Stochastic Modeling and Applications
Vol. 24 No. 1 (January-June, 2020)

Received: 07th March 2020 Revised: 17th April 2020 Accepted: 22nd June 2020

NON-HOMOGENEOUS POISSON QUEUING MODELS WITH LOAD DEPENDENT SERVICE

K. Srinivasa Rao, Department of Statistics Andhra University, Visakhapatnam, India

Ch.Sreelatha, Department of Statistics Andhra University, Visakhapatnam, India

B.Muniswamy, Department of Statistics Andhra University, Visakhapatnam, India

ABSTRACT

This paper addresses the descriptive modeling of a queuing system in which the arrival and service processes are non-stationary. It is assumed that the arrival and service processes follow non-homogeneous Poisson processes. Two models namely, (i) Single server queuing model and (ii) Two node tandem queuing model are developed and analyzed. For both the models the probability generating functions of the queue size distributions are derived using the difference-differential equations. The system characteristics of the model such as the average number of customers in the queue, the throughput of the service station, the average waiting time of a customer in the queue and in the system, the variance of the number of customers in each queue are derived. The sensitivity of the models with respect to the changes in the input parameters are also presented. It is observed that the time dependent nature of arrival and service processes has significant effect on the system performance measures. It is further assumed that the load dependent service can reduce the congestion in queues and delay in service. These models also include some of the earlier models as particular cases.

KEYWORDS: Non-stationary queuing model, time dependent queuing model, tandem queuing model, Non-homogeneous Poisson process, performance measures, sensitivity analysis.

1. INTRODUCTION:

Queuing models provide the basic framework for analyzing several systems arising at places like communication networks, cargo handling, transportation systems, machine repair, supply chain management and production processes. In general in queuing models it is considered that the arrival and service processes are independent of time. But in reality there are several systems in which the arrival and service processes are time dependent. Abry et al. (2002) and Cappe et al. (2002) have mentioned that modeling and analysis of computer network traffic using Poisson processes may not serve the purpose of predicting the traffic. Also they stated that the Poisson like nature assume the arrival of packets are smoother and less bursty but the present communication system the aggregate traffic does not become smooth and remain bursty. Leland et al. (1994) have mentioned that the actual traffic in Ethernet LAN exhibits the property of self similarity (burstness) and long range similarity. Rakesh Singhai et al. (2007) have further studied that metropolitan area network (MAN) traffic, wide area network (WAN) and variable bit rate (VBR) traffic which exhibit self similar and elastic characteristics and established that several of these networks are bursty because of time dependent nature of arrival and service processes.

Crovella et al. (1997), Murali Krishna et al. (2003), Feldman, A. (2000) have demonstrated that in TCP connection arrival or service processes, the inter arrival time of packets / inter service times of packets cannot be characterized by an exponential distribution. Hence, Fischer et al. (2001) have developed G/M/1 model with weibull inter-arrival time distribution. In addition to this a number of measurement studies given by Dinda P.A. et al. (2006) have revealed that the traffic generated by many real world applications exhibits a high degree of burstyness (time varying arrival and service rates). Therefore, there are many situations in real life that the arrival and service processes are time dependent due to various factors like load fluctuations, congestion and flow control, peaks hour overloads, adaptive rooting and others. Much work has been reported in literature regarding queuing models with time dependent arrival and service rates.

Newell (1968) has studied time-dependent arrival rates. Rothkopf and Oren (1979) have given closure approximation for the non-stationary M/M/s queue. Massey et al. (1993) have studied networks of infinite- server queues with non-stationary Poisson input, queuing systems. Massey and Whitt (1994) have analyzed of the modified offered load approximation for the non-stationary Erlang loss model. Mandelbaum and Massey (1995) have studied the approximation for time dependent queues. Davis et al. have (1995) studied the sensitivity to the service time distribution in the non-stationary Erlang loss model. William A. Massey (1996) has studied stability of queues with time varying rates. He analyzed the models using asymptotic method known as Uniform acceleration asymptotic behavior carry based on fluid and diffusion approximations. Duffield et al. (2001) have analyzed a non-stationary load model for packet networks. William A. Massey (2002) has analyzed the queues with time varying rates for telecommunication models. He also reviewed several works which support the arguments that time dependent behavior has an impact on traffic flow models. Ward Whitt (2016) reviewed the recent papers on time varying single server queue. In all these papers they analyzed the queuing models for time varying arrival and service rates using diffusion approximations or Kendal's frame work.

Recently Durga Aparajitha and rajkumar (2014), Srinivasa Rao et al. (2017,2017a) have developed and analyzed the queuing models with time and state dependent service rates. But they assumed that the arrival process follows a Poisson process which implies that the arrival process is independent of time. However, in many practical situations both the arrival and service processes are time dependent and the service rate is dependent on the content of the queue. Very little work has been reported in literature regarding queuing models with non-stationary arrival and service processes having load dependent service rates which are very useful for analyzing several practical situations. Hence, in this paper we develop and analyze queuing models with non-stationary arrival and service processes having load dependent service rates. First we develop a single server queuing model with time dependent arrival and service processes having state dependent service rate. It is assumed that the arrival and service processes follow non-homogeneous Poisson processes with different parameters. It is also assumed that the service rate is linearly dependent on the content of the queue connected to it. This model is extended to the case of two node tandem queuing model with non-stationary arrival and service processes having load dependent service rates. In this model it is assumed that the customers after getting service in the first service station join the second queues which are in series. The arrival process follows a non-homogeneous Poisson processes with parameter $\lambda(t)$. It is also further assumed in both the service stations the service processes follow non-homogeneous Poisson processes with different service rates. In both the models the queue discipline is first in, first out and queue capacity is infinite.

Using the difference-differential equations, the transient behavior of the models are analyzed by deriving the system performance measures such as probability of emptiness of the system, probability of emptiness of the marginal queues, the average number of customers in each queue, the utilization of the service station, the throughput of nodes, the average waiting time of customers in each queue. The sensitivity analyses of the models are also presented. Comparative study of the proposed models with that of models having Poisson arrival and service processes is also discussed.

2. SINGLE SERVER NON-HOMOGENEOUS POISSON QUEUING MODEL

In this section, we briefly present the development of the queuing model, Consider a single server Queuing model with the following assumptions.

a) The arrival process follows a non homogeneous Poisson process with mean arrival rate $\lambda(t)=\lambda_1+\lambda_2 t$.
b) The service process follows a non homogeneous Poisson process with mean service rate $\mu(t)=\mu_1+\mu_2 t$.
c) It is further assumed that the service rate is dependent on the number of customers in the queue.
d) The Queue discipline is first-in-first-out.
e) The Queue capacity is infinity.
f) The schematic diagram representing the Queuing model is shown in Figure-1.

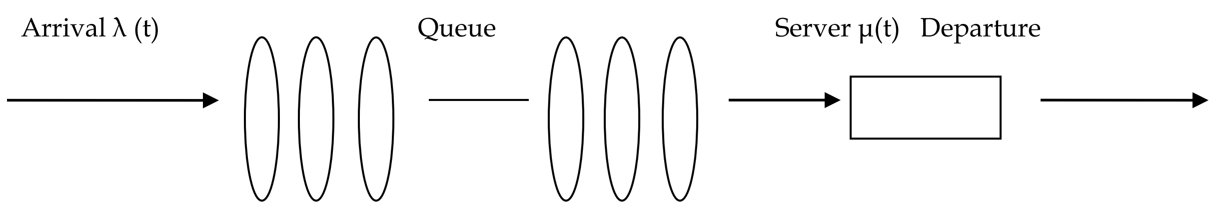


Fig1: Schematic diagram showing the queuing model.

Let n denote the number of customers in the queue, and $P_n(t)$ be the probability that there are n customers in the queue at time t. The difference- differential equations governing the model are:

$$\frac{\partial P_n(t)}{\partial t} = -\big(\lambda(t) + n\mu(t)\big)P_n(t) + \lambda(t)P_{n-1}(t) + (n+1)\mu(t)P_{n+1}(t)\ ; \quad for\ n \geq 1$$

$$\frac{\partial P_0(t)}{\partial t} = -\lambda(t)P_0(t) + \mu(t)P_1(t)\ ; \qquad for\ n = 0 \qquad (2.1)$$

Let the probability generating function of P_n (t) be

$$P(s,t) = \sum_{n=0}^{\infty} P_n(t)s^n \qquad (2.2)$$

Multiplying the equation (2.1) by s^n and summing over all n, we obtain

$$\frac{\partial P(s,t)}{\partial t} = \mu(t)(1-s)\frac{\partial P(s,t)}{\partial s} - \lambda(t)P(s,t)(1-s) \qquad (2.3)$$

Solving the equation (2.3) by the Lagrangian method, the auxiliary equation is

$$\frac{\partial t}{1}=\frac{\partial P}{\lambda(t)(s-1)P}=\frac{\partial s}{\mu(t)(s-1)} \tag{2.4}$$

To solving the first and third terms in equation (2.4), consider the service rate is linear and time dependent and is of the form $\mu(t) = \mu_1 + \mu_2 t$.

This implies,

$$\mathrm{a} = (\mathrm{s}-1)\mathrm{e}^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}, \qquad \text{where a is an arbitrary constant} \tag{2.5}$$

Solving the first and second terms in equation (2.4), consider the arrival rate is linear and time dependent and is of the form $\lambda(t) = \lambda_1 + \lambda_2 t$

$$b = Pexp\left[-(s-1)e^{-\int \mu(t)dt}\int \lambda(t)e^{\int \mu(t)dt}dt\right] \tag{2.6}$$

where b is arbitrary constant. Using the initial conditions $P_j(0)=0$, $P_0(0)=1$, $P_0(t)=0$; $\forall\, t>0$; the general solution of (2.4) gives the probability generating function of the number of customers in the queue at time 't' as

$$P(s,t) = \exp\left[(s-1)+\left[e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\mu_1 v+\mu_2\frac{v^2}{2}}dv+\lambda_2\int_0^t ve^{\mu_1 v+\mu_2\frac{v^2}{2}}dv\right]\right]\right] \quad ;\ s<1 \tag{2.7}$$

3. CHARACTERSTICS OF THE SINGLE SERVER NON-HOMOGENEOUS POISSON QUEUING MODEL

Expanding P(s, t) given in equation (2.7) and collecting the constant terms, we obtain the probability that the queue is empty as

$$P_0(t) = \exp\left[-e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]\right] \tag{3.1}$$

The mean number of customers in the system is

$$L(t) = e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right] \tag{3.2}$$

The mean number of customers in the queue is

$$L_q(t) = L(t) - \left(1-P_0(t)\right)$$

$$= e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]$$
$$-\left[1-\exp\left[-e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]\right]\right] \tag{3.3}$$

The utilization of the service station is

$U(t) = 1 - P_0(t)$

$$= \left[1-\exp\left[-e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]\right]\right] \tag{3.4}$$

The throughput of the service station is

$Thp(t) = \mu(t)U(t)$

$$= (\mu_2+\mu_2 t)\left[1-\exp\left[-e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]\right]\right] \tag{3.5}$$

The average waiting time of a customer in the system is

$W(t) = \dfrac{L(t)}{ThP(t)}$

$$= \frac{e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]}{(\mu_2+\mu_2 t)\left[1-\exp\left[-e^{-\left(\mu_1 t+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv+\lambda_2\int_0^t ve^{\left(\mu_1 v+\mu_2\frac{v^2}{2}\right)}dv\right]\right]\right]} \tag{3.6}$$

The variance of the number of customers in the system is

$$V(t) = e^{-\int\left(\mu_1+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1+\mu_2\frac{v^2}{2}\right)}dv+\lambda_1\int_0^t e^{\left(\mu_1+\mu_2\frac{v^2}{2}\right)}dv\right] \tag{3.7}$$

The coefficient of variation of the number of customers in the system is

$CV = \dfrac{\sqrt{V(t)}}{L(t)} * 100$

$$=\left[e^{-\int\left(\mu_1+\mu_2\frac{t^2}{2}\right)}\left[\lambda_1\int_0^t e^{\left(\mu_1+\mu_2\frac{v^2}{2}\right)}dv+\lambda_1\int_0^t e^{\left(\mu_1+\mu_2\frac{v^2}{2}\right)}dv\right]\right]^{-1/2} * 100 \tag{3.8}$$

4. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

In this paper, the performance of the queuing model is discussed through numerical illustration. The customers arrive to the queue in which the arrival and service processes are non-homogeneous Poisson processes with mean arrival rate $\lambda(t)=\lambda_1+\lambda_2 t$ and service rate $\mu(t)=\mu_1+\mu_2 t$.

