFERNECE LABELING OF GRAPHS

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Abstract. In this paper, we contribute some new results for Super Root cube of Cube Difference Labeling of Graphs. We prove that Cube Difference Labeling of Graphs Super Root Cube of Cube Difference Labeling of Graphs. We use some standard graphs to derive the results for Super Root Cube of Cube of Difference Labeling of Graphs.

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1. Introduction

The graph considered here will be finite, undirected and simple. The vertex set is denoted by $V(G)$ and the edge set is denoted by $E(G)$. The concept of Root Square Mean Labeling was introduced by S.S. Sandhiya et.al[1]. Super Root Square Mean Labeling of graphs was introduced by K.Thirugnanasambandam and K.Venkatesan[5]. R.Gowri and G. Vembarasi introduced the concept of Root Cube Mean Labeling of graph[5]. Root Cube Difference Labeling of graphs was introduced by R.Gowri and G.Vembarasi[9]. In this paper, we introduce the concept of Super Root Cube of Cube Difference Labeling of graphs and investigate Super Root Cube of Cube Difference Labeling of Comb, $P_n \bigodot K_{1,n}, S(B_{n,n})$ and Ladder graphs. The following Definitions are useful for the present study.

2. Preliminaries

Definition 2.1. The graph obtained by joining a simple pendent edge to each vertex of a path is called a Comb graph.

Definition 2.2. A walk in which vertices are called a path. A path on n vertices is denoted by P_n

Definition 2.3. The product graph $P_1 \times P_n$ is called a Ladder and it is denoted by L_n

Definition 2.4. Let G be a graph (p, q) and $f : V(G) \to \{1, 2, \cdots, p+q\}$ be an injective function.For each edge $e = uv$. Let $f^*(e = uv) = \lceil \sqrt{\frac{f(u)^2 + f(v)^2}{2}} \rceil$

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(or) $\lfloor\sqrt{\dfrac{f(u)^2+f(v)^2}{2}}\rfloor$, then f is called a super root square mean labeling if $f(v)\cup\{f^*(e):e\in E(G)\}-\{1,2,3,\cdot,p+q\}$. A graph that admits a super root mean labeling is called as super root mean graph.

Definition 2.5. Let $G=(V(G),E(G))$ be a graph G is said to cube difference labeling if there exists a injective $f:V(G)\to\{0,1,\cdots p-1\}$ such that the induced function $f^*:E(G)\to N$ is given by $f^*(uv)-|[f(u)]^3-[f(v)]^3|$ is injective.

Definition 2.6. Let $G=(V(G),E(G))$ be a graph G is said to cube difference labeling if there exists a injective $f:V(G)\to\{0,1,\cdots,p-1\}$ such that the induced function $f^*:E(G)\to N$ is given by $f^*(uv)=\lceil\sqrt{[f(u)]^3-[f(v)]^3}\rceil$ (or) $\lfloor\sqrt{[f(u)]^3-[f(v)]^3}\rfloor$ is injective.

3. Super Root Cube of Cube Difference Labeling of Graphs

Definition 3.1. Let G be a graph (p,q) and $f:V(G)\to\{1,2,\cdots,p+q\}$ be an injective funtion. For each edge $e=uv$. Let $f^*(e=uv)=\lceil\sqrt{|((f(u))^3-(f(v))^3)^{\frac{1}{3}}|}\rceil$ (or) $\lfloor\sqrt{|((f(u))^3-(f(v))^3)^{\frac{1}{3}}|}\rfloor$, then f is called a super root cube of cube difference labeling if $f(v)\cup\{f^*(e):e\in E(G)\}=\{1,2,\cdots,p+q\}$. A graph that admits a super root cube of cube difference mean labeling is called as super root cube of cube difference labeling of graph.