Because the characteristics of the queuing model are highly sensitive with respect to time, the transient behavior of the model is studied by computing the performance measures with the following sets of values for the model parameters

t = 2, 3, 4, 5, 6 ; λ_1=2, 4, 6, 8, 10, 13; λ_2=1, 4, 5, 7, 8, 9

μ_1=3, 9, 10, 11, 12, 13; μ_2= 1, 2, 2.5, 3, 3.5, 4;

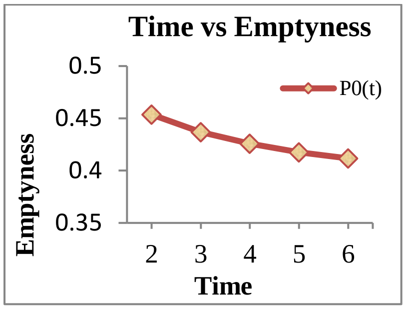
The probability of emptiness of the queue, the mean number of customers, the utilization of service station, the throughput of the service station, the variance of the number of customers in the queue and the coefficient of the variation of the number of customers in the queue are computed for different values of the parameters t, λ_1, λ_2, μ_1, μ_2 and are presented in Table 1. The relationships between the parameters and performance are shown in Figure 2a, 2b.

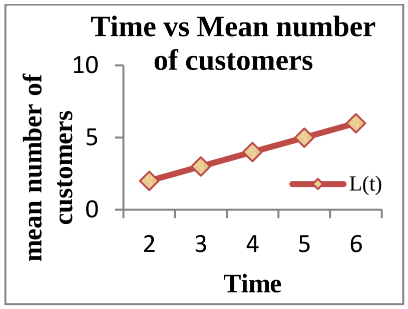
Table 1

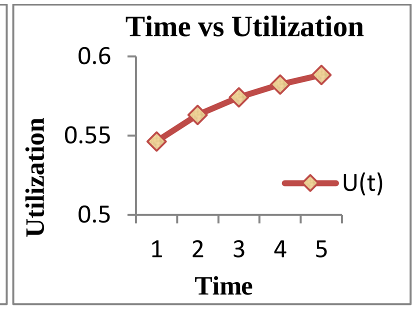
Values of $P_0(t)$, L(t), U(t), ThP(t),W(t),V(t),CV(t) for different values of parameters

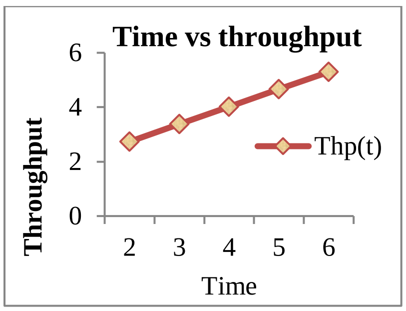
t	λ_1	λ_2	μ_1	μ_2	$P_0(t)$	L(t)	U(t)	ThP(t)	W(t)	V(t)	CV(t)
2	2	1	3	1	0.4536	0.7906	0.5464	2.7320	0.2894	0.7906	1.1247
3	2	1	3	1	0.4368	0.8282	0.5632	3.3791	0.2451	0.8282	1.0988
4	2	1	3	1	0.4257	0.8540	0.5743	4.0201	0.2124	0.8540	1.0821
5	2	1	3	1	0.4177	0.8729	0.5823	4.6583	0.1874	0.8729	1.0703
6	2	1	3	1	0.4117	0.8875	0.5883	5.2947	0.1676	0.8875	1.0615
2	**4**	1	3	1	0.2986	1.2088	0.7014	3.5072	0.3447	1.2088	0.9096
2	**6**	1	3	1	0.1965	1.6270	0.8035	4.0174	0.4050	1.6270	0.7840
2	**8**	1	3	1	0.1293	2.0452	0.8707	4.3533	0.4698	2.0452	0.6992
2	**10**	1	3	1	0.0851	2.4635	0.9149	4.5743	0.5385	2.4635	0.6371
2	**13**	1	3	1	0.0455	3.0908	0.9545	4.7727	0.6476	3.0908	0.5688
2	2	**4**	3	1	0.1484	1.9075	0.8516	4.2578	0.4480	1.9075	0.7240
2	2	**5**	3	1	0.1023	2.2798	0.8977	4.4885	0.5079	2.2798	0.6623
2	2	**7**	3	1	0.0486	3.0245	0.9514	4.7571	0.6358	3.0245	0.5750
2	2	**8**	3	1	0.0335	3.3968	0.9665	4.8326	0.7029	3.3968	0.5426
2	2	**9**	3	1	0.0231	3.7691	0.9769	4.8846	0.7716	3.7691	0.5151
2	2	1	**9**	1	0.6989	0.3582	0.3011	3.3120	0.1082	0.3582	1.6708
2	2	1	**10**	1	0.7199	0.3286	0.2801	3.3609	0.0978	0.3286	1.7445
2	2	1	**11**	1	0.7382	0.3035	0.2618	3.4023	0.0892	0.3035	1.8151
2	2	1	**12**	1	0.7543	0.2820	0.2457	3.4403	0.0820	0.2820	1.8831
2	2	1	**13**	1	0.7685	0.2634	0.2315	3.4731	0.0758	0.2634	1.9486
2	2	1	3	**2**	0.5628	0.5748	0.4372	3.0602	0.1878	0.5748	1.3190
2	2	1	3	**2.5**	0.6038	0.5045	0.3962	3.1695	0.1592	0.5045	1.4079
2	2	1	3	**3**	0.6382	0.4491	0.3618	3.2564	0.1379	0.4491	1.4922

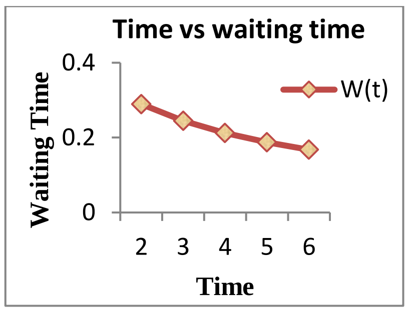
2	2	1	3	**3.5**	0.6673	0.4045	0.3327	3.3270	0.1216	0.4045	1.5723
2	2	1	3	**4**	0.6922	0.3678	0.3078	3.3856	0.1087	0.3678	1.6488

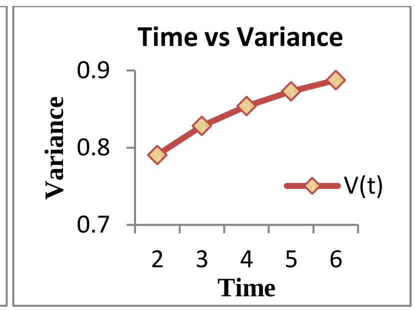


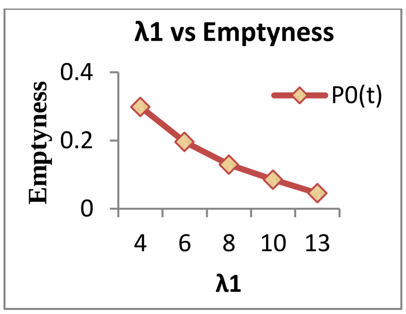


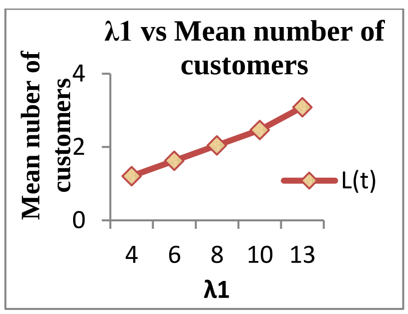


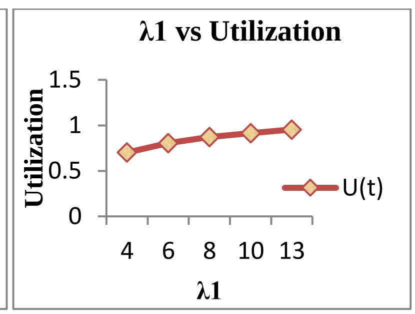


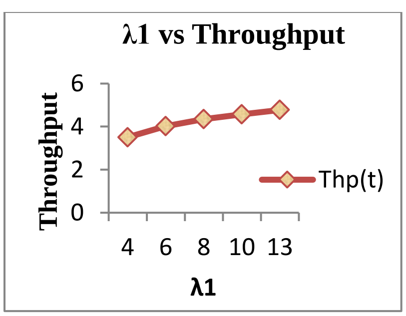


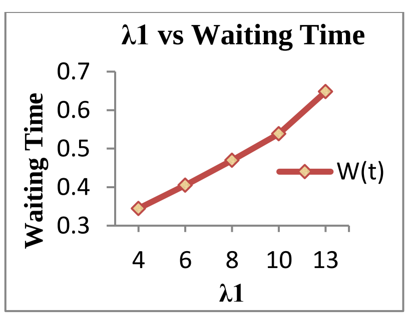


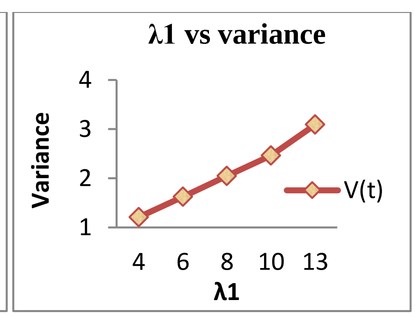












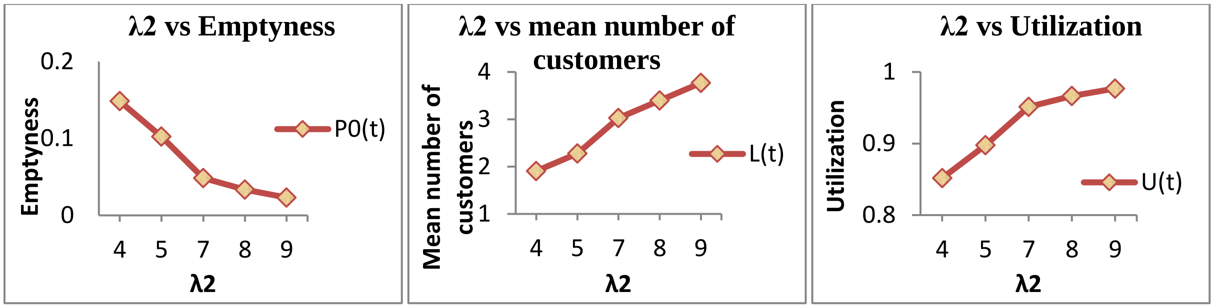
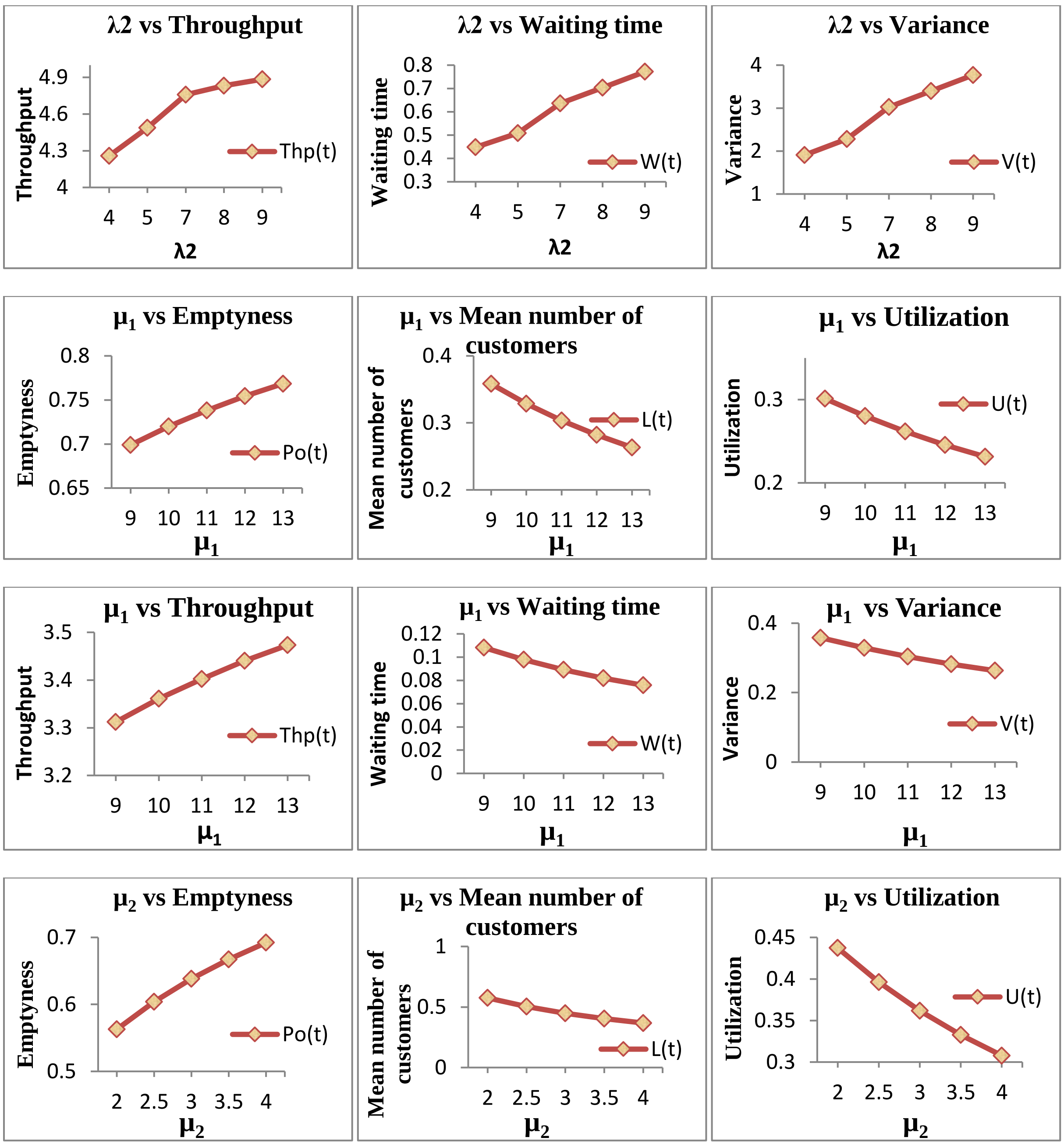


Figure 2a: The relationships between the parameters and performance measures.



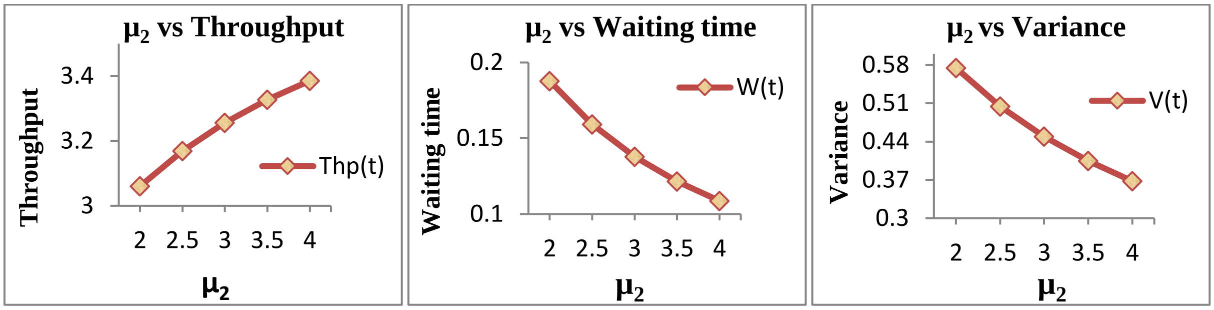


Figure 2b: The relationships between the parameters and performance measures.

From Table-1, it is observed that as time (t) increases, the probability of emptiness of the queue decreases, the mean number of customers in the queue increases, the utilization of the service station increases, the throughput of the service station increases, the average waiting time of the customers in the queue decreases, the variance of the number of customers in the queue increases, and the coefficient of variation of the number of customers in the queue decreases, when the other parameters are fixed.

It is observed that as the arrival rate parameter (λ_1) increases, the probability of emptiness of the queue decreases, the mean number of customers in the queue increases, the utilization of the service station increases, the throughput of the service station increases, the average waiting time of the customers in the queue increases, the variance of the number of customers in the queue increases, and the coefficient of variation of the number of customers in the queue decreases, when the other parameters are fixed.

It is also observed that as the arrival rate parameter (λ_2) increases, the probability of emptiness of the queue decreases, the mean number of customers in the queue increases, the utilization of the service station increases, the throughput of the service station increases, the average waiting time of the customers in the queue increases, the variance of the number of customers in the queue increases, and the coefficient of variation of the number of customers in the queue decreases, when the other parameters are fixed.

It is observed that as the service rate parameter (μ_1) increases, the probability of emptiness of the queue increases, the mean number of customers in the queue decreases, the utilization of the service station decreases, the throughput of the service station increases, the average waiting time of the customers in the queue decreases, the variance of the number of customers in the queue decreases, and the coefficient of variation of the number of customers in the queue increases, when the other parameters are fixed.

It is observed that as the service rate parameter (μ_2) increases, the probability of emptiness of the queue increases, the mean number of customers in the queue decreases, the utilization of the service station decreases, the throughput of the service station increases, the average waiting time of the customers in the queue decreases, the variance of the number of customers in the queue decreases, and the coefficient of variation of the number of customers in the queue increases, when the other parameters are fixed.

Sensitivity analysis of the model is performed with respect to the value of time (t), arrival rate parameters λ_1 and λ_2, service rate parameters $\mu_1$and μ_2 and all parameters together on the mean number of customers (L), Utilization (U), the mean delay (W) and the throughput (Thp), are computed and presented in Table 2 with variation of -15%, -10%, -5%, 0%, +15%, +10%, +5% on the model parameters.

Table 2: Sensitivity Analysis of the model
(t =4, λ_1 =2, λ_2 =1, μ_1=3, μ_2 =1**)**

Parameter	**Performance measure**	**% change in parameters**						
		-15%	-10%	-5%	0%	5%	10%	15%
t=4	L(t)	0.8396	0.8447	0.8495	0.8540	0.8583	0.8622	0.8660
	U(t)	0.5681	0.5703	0.5724	0.5743	0.5761	0.5778	0.5794
	W(t)	0.2309	0.2244	0.2183	0.2124	0.2069	0.2017	0.1967
	Thp(t)	3.6360	3.7642	3.8922	4.0201	4.1479	4.2756	4.4032
λ_1 =2	L(t)	0.8102	0.8248	0.8394	0.8540	0.8686	0.8832	0.8978
	U(t)	0.5552	0.5617	0.5680	0.5743	0.5805	0.5866	0.5925
	W(t)	0.2085	0.2098	0.2111	0.2124	0.2138	0.2151	0.2165
	Thp(t)	3.8867	3.9318	3.9763	4.0201	4.0633	4.1059	4.1478
λ_2=1	L(t)	0.7697	0.7978	0.8259	0.8540	0.8821	0.9102	0.9398
	U(t)	0.5369	0.5497	0.5622	0.5743	0.5861	0.5976	0.6087
	W(t)	0.2048	0.2073	0.2099	0.2124	0.2150	0.2176	0.2202
	Thp(t)	3.7580	3.8478	3.9352	4.0201	4.1027	4.1830	4.2611
μ_1=3	L(t)	0.9139	0.8930	0.8731	0.8540	0.8358	0.8184	0.8017
	U(t)	0.5990	0.5906	0.5823	0.5743	0.5665	0.5588	0.5514
	W(t)	0.2329	0.2257	0.2189	0.2124	0.2064	0.2006	0.1951
	Thp(t)	3.9238	3.9569	3.9890	4.0201	4.0503	4.0796	4.1080
μ_2=1	L(t)	0.9322	0.9046	0.8786	0.8540	0.8308	0.8087	0.7878
	U(t)	0.6063	0.5953	0.5846	0.5743	0.5643	0.5546	0.5451
	W(t)	0.2402	0.2302	0.2210	0.2124	0.2045	0.1971	0.1901
	Thp(t)	3.8804	3.9290	3.9756	4.0201	4.0628	4.1038	4.1431

The performance measures are highly affected by time (t) as t increases from -15% to +15%, the average number of customers in the queue along with the average delay are increasing. As λ_1 and λ2 are increasing the average number of customers in the queue along with the average delay, the utilization and throughput are increasing. Similarly as the service rate parameters μ_1 and μ_2 decrease from -15% to +15%, the same phenomenon is observed. Overall analysis of the parameters reflects that the load dependent strategy reduces the congestion in queues, the delay in transmission and improves the quality of service.

5. COMPARATIVE STUDY OF SINGLE SERVER NON-STATIONARY QUEUING MODEL

The comparative study between the non-homogeneous and homogeneous Poisson arrival and service rates is presented in this session. The computed performance measure of both models are presented in the Table-3 for different values of t = 2, 3, 4, 5, 6 seconds.

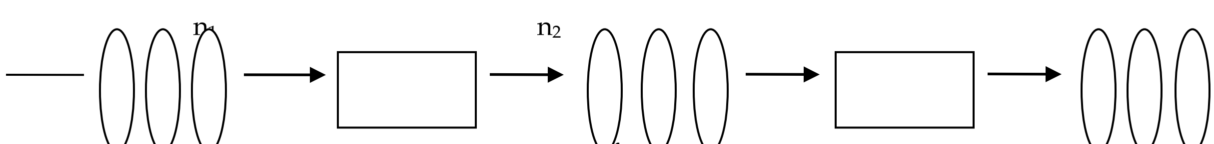
Table 3: Comparative study if model with non-homogeneous and homogeneous Poisson Arrival and Service rates

t	Parameter measure	Non-homogeneous arrival and service process	Homogeneous arrival and service process	Difference	Percentage of Variation
2	L(t)	1.20878	1.3303	0.12152	9.134782
	U(t)	0.70144	0.73553	0.03409	4.634753
	W(t)	0.34466	0.60275	0.25809	42.81875
3	L(t)	1.17175	1.33317	0.16142	12.10798
	U(t)	0.69018	0.73636	0.04618	6.271389
	W(t)	0.28296	0.6035	0.32054	53.1135
4	L(t)	1.14596	1.33333	0.18737	14.05279
	U(t)	0.68209	0.7364	0.05431	7.375068
	W(t)	0.24001	0.60353	0.36352	60.2323
5	L(t)	1.12705	1.33333	0.20628	15.47104
	U(t)	0.67601	0.7364	0.06039	8.200706
	W(t)	0.2084	0.60353	0.39513	65.46982
6	L(t)	1.11254	1.33333	0.22079	16.55929
	U(t)	0.67128	0.7364	0.06512	8.84302
	W(t)	0.18415	0.60353	0.41938	69.48785

From Table 3, it is observed that as time increases the percentage of variation of the performance measures between the two models are increasing. The model with homogeneous Poisson arrival and service process has more L(t) ant W(t) than the model with non-homogeneous Poisson arrival and service processes. It is also observed that the assumption of non-homogeneous Poisson arrival and service processes has a significant influence on all the performance measures of the queuing model. As t increases the percentage of variation is increasing. Time also has a significant effect on the system performance and the proposed model can predict the performance more accurately.

6. TWO NODE TANDEM NON-HOMOGENEOUS POISSON QUEUING MODEL

Consider a queuing system in which the customers arrive to the first queue and after getting service through the first server they join the second queue, which is serially connected to the first service station. The arrival of the customers follow non-homogeneous Poisson process with mean arrival rate $\lambda(t)= \lambda_1+ \lambda_2 t$. and the service processes in the first and second service stations follow non-homogeneous Poisson processes with mean service rates $\mu_1(t) = \alpha_1+\beta_1 t$ and $\mu_2(t) = \alpha_2+\beta_2 t$ respectively. The schematic diagram representing the queuing model is shown in Figure-3.