Theorem 3.2. *The graph Comb is a Super Root cube of Cube Difference Labeling of graph.*

Proof. Let G be a Comb graph with vertex set $V(G)=\{s_1,s_2,\cdots,s_n,t_1,t_2,\cdots,t_n\}$. Let P_n be the path $s_1,s_2,\cdots,s_n$ and join a vertex t_i to $s_i, 1\le i\le n$. Define the function $f:V(G)\to\{1,2,\cdots,p+q\}$ by $f(u_i)=2i, 1\le i\le n$ and $f(v_i)=2i-1, 1\le i\le n$. Then the edge labeled as, $f^*(u_iu_{i+1})=\sqrt{|(24i^2+24i+8)^{\frac{1}{3}}|}, 1\le i\le n-1$

$f^*(u_iv_i)=\sqrt{|(12i^2-6i+1)^{\frac{1}{3}}|}, 1\le i\le n-1$

Hence the graph G is a Super Root Cube of Cube Difference Labeling graph. □

Theorem 3.3. *$P_n\bigodot K_{1,2}$ is a Super Root Cube of Cube Difference Labeling of graph.*

Proof. Let P_n be the path $u_1u_2\cdots u_n$. Let c_i and $d_i, 1\le i\le n$ be the vertices of $K_{1,2}$ attached to u_i. Define the function $f:V(P_n\bigodot K_{1,2})\to\{1,2,\cdots,p+q\}$ by $f(u_i)=i$ for $1\le i\le n, f(c_i)=2i+1$ for $2\le i\le n, f(d_i)=2i+4$ for $1\le i\le n$. Then the induced edge labeled as

$f^*(u_iu_{i+1})=\sqrt{|(3i^2+3i+1)^{\frac{1}{3}}|}, 1\le i\le n-1$

$f^*(u_ic_i)=\sqrt{|\ (7i^3+12i^2+6i+1)^{\frac{1}{3}}\ |}, 2\le i\le n$

$f^*(u_id_i)=\sqrt{|(7i^3+48i^2+96i+64)^{\frac{1}{3}}|}, 1\le i\le n$

Hence the graph $P_n\bigodot K_{1,2}$ is a Super Root Cube of Cube Difference Labeling of graph. □

Remark 3.4. $P_n \bigodot K_{1,2}$ is not a Super Root Cube of Cube Difference Labeling. Then the edges values are repeated in this vertices (8,5) and (11,12).

Theorem 3.5. *The graph $S(B_{n,n})$ is a Super Root Cube of Cube Difference Labeling of graph.*

Proof. Let $V(B_{n,n}) = \{u, u_i, v, v_i : 1 \leq i \leq n\}$ and $E(B_{n,n}) = \{uu_i, uv, vv_i : 1 \leq i \leq n\}$. Define the function $f : V(S(B_{n,n})) \to \{1, 2, \cdots, p+q\}$ by $f(u) = 1, f(u_i) = i+1, 2 \leq i \leq n$ and $f(v) = 2, f(v_i) = i+7, 1 \leq i \leq n$. Then the edge labeled as,
$f^*(uu_i) = \sqrt{|(i^3 + 3i^2 + 3i)^{\frac{1}{3}}|}, 2 \leq i \leq n$
$f^*(uv) = 1$
$f^*(vv_i) = \sqrt{|(i^3 + 21i^2 + 147i + 335)^{\frac{1}{3}}|}, 1 \leq i \leq n$
Hence graph $S(B_{n,n})$ is a Super Root Cube of Cube Differece Labeling of graph. □

Theorem 3.6. *The Ladder $P_n \times P_2$ is a Super Root Cube of Cube difference Labeling of graph.*

Proof. Let G be a Ladder graph. Let $V(G) = \{x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_n\}$. Define the function $f : V(G) \to \{1, 2, \cdots p+q\}$ by $f(u_i) = i, 1 \leq i \leq n$ and $f(v_i) = i+5, 1 \leq i \leq n$. The edges labels are,
$f^*(u_i u_{i+1}) = \sqrt{|(3i^2 + 3i + 1)\frac{1}{3}|}, 1 \leq i \leq n-1$
$f^*(v_i v_{i+1}) = \sqrt{|(3i^2 + 33i + 91)\frac{1}{3}|}, 1 \leq i \leq n-1$
$f^*(u_i v_i) = \sqrt{|(15i^2 + 75i + 125)\frac{1}{3}|}, 1 \leq i \leq n$ □

Acknowledgment. In this paper we discuss Super Root Cube of Cube Difference Labeling of Comb, $P_n \bigodot K_{1,2}, S(B_{n,n})$ and Ladder graphs. Here we investigate four results corresponding to labeled graphs. Similar work can be carried out of other families also.

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