λ(t) μ_1(t) μ_2(t)

Figure -3: Schematic diagram of the queuing model

Let $P_{n_1 n_2}(t)$ be the probability that there are n_1 customers in the first queue and n_2 customers in the second queue at time't'. The difference differential equations governing the model are:

$$\frac{\partial P_{n_1,n_2}(t)}{\partial t} = -\big(\lambda(t) + n_1\mu_1(t) + n_2\mu_2(t)\big)P_{n_1,n_2}(t) + \lambda(t)P_{n_1-1,n_2}(t)$$

$$+(n_1+1)\mu_1(t)P_{n_1+1,n_2-1}(t) + (n_2+1)\mu_2(t)P_{n_1,n_2+1}(t)\ ; \qquad \forall\ n_1, n_2 \geq 0$$

$$\frac{\partial P_{n_1,0}(t)}{\partial t} = -\big(\lambda(t) + n_1\mu_1(t)\big)P_{n_1,0}(t) + \lambda(t)P_{n_1-1,0}(t) + \mu_2(t)P_{n_1,1}(t)\ ;$$

$$\forall\ n_1 > 0, n_2 = 0$$

$$\frac{\partial P_{0,n_2}(t)}{\partial t} = -\big(\lambda(t) + n_2\mu_2(t)\big)P_{0,n_2}(t) + \mu_1(t)P_{1,n_2-1}(t) + (n_2+1)\mu_2(t)P_{0,n_2+1}(t);$$

$$\forall\ n_2 > 0, n_1 = 0$$

$$\frac{\partial P_{0,0}(t)}{\partial t} = -\lambda(t)P_{0,0}(t) + \mu_2(t)P_{0,1}(t);\ \ n_1 = 0, n_2 = 0 \tag{6.1}$$

The probability generating function of $P_{n_1 n_2}(t)$ is

$$P(S_1, S_2, t) = \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty} P_{n_1,n_2}(t) s_1^{n_1} s_2^{n_2} \tag{6.2}$$

Multiplying the equation (6.1) with $s_1^{n_1}, s_2^{n_2}$ and summing over all n_1,n_2, we get

$$\frac{\partial P(t)}{\partial t} = -\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}(\lambda(t) + n_1\mu_1(t) + n_2\mu_2(t))\, P_{n_1,n_2}(t)s_1^{n_1}s_2^{n_2} + \sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}\lambda(t)\, P_{n_1-1,n_2}(t)s_1^{n_1}s_2^{n_2}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}(n_1+1)\,\mu_1(t)P_{n_1+1,n_2-1}(t)s_1^{n_1}s_2^{n_2}$$
$$+\sum_{n_1=0}^{\infty}\sum_{n_2=0}^{\infty}(n_2+1)\mu_2(t)\, P_{n_1,n_2+1}(t)s_1^{n_1}s_2^{n_2}\ ; \tag{6.3}$$

After simplifying, we get

$$\frac{\partial P(S_1, S_2, t)}{\partial t} = \mu_1(t)(s_2 - s_1)\frac{\partial P(s_1, s_2, t)}{\partial s_1} + \mu_2(t)(1 - s_2)\frac{\partial P(s_1, s_2, t)}{\partial s_2}$$
$$-\lambda(t)(1 - s_1)P(s_1, s_2, t) \tag{6.4}$$

Solving the equation (6.4) by Lagrangian's method, the auxiliary equations are

$$\frac{dt}{1} = \frac{ds_1}{-\mu_1(t)(s_2 - s_1)} = \frac{ds_2}{-\mu_2(t)(1 - s_2)} = \frac{dP}{-\lambda(t)(1 - s_1)P(s_1, s_2, t)} \tag{6.5}$$

Let the arrival rate and service rates are linear and time dependent and is of the form

$\lambda(t) = \lambda_1 + \lambda_2 t;$

$\mu_1(t) = \alpha_1 + \beta_1 t; \qquad 0 \le \beta_1 \le 1$

$and\ \mu_2(t) = \alpha_2 + \beta_2 t; \qquad 0 \le \beta_2 \le 1, \alpha_1 \ne \alpha_2$

Solving the first and third terms in equation (6.5), we get

$$a = (s_2 - 1)e^{-\int \mu_2(t)dt} \tag{6.6}$$

Solving the first and second terms in equation (6.5), we get

$$b = s_1 e^{-\int \mu_1(t)dt} + (s_2 - 1)e^{-\int \mu_2(t)dt}\left(\int \mu_1(t)e^{\int[\mu_2(t)-\mu_1(t)]dt}\,dt\right) + \int \mu_1(t)e^{-\int \mu_1(t)dt}dt) \tag{6.7}$$

Solving the first and fourth terms in equation (6.5), we get

$$\begin{aligned} c = P(s_1, s_2, t)\exp\Bigg(-\Bigg[&s_1 e^{-\int \mu_1(t)dt} + (s_2 - 1)e^{-\int \mu_2(t)dt}\left(\int \mu_1(t)e^{\int[\mu_2(t)-\mu_1(t)]dt}\,dt\right) \\ &+ \int \mu_1(t)e^{-\int \mu_1(t)dt}dt\Bigg]\left[\int \lambda(t)\, e^{\int \mu_1(t)dt}\right]\Bigg) \\ &+ \left[(s_2 - 1)e^{-\int \mu_2(t)dt}\int \lambda(t)\, e^{\int \mu_1(t)dt}\left(\int \mu_1(t)e^{\int[\mu_2(t)-\mu_1(t)]dt}\,dt\right)dt\right] \\ &+ \left[\int \lambda(t)\, e^{\int \mu_1(t)dt}\left(\mu_1(t)e^{-\int \mu_1(t)dt}dt\right)dt\right] + \int \lambda(t)dt \end{aligned} \tag{6.8}$$

Where a, b and c are arbitrary constants. Using the initial conditions $P_{00}(0)=1$, $P_{00}(t)=0$, $\forall$ t >0.

The probability generating function of the number of customers in the first queue and the number of customers in the second queue at time't' is

$$\begin{aligned} P(s_1, s_2, t) = \exp\Bigg[\lambda_1\Bigg[&(s_1 - 1)e^{-\left(\alpha_1 t + \beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1 + \lambda_2 \vartheta)e^{\left(\alpha_1\vartheta + \beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right) \\ &+ (s_2 - 1)e^{-\left(\alpha_2 t + \beta_2 \frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2 - \alpha_1} - \frac{\int_0^t (\alpha_1 + \beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta + (\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right) \\ &+ (s_2 - 1\ )e^{-\left(\alpha_2 t + \beta_2 \frac{t^2}{2}\right)}\Bigg(\frac{\int_0^t (\lambda_1 + \lambda_2\vartheta)e^{\alpha_1\vartheta + \beta_1\frac{\vartheta^2}{2}}d\vartheta \int_0^t (\alpha_1 + \beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta + (\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1} \\ &- \frac{\int_0^t (\lambda_1 + \lambda_2\vartheta)e^{\alpha_1\vartheta + \beta_1\frac{\vartheta^2}{2}}\left(\int_0^t (\alpha_1 + \beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta + (\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1} - \frac{1}{\alpha_2}\Bigg)\Bigg]\Bigg] \end{aligned} \tag{6.9}$$

7. CHARACTERSTICS OF THE TWO NODE TANDEM NON-HOMOGENEOUS POISSON QUEUING MODEL

Expanding P(s1,s2,t) given in equation (9) and collecting the constant terms, we obtain the probability that the queue is empty as

$$P_{0,0}(t) = exp\left[-\lambda_1\left[e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)+e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}\right.\right.\right.$$
$$\left.-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)$$
$$+e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$
$$\left.\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right)\right]\right] \tag{7.1}$$

Taking $s_2 = 1$ in P(s1,s2,t) , we obtain the probability generating function of the first queue size as

$$P(s_1,t)=exp\left[\lambda_1(s_1-1)e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)\right] \tag{7.2}$$

By expanding P(s1,t), and collecting the constant terms, we obtain the probability that the first queue is empty as

$$P_{0.}(t) = exp\left[-\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)\right] \tag{7.3}$$

The mean number of customers in the first queue is

$$L_1(t) = \lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right) \tag{7.4}$$

The utilization of the first service station is

$$U_1(t) = 1 - P_{0.}(t)$$

$$= 1 - exp\left[-\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)\right] \tag{7.5}$$

The throughput of the first service station is

$$ThP_1(t) = \mu_1(t)U_1(t)$$

$$= (\alpha_1 + \beta_1 t)\left[1 - exp\left[-\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right)\right]\right] \quad (7.6)$$

The average waiting time of a customer in the first queue is

$$W_1(t) = \frac{L_1(t)}{ThP_1(t)}$$

$$= \frac{\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right)}{(\alpha_1+\beta_1 t)\left[1 - exp\left[-\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right)\right]\right]} \quad (7.7)$$

The variance of the number of customers in the first queue is

$$V_1(t) = \lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right) \quad (7.8)$$

The coefficient of variation of the number of customers in the first queue is

$$CV_1(t) = \frac{\sqrt{V_1(t)}}{L_1(t)}$$

$$= \left[\lambda_1 e^{-\left(\alpha_1 t+\beta_1 \frac{t^2}{2}\right)}\left(\frac{\int_0^t (\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1} - \frac{1}{\alpha_1}\right)\right]^{\frac{-1}{2}} \quad (7.9)$$

Taking $s_1 = 1$ in $P(s_1,s_2,t)$, we obtain the probability generating function for the Second queue size as

$$P(s_2,t) = exp\left[\lambda_1\left[(s_2-1)e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1} - \frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right) + (s_2 - 1)e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_2\vartheta+\beta_2\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1} - \frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1} - \frac{1}{\alpha_2}\right)\right]\right] \quad (7.10)$$

By expanding P(s2,t), and collecting the constant terms, we obtain the probability that the first queue is empty as

$$P_{.0}(t) = exp\left[-\lambda_1\left[e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)\right.\right.$$

$$+e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]\right]\right] \quad (7.11)$$

The mean number of customers in the Second queue is

$$L_2(t) = \lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right] \quad (7.12)$$

The utilization of the Second service station is

$U_2(t) = 1 - P_{.0}(t)$

$$= 1 - exp\left[-\lambda_1\left[e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)\right.\right.$$

$$+ e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]\right]\right] \tag{7.13}$$

The throughput of the Second service station is

$ThP_2(t) = \mu_2(t)U_2(t)$

$$= (\alpha_2+\beta_2 t)\left[1 - exp\left[-\lambda_1\left[e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)\right.\right.\right.$$

$$+ e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.\left.\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]\right]\right]\right] \tag{7.14}$$

The average waiting time of a customer in the Second queue is

$$W_2(t) = \frac{L_2(t)}{ThP_2(t)}$$

Where, $L_2(t)$ and $ThP_2(t)$ are given in equation (7.12) and (7.14) respectively (7.15)

The variance of the number of customers in the Second queue is

$$V_2(t) = \lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right] \tag{7.16}$$

The coefficient of variation of the number of customers in the Second queue is

$$CV_2(t)=\frac{\sqrt{V_2(t)}}{L_2(t)}$$

$$=\left[\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)\right.$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]\right]^{\frac{-1}{2}} \tag{7.17}$$

The mean number of customers in the queuing system at time t is

$$L(t)=L_1(t)+L_2(t)$$

$$=\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)+$$

$$\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}-\right.$$

$$\left.\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right] \tag{7.18}$$

The variance of the number of customers in the system is

$$V(t) = V_1(t) + V_2(t)$$

$$= \lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)+$$

$$\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}-\right.$$

$$\left.\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right] \tag{7.19}$$

The mean number of customers waiting in the queue is

$$L_q(t) = L_1(t) - (1 - P_{0.}(t)) + L_2(t) + (1 - P_{.0}(t))$$

$$= \lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)$$

$$+exp\left[-\lambda_1 e^{-\left(\alpha_1 t+\beta_1\frac{t^2}{2}\right)}\left(\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\left(\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}\right)}d\vartheta}{\lambda_1}-\frac{1}{\alpha_1}\right)\right]$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)$$

$$+\lambda_1 e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}\right.$$

$$\left.-\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]$$

$$+exp\left[-\lambda_1\left[e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left(\frac{1}{\alpha_2-\alpha_1}-\frac{\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\alpha_1}\right)+\right.\right.$$

$$e^{-\left(\alpha_2 t+\beta_2\frac{t^2}{2}\right)}\left[\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}d\vartheta\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta}{\lambda_1}-\right.$$

$$\left.\left.\left.\frac{\int_0^t(\lambda_1+\lambda_2\vartheta)e^{\alpha_1\vartheta+\beta_1\frac{\vartheta^2}{2}}\left(\int_0^t(\alpha_1+\beta_1\vartheta)e^{(\alpha_2-\alpha_1)\vartheta+(\beta_2-\beta_1)\frac{\vartheta^2}{2}}d\vartheta\right)d\vartheta}{\lambda_1}-\frac{1}{\alpha_2}\right]\right]\right]-2 \tag{7.20}$$

8. NUMERICAL ILLUSTRATION AND SENSITIVITY ANALYSIS

In this section, the performance of the queuing model is discussed through numerical illustration. The characteristics of the queuing model are highly sensitive with respect to time, the transient behavior of the model is studied by computing the performance measures with the following sets of values for the model parameters.

t = 0.11, 0.13, 0.15, 0.17 $\lambda 1$ =4, 6, 8, 9, 9.5 λ_2 =5, 5.5, 5.9, 6.4, 6.8 α_1 = 9, 10, 10.2, 10.4, 10.6

$\beta 1$ = 6, 8, 10, 13, 16 α_2 = 11, 11.4, 11.8, 12.2, 12.6 β_2 = 11, 15, 20, 25, 30

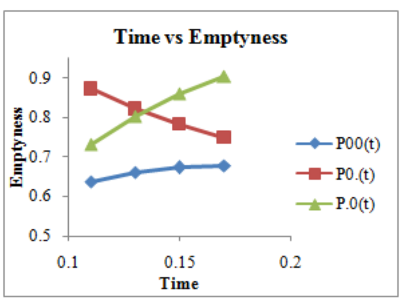
For different values of the parameters t, λ_1, λ_2, α_1, β_1, α_2, and β_2 , the computed values of the probability of emptiness of the queue, the mean number of customers, the utilization of the service station, the throughput of the service station, the variance of the number of customers in the system and the coefficient of variation of the number of customers in the system are presented in Table 4. The relationship between the parameters and the performance measures are represented in the Figures 4a, 4b.

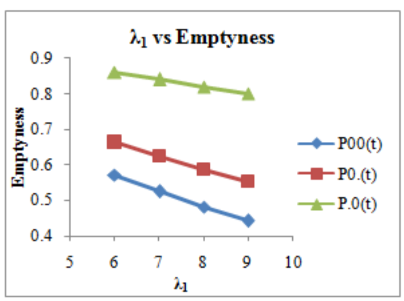
From Table 4, it is observed that as time (t) varies from 0.11 to 0.17, the probability of emptiness of the queuing system increases from 0.63850 to 0.67847 .Similarly, the probability of emptiness of the first queue decreases from 0.87268 to 0.74922 and the probability of emptiness of the second queue increases from 0.73165 to 0.90557, the mean number of customers in the first queue increases from 0.13618 to 0.28872, and in the second queue it decreases from 0.31246 to 0.09919 and the mean number of customers in the system decreases from 0.44864 to 0.38791, when all other parameters are fixed.

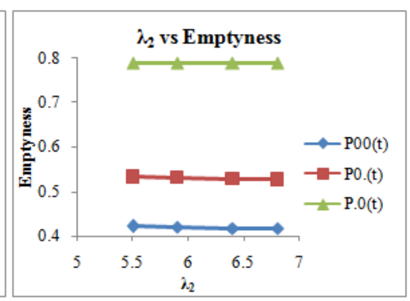
Table.4
Values of $P_{00}(t)$, $P_{0.}(t)$, $P_{.0}(t)$, $L_1(t)$, $L_2(t)$ for different values of parameters

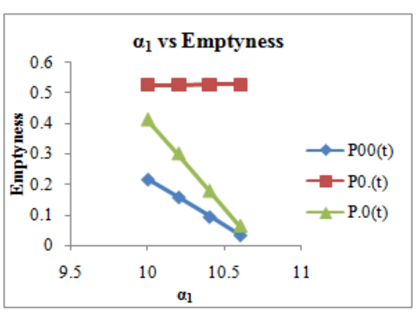
T	λ_1	λ_2	α_1	β_1	α_2	β_2	$P_{00}(t)$	$P_{0.}(t)$	$P_{.0}(t)$	$L_1(t)$	$L_2(t)$	$L(t)$
0.11	4	5	9	6	11	11	0.63850	0.87268	0.73165	0.13618	0.31246	0.44864
0.13	4	5	9	6	11	11	0.66026	0.82222	0.80301	0.19574	0.21938	0.41512
0.15	4	5	9	6	11	11	0.67287	0.78183	0.86063	0.24612	0.15009	0.39621
0.17	4	5	9	6	11	11	0.67847	0.74922	0.90557	0.28872	0.09919	0.38791
0.17	**6**	5	9	6	11	11	0.57143	0.66310	0.86176	0.41083	0.14878	0.55961
0.17	**7**	5	9	6	11	11	0.52442	0.62383	0.84065	0.47188	0.17358	0.64546
0.17	**8**	5	9	6	11	11	0.48127	0.58688	0.82006	0.53294	0.19838	0.73132

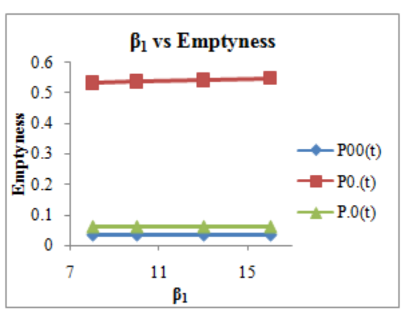
0.17	**9**	5	9	6	11	11	0.44168	0.55212	0.79997	0.59399	0.22318	0.81717
0.17	9.5	**5.5**	9	6	11	11	0.42124	0.53314	0.79012	0.62897	0.23557	0.86454
0.17	9.5	**5.9**	9	6	11	11	0.41975	0.53125	0.79012	0.63253	0.23557	0.86810
0.17	9.5	**6.4**	9	6	11	11	0.41788	0.52889	0.79012	0.63698	0.23557	0.87255
0.17	9.5	**6.8**	9	6	11	11	0.41640	0.52701	0.79012	0.64054	0.23557	0.87611
0.17	9.5	6.8	**10**	6	11	11	0.21728	0.52671	0.41252	0.64111	0.88547	1.52658
0.17	9.5	6.8	**10.2**	6	11	11	0.15843	0.52728	0.30047	0.64002	1.20240	1.84242
0.17	9.5	6.8	**10.4**	6	11	11	0.09386	0.52802	0.17775	0.63862	1.72737	2.36599
0.17	9.5	6.8	**10.6**	6	11	11	0.03306	0.52892	0.06250	0.63692	2.77264	3.40956
0.17	9.5	6.8	10.6	**8**	11	11	0.03333	0.53272	0.06257	0.62977	2.77139	3.40116
0.17	9.5	6.8	10.6	**10**	11	11	0.03361	0.53649	0.06265	0.62271	2.77020	3.39291
0.17	9.5	6.8	10.6	**13**	11	11	0.03402	0.54209	0.06275	0.61233	2.76854	3.38087
0.17	9.5	6.8	10.6	**16**	11	11	0.03442	0.54762	0.06285	0.60218	2.76701	3.36919
0.17	9.5	6.8	10.6	16	**11.4**	11	0.17831	0.54762	0.32561	0.60218	1.12206	1.72424
0.17	9.5	6.8	10.6	16	**11.8**	11	0.30377	0.54762	0.55471	0.60218	0.58931	1.19149
0.17	9.5	6.8	10.6	16	**12.2**	11	0.39236	0.54762	0.71648	0.60218	0.33340	0.93558
0.17	9.5	6.8	10.6	16	**12.6**	11	0.45405	0.54762	0.82914	0.60218	0.18736	0.78954
0.17	9.5	6.8	10.6	16	12.6	**15**	0.46089	0.54762	0.84163	0.60218	0.17241	0.77459
0.17	9.5	6.8	10.6	16	12.6	**20**	0.46897	0.54762	0.85638	0.60218	0.15504	0.75722
0.17	9.5	6.8	10.6	16	12.6	**25**	0.47655	0.54762	0.87022	0.60218	0.13901	0.74119
0.17	9.5	6.8	10.6	16	12.6	**30**	0.48364	0.54762	0.88317	0.60218	0.12423	0.72641

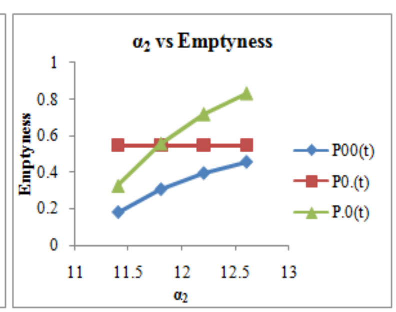












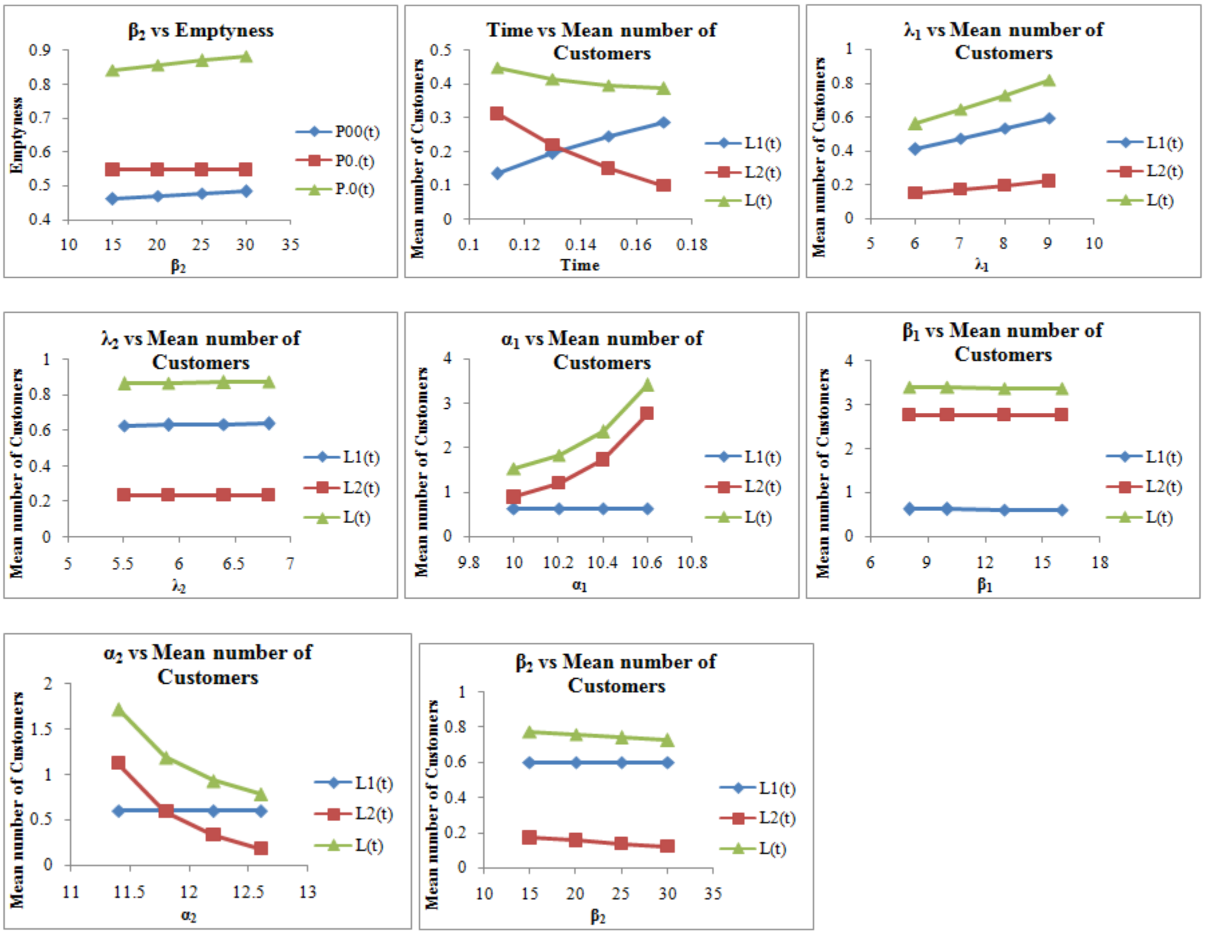


Figure 4a: The relationship between the parameters and performance measures

It is further observed that as the arrival rate parameter (λ_1) varies from 6 to 9, the probability of emptiness of the queuing system decreases from 0.57143 to 0.44168 .Similarly, the probability of emptiness of the first and second queues decrease from 0.66310 to 0.55212 and 0.86176 to 0.79997 respectively, the mean number of customers in the first and second queues increase from 0.41083 to 0.59399 and 0.14878 to 0.22318 respectively, and the mean number of customers in the system increases from 0.55961 to 0.81717, when all other parameters are fixed.

It is also observed that as the arrival rate parameter (λ_2) varies from 5.5 to 6.8, the probability of emptiness of the queuing system decreases from 0.42124 to 0.41640. Similarly, the probability of emptiness of the first queue decreases from 0.53314 to 0.52701, and in the second queue it remains constant, the mean number of customers in the first queue increases from 0.62897 to 0.64054, and in the second queue it remains constant, the mean number of customers in the system increases from 0.86454 to 0.87611, when all other parameters are fixed.

It is observed that as the service rate parameter (α_1) varies from 10 to 10.6, the probability of emptiness of the queuing system decreases from 0.21728 to 0.03306. Similarly, the probability of emptiness of the first queue increases from 0.52671 to 0.52892, the probability of emptiness of the second queue decreases from 0.41252 to 0.06250 and the mean number of customers in the first queue decreases from 0.64111 to 0.63692, the mean number of customers in the second queue increases from 0.88547 to

2.77264 and the mean number of customers in the system increases from 1.52658 to 3.40956, when all other parameters are fixed.

It is also observed that as the service rate parameter (β_1) varies from 8 to 16, the probability of emptiness of the queuing system increases from 0.03333 to 0.03442. Similarly, the probability of emptiness of the first queue increases from 0.53272 to 0.54762, the probability of emptiness of the second queue increases from 0.06257 to 0.06285 and the mean number of customers in the first queue decreases from 0.62977 to 0.60218, the mean number of customers in the second queue decreases from 2.77139 to 2.76701 and the mean number of customers in the system decreases from 3.40116 to 3.36919, when all other parameters are fixed.

It is observed that as the service rate parameter (α_2) varies from 11.4 to 12.6, the probability of emptiness of the queuing system increases from 0.17831 to 0.45405. Similarly, the probability of emptiness of the second queue increases from 0.32561 to 0.82914, and in the first queue it remains constant, the mean number of customers in the second queue decreases from 1.12206 to 0.18736, and in the first queue it remains constant, the mean number of customers in the system decreases from 1.72424 to 0.78954, when all other parameters are fixed.

It is also observed that as the service rate parameter (β_2) varies from 15 to 30, the probability of emptiness of the queuing system increases from 0.46089 to 0.48364. Similarly, the probability of emptiness of the second queue increases from 0.84163 to 0.88317, and in the first queue remains constant, the mean number of customers in the second queue decreases from 0.17241 to 0.12423, and in the first queue remains constant, the mean number of customers in the system decreases from 0.77459 to 0.72641, when all other parameters are fixed.

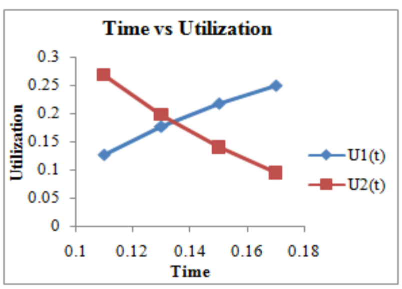
From Table 5, as time (t) varies from 0.11 to 0.17, the utilization of the first service station, the throughput of the first service station, and the average waiting time of a customer in the first queue increase from 0.12732 to 0.25078, 1.22987 to 2.51280 and 0.11073 to 0.11490 respectively. But in the second queue they decrease from 0.26835 to 0.09443, 3.27659 to 1.21530 and 0.09536 to 0.0.08162 respectively. When all other parameters are fixed. It is observed that the utilization of the service stations, throughput of the service stations and the waiting time of customers in each queue are highly sensitive with respect to time.

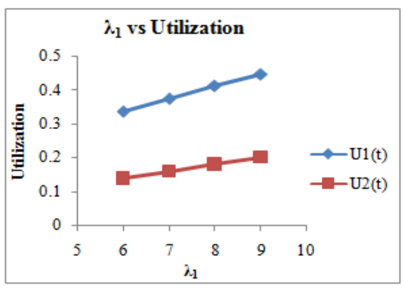
Table 5

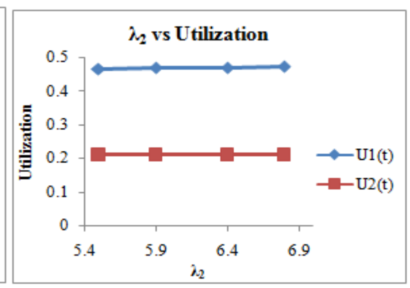
Values of $U_1(t)$, $U_2(t)$, $Thp_1(t)$, $Thp_2(t)$, $W_1(t)$, $W_2(t)$ for different values of parameters

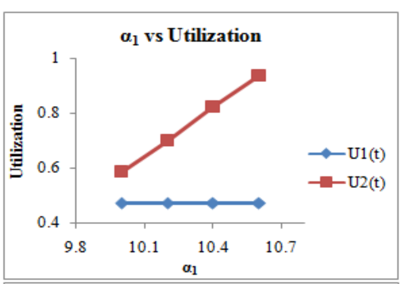
T	λ_1	λ_2	α_1	β_1	α_1	β_2	$U_1(t)$	$U_2(t)$	$Thp_1(t)$	$Thp_2(t)$	$W_1(t)$	$W_2(t)$
0.11	4	5	9	6	11	11	0.12732	0.26835	1.22987	3.27659	0.11073	0.09536
0.13	4	5	9	6	11	11	0.17778	0.19699	1.73866	2.44854	0.11258	0.08960
0.15	4	5	9	6	11	11	0.21817	0.13937	2.15987	1.76301	0.11395	0.08513
0.17	4	5	9	6	11	11	0.25078	0.09443	2.51280	1.21530	0.11490	0.08162
0.17	**6**	5	9	6	11	11	0.33690	0.13824	3.37574	1.77921	0.12170	0.08362
0.17	**7**	5	9	6	11	11	0.37617	0.15935	3.76927	2.05085	0.12519	0.08464
0.17	**8**	5	9	6	11	11	0.41312	0.17994	4.13949	2.31584	0.12875	0.08566
0.17	**9**	5	9	6	11	11	0.44788	0.20003	4.48778	2.57434	0.13236	0.08669

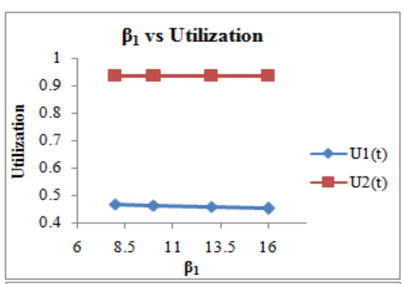
0.17	9.5	**5.5**	9	6	11	11	0.46686	0.20988	4.67794	2.70120	0.13445	0.08721
0.17	9.5	**5.9**	9	6	11	11	0.46875	0.20988	4.69692	2.70120	0.13467	0.08721
0.17	9.5	**6.4**	9	6	11	11	0.47111	0.20988	4.72056	2.70120	0.13494	0.08721
0.17	9.5	**6.8**	9	6	11	11	0.47299	0.20988	4.73939	2.70120	0.13515	0.08721
0.17	9.5	6.8	**10**	6	11	11	0.47329	0.58748	5.21569	7.56085	0.12292	0.11711
0.17	9.5	6.8	**10.2**	6	11	11	0.47272	0.69953	5.30391	9.00291	0.12067	0.13356
0.17	9.5	6.8	**10.4**	6	11	11	0.47198	0.82225	5.38999	10.58234	0.11848	0.16323
0.17	9.5	6.8	**10.6**	6	11	11	0.47108	0.93750	5.47398	12.06567	0.11635	0.22980
0.17	9.5	6.8	10.6	**8**	11	11	0.46728	0.93743	5.58871	12.06466	0.11269	0.22971
0.17	9.5	6.8	10.6	**10**	11	11	0.46351	0.93735	5.70122	12.06371	0.10922	0.22963
0.17	9.5	6.8	10.6	**13**	11	11	0.45791	0.93725	5.86588	12.06236	0.10439	0.22952
0.17	9.5	6.8	10.6	**16**	11	11	0.45238	0.93715	6.02573	12.06112	0.09993	0.22942
0.17	9.5	6.8	10.6	16	**11.4**	11	0.45238	0.67439	6.02573	8.94917	0.09993	0.12538
0.17	9.5	6.8	10.6	16	**11.8**	11	0.45238	0.44529	6.02573	6.08714	0.09993	0.09681
0.17	9.5	6.8	10.6	16	**12.2**	11	0.45238	0.28352	6.02573	3.98907	0.09993	0.08358
0.17	9.5	6.8	10.6	16	**12.6**	11	0.45238	0.17086	6.02573	2.47232	0.09993	0.07578
0.17	9.5	6.8	10.6	16	12.6	**15**	0.45238	0.15837	6.02573	2.39931	0.09993	0.07186
0.17	9.5	6.8	10.6	16	12.6	**20**	0.45238	0.14362	6.02573	2.29784	0.09993	0.06747
0.17	9.5	6.8	10.6	16	12.6	**25**	0.45238	0.12978	6.02573	2.18678	0.09993	0.06357
0.17	9.5	6.8	10.6	16	12.6	**30**	0.45238	0.11683	6.02573	2.06783	0.09993	0.06008

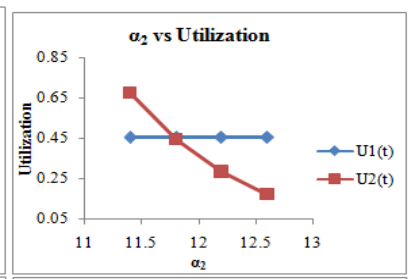


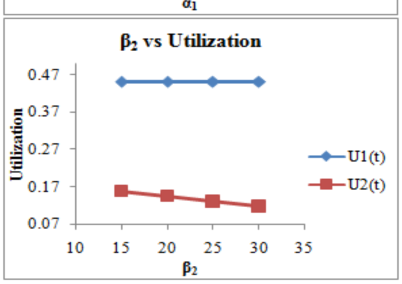


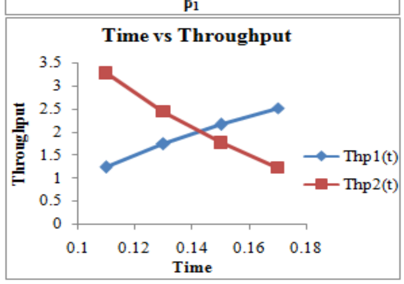


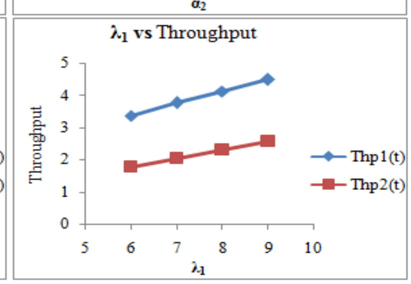












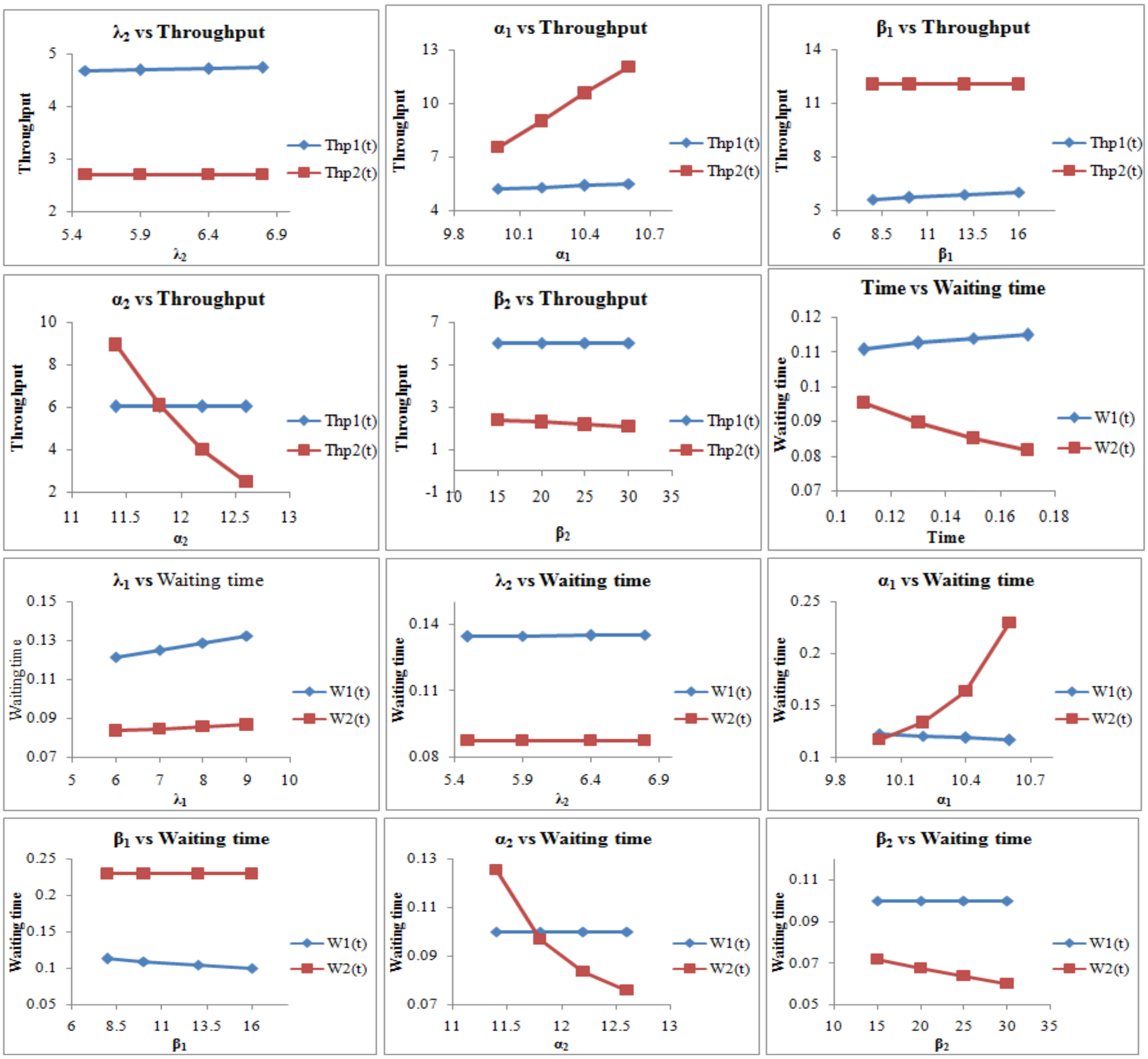


Figure 4b: The relationship between the parameters and performance measure

It is further observed that as the arrival rate parameter (λ_1) varies from 6 to 9, the utilization of the service station, the throughput of service station, and the average waiting time of a customers in the first and second queue increase from 0.33690 to 0.44788, 0.13824 to 0.20003, 3.37574 to 4.48778, 1.77921 to 2.57434, 0.12170 to 0.13236 and 0.08362 to 0.08669 respectively, when all other parameters are fixed.

It is also observed that as the arrival rate parameter (λ_2) varies from 5.5 to 6.8 , the utilization of the first service station, the throughput of the first service station, and the average waiting time of a customers in the first queue increase from 0.46686 to 0.47299, 4.67794 to 4.73939 and 0.13445 to 0.13515 respectively.

It is observed that as the service rate parameter (α_1) varies from 10 to 10.6, the utilization of the first service station decreases from 0.47329 to 0.47108, the utilization of the second service station increases from 0.58748 to 0.93750, the throughput of the first and second service stations increase from 5.21569 to 5.47398, 7.56085 to 12.06567 respectively, and the average waiting time of a customers in the first queue decreases from 0.12292 to 0.11635, the average waiting time of a customers in the second queue increases from 0.13356 to 0.22980, when all other parameters are fixed.

It is also observed that as the service rate parameter (β_1) varies from 8 to 16, the utilization of the first and second service stations decrease from 0.46728 to 0.45238 and 0.93743 to 0.93715 respectively, the throughput of the first service station increases from 5.58871 to 6.02573 and the second service station decreases from 12.06466 to 12.06112, and the average waiting time of a customers in the first and second queue decrease from 0.11269 to 0.09993 and 0.22971 to 0.22942 respectively, when all other parameters are fixed.

It is observed that as the service rate parameter (α_2) varies from 11.4 to 12.6, the utilization of the second service station, the throughput of the second service station, and the average waiting time of a customer in the second queue decrease from 0.67439 to 0.17086, 8.94917 to 2.47232 and 0.12538 to 0.07578 respectively.

It is also observed that as the service rate parameter (β_2) varies from 15 to 30, the utilization of the second service station, the throughput of the second service station, and the average waiting time of a customer in the second queue decrease from 0.15837 to 0.11683, 2.39931 to 2.06783 and 0.07186 to 0.06008 respectively. And the performance measures remain constant in first queue, when all other parameters are fixed.

From Table 6, as time (t) varies from 0.11 to 0.17, the variance of the number of customers in first queue increases from 0.13618 to 0.28872, in second queue it decreases from 0.31246 to 0.09919 and in the system it decreases from 0.44864 to 0.38791, the coefficient of variation of number of customers in first queue decreases from 2.70983 to 1.86106 and in second queue it increases from 1.49297 to 1.60559, when all other parameters are fixed. It is observed that the variance and coefficient of variation of number of customers in each queue are highly sensitive with respect to time.

It is further observed that as the arrival rate parameter (λ_1) varies from 6 to 9, the variance of the number of customers in each queue and in the system increase from 0.41083 to 0.59399, 0.14878 to 0.22318 and 0.55961 to 0.81717 respectively the coefficient of variation of number of customers in each queue decrease from 1.56016 to 1.29750 and 1.33677 to 1.10623 respectively for first and second queue, when all other parameters are fixed.

It is also observed that as the arrival rate parameter (λ_2) varies from 5.5 to 6.8, the variance of the number of customers in first queue increases from 0.62897 to 0.64054, in second queue remains be constant and in the system it increases from 0.86454 to 0.87611, the coefficient of variation of number of customers in each queue decreases from 1.26091 to 1.24947 and 1.07549 to 1.06836 for first and second queue respectively, when all other parameters are fixed.

It is observed that as the service rate parameter (α_1) varies from 10 to 10.6, the variance of the number of customers in first queue decreases from 0.64111 to 0.63692, in second queue it increases from 0.88547 to 2.77264 and in the system it increases from 1.52658 to 3.40956, the coefficient of variation of number of customers in first queue increases from 1.24892 to 1.25302 and in second queue it decreases from 0.80936 to 0.54156, when all other parameters are fixed.

It is also observed that as the service rate parameter (β_1) varies from 8 to 16, the variance of the number of customers in the first, second queues and in the system decrease from 0.62977 to 0.60218, 2.77139 to 2.76701 and 3.40116 to 3.36919 respectively, the coefficient of variation of number of customers in first and second queue increase from 1.26012 to 1.28866 and 0.54223 to 0.54480 respectively, when all other parameters are fixed.

It is observed that as the service rate parameter (α_2) varies from 11.4 to 12.6, the variance of the number of customers in the second queue and in the entire queue decrease from 1.12206 to 0.18736 and 1.72424 to 0.78954 respectively and in the first queue they remain constant,

Table.6

Values of $V_1(t)$, $V_2(t)$,$V(t)$, $CV_1(t)$, $CV_2(t)$ for different values of parameters

T	λ_1	λ_2	α_1	β_1	α_2	β_2	$V_1(t)$	$V_2(t)$	$V(t)$	$CV_1(t)$	$CV_2(t)$
0.11	4	5	9	6	11	11	0.13618	0.31246	0.44864	2.70983	1.49297
0.13	4	5	9	6	11	11	0.19574	0.21938	0.41512	2.26024	1.55206
0.15	4	5	9	6	11	11	0.24612	0.15009	0.39621	2.01572	1.58869
0.17	4	5	9	6	11	11	0.28872	0.09919	0.38791	1.86106	1.60559
0.17	**6**	5	9	6	11	11	0.41083	0.14878	0.55961	1.56016	1.33677
0.17	**7**	5	9	6	11	11	0.47188	0.17358	0.64546	1.45573	1.24470
0.17	**8**	5	9	6	11	11	0.53294	0.19838	0.73132	1.36981	1.16936
0.17	**9**	5	9	6	11	11	0.59399	0.22318	0.81717	1.29750	1.10623
0.17	9.5	**5.5**	9	6	11	11	0.62897	0.23557	0.86454	1.26091	1.07549
0.17	9.5	**5.9**	9	6	11	11	0.63253	0.23557	0.8681	1.25736	1.07328
0.17	9.5	**6.4**	9	6	11	11	0.63698	0.23557	0.87255	1.25296	1.07054
0.17	9.5	**6.8**	9	6	11	11	0.64054	0.23557	0.87611	1.24947	1.06836
0.17	9.5	6.8	**10**	6	11	11	0.64111	0.88547	1.52658	1.24892	0.80936
0.17	9.5	6.8	**10.2**	6	11	11	0.64002	1.20240	1.84242	1.24998	0.73673
0.17	9.5	6.8	**10.4**	6	11	11	0.63862	1.72737	2.36599	1.25135	0.65012
0.17	9.5	6.8	**10.6**	6	11	11	0.63692	2.77264	3.40956	1.25302	0.54156
0.17	9.5	6.8	10.6	**8**	11	11	0.62977	2.77139	3.40116	1.26012	0.54223
0.17	9.5	6.8	10.6	**10**	11	11	0.62271	2.77020	3.39291	1.26723	0.54289
0.17	9.5	6.8	10.6	**13**	11	11	0.61233	2.76854	3.38087	1.27793	0.54386
0.17	9.5	6.8	10.6	**16**	11	11	0.60218	2.76701	3.36919	1.28866	0.54480
0.17	9.5	6.8	10.6	16	**11.4**	11	0.60218	1.12206	1.72424	1.28866	0.76156
0.17	9.5	6.8	10.6	16	**11.8**	11	0.60218	0.58931	1.19149	1.28866	0.91612
0.17	9.5	6.8	10.6	16	**12.2**	11	0.60218	0.33340	0.93558	1.28866	1.03386
0.17	9.5	6.8	10.6	16	**12.6**	11	0.60218	0.18736	0.78954	1.28866	1.12541
0.17	9.5	6.8	10.6	16	12.6	**15**	0.60218	0.17241	0.77459	1.28866	1.13622
0.17	9.5	6.8	10.6	16	12.6	**20**	0.60218	0.15504	0.75722	1.28866	1.14919
0.17	9.5	6.8	10.6	16	12.6	**25**	0.60218	0.13901	0.74119	1.28866	1.16155
0.17	9.5	6.8	10.6	16	12.6	**30**	0.60218	0.12423	0.72641	1.28866	1.17330

the coefficient of variation of number of customers in second queue increases from 0.76156 to 1.12541 and in the first queue they remain constant, when all other parameters are fixed.

It is also observed that as the service rate parameter (β_2) varies from 15 to 30, the variance of the number of customers in the second queue and entire queue decrease from 0.17241 to 0.12423 and 0.77459 to 0.72641 respectively and in the first queue they remain constant, the coefficient of variation of number of customers in second queue increases from 1.13622 to 1.17330 and in the first queue they remain constant, when all other parameters are fixed.

The sensitivity analysis of the model with respect to the mean number of customers, the utilization of service stations, the mean delay in transmission, and the throughput are computed and presented in Table 7 with variation of -10%, -5%, 0%, 5%, 10%, on the model parameter. The performance measure are highly sensitive with respect to time (t) as t increases, the average number of customers in the first queue increase along with the average delay, the utilization and the throughput of the first queue. Arrival rate parameter λ_1 increases, the average number of customers in first queue increase along with the average delay in transmission, the utilization of service stations and throughput of each queue.

Arrival rate parameter λ_2 increases, the average number of customers in the first queue increase along with the average delay, the utilization of first queue and the throughput of the first queue and there is no change with respect to the second queue.

Service rate parameter α_1 increases, the average number of customers in the first queue decreases and second queue increases along with the average delay, the utilization of first queue decreases and second queue increases, the throughput of the each queue is increases. Service rate parameter β_1 increases, the average number of customers in each queue is decreasing along with the average delay in each queue, the utilization of each queue decreases. The throughput of the first queue increases and in the second queue it decreases.

Table.7

The Values of $L_1(t)$, $L_2(t)$, $U_1(t)$, $U_2(t)$, $Thp_1(t)$, $Thp_2(t)$, $W_1(t)$, $W_2(t)$ for different values of t, λ_1, λ_2, α_1, β_1, α_2, β_2 parameters

Parameter	% change in parameters	Performance Measures							
		$L_1(t)$	$L_2(t)$	$U_1(t)$	$U_2(t)$	$W_1(t)$	$W_2(t)$	$Thp_1(t)$	$Thp_2(t)$
t=0.2	-10%	0.31916	0.17854	0.27324	0.16351	0.11799	0.08877	2.70506	2.01116
	-5%	0.33765	0.15060	0.28656	0.13980	0.11842	0.08687	2.85123	1.73357
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.37073	0.10555	0.30977	0.10017	0.11908	0.08363	3.11316	1.26220
	+10%	0.38552	0.08760	0.31990	0.08388	0.11932	0.08224	3.23102	1.06523
λ_1=4	-10%	0.32620	0.11378	0.27834	0.10754	0.11720	0.08464	2.78340	1.34427
	-5%	0.34050	0.12010	0.28859	0.11316	0.11799	0.08490	2.88587	1.41456
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.36910	0.13274	0.30865	0.12431	0.11959	0.08543	3.08647	1.55382
	+10%	0.38341	0.15170	0.31846	0.12982	0.12039	0.08622	3.18463	1.62279

λ_2=6	**-10%**	0.34792	0.12642	0.29385	0.11875	0.11840	0.08516	2.93848	1.48441
	-5%	0.35136	0.12642	0.29627	0.11875	0.11859	0.08516	2.96272	1.48441
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.35824	0.12642	0.30110	0.11875	0.11898	0.08516	3.01096	1.48441
	+10%	0.36168	0.12642	0.30350	0.11875	0.11917	0.08516	3.03496	1.48441
α_1=9	**-10%**	0.35927	0.01475	0.30182	0.01464	0.13081	0.08059	2.74653	0.18302
	-5%	0.35761	0.05922	0.30065	0.05750	0.12455	0.08239	2.87122	0.71880
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.35113	0.24612	0.29611	0.21817	0.11348	0.09025	3.09434	2.72711
	+10%	0.34680	0.53734	0.29305	0.41570	0.10857	0.10341	3.19429	5.19627
β_1=5	**-10%**	0.35619	0.12661	0.29966	0.11892	0.12007	0.08517	2.96665	1.48650
	-5%	0.35550	0.12651	0.29917	0.11884	0.11942	0.08517	2.97679	1.48545
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.35411	0.12632	0.29820	0.11867	0.11816	0.08516	2.99695	1.48338
	+10%	0.35342	0.12632	0.29772	0.11859	0.11753	0.08516	3.00697	1.48236
α_2=10.5	**-10%**	0.35480	0.93377	0.29869	0.60693	0.11879	0.13437	2.98689	6.94934
	-5%	0.35480	0.30435	0.29869	0.26239	0.11879	0.09686	2.98689	3.14217
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.35480	0.04758	0.29869	0.04647	0.11879	0.07862	2.98689	0.60527
	+10%	0.35480	0.00584	0.29869	0.00582	0.11879	0.07402	2.98689	0.07886
β_2=10	**-10%**	0.35480	0.12976	0.29869	0.12169	0.11879	0.08669	2.98689	1.49679
	-5%	0.35480	0.12808	0.29869	0.12021	0.11879	0.08592	2.98689	1.49066
	0%	0.35480	0.12642	0.29869	0.11875	0.11879	0.08516	2.98689	1.48441
	+5%	0.35480	0.12477	0.29869	0.11730	0.11879	0.08442	2.98689	1.47804
	+10%	0.35480	0.12315	0.29869	0.11587	0.11879	0.08369	2.98689	1.47154

($t = 0.2$, $\lambda_1 = 4$, $\lambda_2 = 6$, $\alpha_1 = 9$, $\beta_1 = 5$, $\alpha_2 = 10.5$, $\beta_2 = 10$)

Service rate parameters α_2 and β_2 increase, the average number of customers in the second queue decreases along with the average delay in the second queue, the utilization and throughput of the second service station decrease and there is no change with respect to the first queue.

9. COMPARATIVE STUDY OF THE TWO NODES TANDEM NON-STATIONARY QUEUING MODEL

The comparative study of the developed model with that of homogeneous poison arrival and service rates is performed by computing the performance measure. The results are presented in the Table 8 for different values of t = 0.18, 0.20, 0.22, 0.24.

From Table 8, it is observed that as the time increases the percentage variation of the performance measures between the two models is significant. The model with non-homogeneous Poisson arrival and service processes has higher performance than that of the model with homogeneous Poisson arrival and service process. It can also be observed that the assumption of non-homogeneous Poisson arrival and service processes has a significant influence on all the performance measures of the queuing model. This model includes the following queuing models as particular cases:

(i) M/M/1 Queuing with state dependent service model when $\lambda_2 = 0$, $\beta_1 = 0$, $\alpha_2 = 0$, $\beta_2 = 0$.

(ii) Two node tandem Poisson queuing with load dependent model when $\lambda_2 = 0$, $\beta_1 = 0$, and $\beta_2 = 0$.

(iii) Single server of queuing model with non-homogeneous Poisson arrivals and state dependent service. When, $\beta_1 = 0$, $\alpha_2 = 0$, $\beta_2 = 0$.

(iv) Two node tandem queuing model with non-homogeneous Poisson arrivals and state dependent service. When, $\beta_1 = 0$, $\beta_2 = 0$.

(v) Single node Poisson arrival and non-homogeneous Poisson service queuing model with load dependent when $\lambda_2 = 0$, $\alpha_2 = 0$, $\beta_2 = 0$.

(vi) Two node tandem queuing model with Poisson arrival and non-homogeneous Poisson service processes with load dependent when $\lambda_2 = 0$.

Table 8

Comparative study of models with Non-Homogeneous and Homogeneous Poisson arrival and service rates

T	Parameter Measure	Models with Non-Homogeneous arrival and service	Models with Homogeneous arrival and service	Difference	Pesrcentage of Variation
0.18	**$L_1(t)$**	0.62670	0.63027	0.00357	0.566424
	$L_2(t)$	0.13444	0.20059	0.06615	32.97772
	$U_1(t)$	0.46565	0.46755	0.0019	0.406374
	$U_2(t)$	0.12579	0.18175	0.05596	30.78955
	$W_1(t)$	0.09984	0.12717	0.02733	21.49092
	$W_2(t)$	0.06985	0.08759	0.01774	20.25345
0.20	**$L_1(t)$**	0.66593	0.68108	0.01515	2.224408
	$L_2(t)$	0.07777	0.13355	0.05578	41.76713
	$U_1(t)$	0.48620	0.49393	0.00773	1.564999
	$U_2(t)$	0.07482	0.12502	0.0502	40.15358
	$W_1(t)$	0.09925	0.13008	0.03083	23.70080
	$W_2(t)$	0.06663	0.08478	0.01815	21.40835
0.22	**$L_1(t)$**	0.69436	0.72218	0.02782	3.852225
	$L_2(t)$	0.04055	0.08572	0.04517	52.69482
	$U_1(t)$	0.50060	0.51431	0.01371	2.665707
	$U_2(t)$	0.03974	0.08215	0.04241	51.62508
	$W_1(t)$	0.09823	0.13247	0.03424	25.84736
	$W_2(t)$	0.06418	0.08282	0.01864	22.50664
0.24	**$L_1(t)$**	0.71429	0.75543	0.04114	5.445905
	$L_2(t)$	0.01695	0.05199	0.03504	67.39758
	$U_1(t)$	0.51046	0.53019	0.01973	3.721307

	U_2(t)	0.01681	0.05066	0.03385	66.81800
	W_1(t)	0.09690	0.13442	0.03752	27.91251
	W_2(t)	0.06225	0.08145	0.0192	23.57274

7. Conclusion

This paper deals with design and development of a new novel queuing models with time dependent arrival and service processes and state dependent service which is very useful for analyzing the communication network such as LAN, WAN, MAN, data voice transformations during the peak time analysis. Here the arrival and service processes are characterized with non-homogeneous Poisson processes. The explicit expressions for the system performance measures such as the content of the buffers, the average waiting time of the customers in the queue and in the system, the throughput of the nodes and the variance are useful for predicting the system behavior under transient conditions. It is observed that the time dependent and state dependent nature of the arrival and service processes have significant effect on the system performance measures. There is a scope for further extension of this paper by developing and analyzing a two node tandem queueing model with bulk arrivals having time and state dependent arrival and service rates.

References:

[1] Choi, B.D.and Choi, D.I (1996),"Queuing system with length dependence service times and its applications to cell discarding scheme in ATM networks", IEEE proceedings communications, Vol.143, pp:5-11.

[2.] Erlang, A.K.(1909),"Probability and telephone calls", Nyt. Tidsskr Krarup Mat.Ser.B, Vol.20,33-39.

[3.] Kin K.Leung (2002), " Load-dependent service queues with application to congestion control in broadband networks", Performance Evaluation, Vol.50, No4,pp:27-40.

[4.] Nageswararao, K., Srinivasa Rao, K. an Srinivasarao, P. (2010), "A tandom communication net work with dynamic band width allocation and modified phase type transmission having bulk arrivals", International journal of Computer Sciences, Issues, Vol.7,No.2,pp:18-26.

[5.] Padmavathi, G., Srinivasa Rao, K and Reddy, K.V.V.S (2009), "Performance Evaluation of parallel and series communication network with dynamic band width allocation CIIT" International journal of Networking and Communication, Vol.1, No.7,pp:433-454.

[6.] Parthasarathy, P.R and Selvaraju, N. (2001)," Transient analysis of queue where potential customers are discouraged by queue length", Mathematical problem in Engineering, Vol. 7,pp: 433-454.

[7.] M.V. Ramasundari, K. Srinivasa Rao, P. Srinivasa Rao and P. Suresh Varma (2011) ," Three node communication network model with dynamic bandwidth allocation and Non-homogeneous Poisson arrivals", International journal of Computer Applictions, Vol. 7, No.1,pp:19-27.

[8.] Srinivasa Rao, K. Shoba,T. Srinivasa Rao, P.(2000), "The M/M/1 interdependent queuing model with controllable arrival rates", Opsearch Vol 37(1) pp:14-24.

[9.] Srinivasa Rao, K. Srinivasa Rao, P. Lakshminarayana , j. " M/M/1 interdependent queueing model with servers vacaion" Proc.of AP Akademi of Sciences Vol7, No.3(2003),pp:191-196.

[10.] Srinivasa Rao, K., Suresh varma, P. and Srinivas, Y.(2008), "Interdependent Queuing model with startup delay". Statistical theory and App.7(2), 219-228.

[11.] Srinivasa Rao, K. and Vijay kumar CVRS (2003), " interdependent tandem queuing model". International journal of Management and Systems Vol 16(2) pp:157-168.

[12.] Suresh Varma, P. Srinivasa Rao, K.(2007), " A communication Network with load dependent transmission", International Journal of Mathematical Sciences, Issues, Vol.7, No.2,pp:199-210.

[13.] A.V.S Suhasini, K. Srinivasa Rao and P.R.S Reddy.(2013), "On parallel and series Non-homogeneous bulk arrival queueing model". OPSEARCH 50(4),521-547.

[14.] Suhasini, A.V.S, K. Srinivasa Rao, P.R.S. Reddy.(2014),"Queuing model with Non-homogeneous bulk arrivals having state dependent service rates" int. J. Operation research Vol:21(1),84-99.

[15.] Suhasini, A.V.S, K. Srinivasa Rao, P.R.S. Reddy.(2013)," Transient analysis of parallel and series queuing model with Non-homogeneous Poisson binomial bulk arrival and load dependent service rates" Neural , parallel and scientific computations, Vol:21(2),235-262.

[16.] Suhasini, A.V.S, K. Srinivasa Rao, P.R.S. Reddy.(2012)," Transient analysis of tandem queuing model with Non-homogeneous Poisson bulk arrivals having state dependent service rates", Int.J.Adv.Comp.math.Sci.Vol:3(3),272-289.

[17.] P. Trinath Rao, K. Srinivasa Rao and K.V.V.S Reddy(2012), "Performance of Non-homogeneous communication with Poisson arrivals and dynamic bandwidth allocation" accepted in International Journal of Systems, control and communication,Vol.4(3),164-182.

[18.] J.Duraga Aparajitha, G.V.S. Rajkumar(2015), "Single Server Queuing model with time and state dependent service rate", Journal of the Indian Society for Probability and Statistics Vol.15,67-78.

[19.] Davis, J.L., Massey W.A., and Whitty, W., (1995) "Sensitivity to the service- time distribution in the non- stationary Erlang loss model‟. Management Science. Vol.41, No 6, PP 1107-1116.

[20.] Duffield, N.G., Massey, W.A., and Whitt,W., (2001) " A nonstationary offered load model for packet networks‟. Telecommunication systems. Vol.13, Issue 3/4, PP 271- 296.

[21.] Durga aparajitha J.,Rajkumak G.V.S, (2014) "single server queueing model with time and state dependent service rate". Journal of the Indian society for probability and statistics. Vol.15, PP 67-77.

[22.] Mandelbaum, A. and Massey, W.A., (1995) ‟ Strong approximations for time dependent queues". MOR. Vol.20, No1, PP 33-64.

[23.] Massey, W.A. and Whitt, W. (1993), "Networks of infinite-server queues with nonstationary Poisson input queueing systems". Queueing systems and their applications. Vol.13, No1, PP 183-250.

[24.] Massey, W.A. and Whitt, W. (1994), "An analysis of the modified load approximation for the nonstationary Erlang loss model". Annals of applied probability Vol.4, No 4, PP 1145-1160.

[25.] Newell, G.F. (1968), "Queues with time-dependent arrival rates (parts I-III)". Journal of Applied probability. Vol.5, PP 436-451(I), 579-590 (II), 591- 606 II).

[26.] Rothkopf, M.H. and Oren, S.S, (1979) "A closure approximation for the nonstationary M/M/s queue". Mangement Science Vol.25, PP 522:534.

[27.] Sadu, A. R., Srinivasa rao, K., Nirupama devi, K, "Forked queueing model with load dependent service rate bulk arrivals". International journal of operation research, Vol.30, No1, PP 1-32.

[28.] Srinivasa Rao, K., M.Govinda Rao and K.Naveen Kuamr (2011) - Transient analysis of an interdependent forked Tandem queueing model with load dependent service rate, International Journal of Computer Applications (IJCA), Volume 34, No. 3, pp: 33 – 40.

[29.] Srinivasa Rao,K., VAsanta, M.R., and Vijaya Kumar, C. V. R.S., (2000), on an interdependent communication Network, opsearch.37(2):134-143.

[30.] K.Srinivasa Rao, J. Durga aparajitha (2017) " Two node tandem queuing model with phase type state and time dependent service rates" International Journal of Computer Applications (0975 – 8887) Volume 177 – No.3, November 2017

[31.] Ward Whitt (2016), "Recent papers on the time-varying single-server queue".//http.pdfs.semanticsscholar.org//

[32.] William A. Massey (1996), "Stability for queues with time varying rates". Stochastic Networks of the series lecture notes in statistics 117: 95-107.

[33.] William A. Massey (2002), "The analysis of queues with time varying rates for telecommunication models". Telecommunication system 21:2-4,173-204.

Ch.Sreelatha, Department of Statistics Andhra University, Visakhapatnam, India
Email: srilathastats@gmail.com

K. Srinivasa Rao, Department of Statistics Andhra University, Visakhapatnam, India

B.Muniswamy, Department of Statistics Andhra University, Visakhapatnam, India