Journal of Mathematical Control Science and Applications
Vol. 5 No. 1 (January-June, 2019)

Novel Delay-dependent Absolute Stability Criteria for Neutral Lurie Control System With Time-delays

Jianxing Liu, Junkang Tian*, Xiangjun Xie

Abstract: The problem of delay-dependent absolute stability for neutral Lurie control system with delays is investigated. An improved linear matrix inequality-based delay-dependent absolute stability test is introduced to ensure a large upper bound for time-delay. A new class of Lyapunov-Krasovskii functionals combined with the descriptor model transformation and the decomposition technique of coefficient matrix is constructed to derive some novel delay-dependent absolute stability criteria. Finally, a numerical example is given to demonstrate the derived condition is less conservative than those given in the literature.

Keywords: Lurie control system; absolute stability; delay-dependent; linear matrix inequality

1 INTRODUCTION

Lurie control system with time-delay is an important nonlinear control system. The problem of the absolute stability of Lurie control system has been widely studied for several decades [1]-[7]. Since time-delays are frequently encountered in such systems and are often a source of instability, a considerable number of studies have also been done on the stability of Lurie control systems [8]-[19]. [4,7] studied the stability of this kind of systems and derived some stability criteria, but the existing criteria are all delay-independent which do not include the information on delay. Generally, abandonment of information on the delay causes conservativeness of the criteria especially when the time-delay is comparatively small. Recently, by employing the approach of linear matrix inequality (LMI), many novel conditions for delay-dependent absolute stability of Lurie control system are obtained in[16-18]. The advantage of this method is that it uses free weighting matrices to express those relation ships and using the decomposition technique of coefficient matrix, the delay-dependent absolutely stable condition for lurie control systems with multiple time-delays, and the improved results were given in the form of linear matrix inequality. In [19], some new less conservative delay-dependent absolute stability criteria for delay Lur'e control systems with multiple nonlinearities are derived that employ free weighting matrices to express the relation ships between the trems in the Leibniz-Newton formula. Furthermore, time-varying are not considered in [12-19]. Recently, some less conservativee absolute stability criteria for time-delay Lurie systems with sector-bounded nonlinearity is obtained in [24].

In this paper, the delay-dependent absolute stability for neutral Lurie control systems with timedelays is investigated. The object of the paper is to seek for and introduce an improved LMI test to ensure a large bound for the time-delay. Based on the descriptor model transformation and the decomposition technique of coefficient matrix, a new class of Lyapunov-Krasovskii functional is constructed to get improved delay-dependent absolute stability criteria for the considered system. Using these criteria, an upper bound of time-delay can be estimate such that the considered system is absolutely stable. The proposed results for the absolute stability analysis has two advantages. The first is to constructed a novel Lyapunov-Krasovskii functional in which free-weighting matrices are embedded to get less conservative absolute stability conditions, and the second is to combine the

* College of Science, Southwest Petroleum University, Nanchong, Sichuan, 637100, P.R.China. *Email: tianjunkang1980@163.com*

proposed Lyapunov-Krasovskii functional with the descriptor model transformation and the decomposition technique of coefficient matrix. Finally, a numerical example is given to indicate significant improvements over the existing results.

2. PROBLEM FORMULATION AND PRELIMINARIES

Consider the Lurie control systems with time-varying delay.

$$\begin{cases} \dot{x}(t) - D\dot{x}(t-h) = Ax(t) + Bx(t-\tau(t)) + bf(\sigma(t)) \\ \sigma(t) = c^T x(t) \\ x(\theta) = \phi(\theta), \quad \theta \in [-\max(\bar{\tau}, h), 0] \end{cases} \tag{1}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_m(t))^T$ is the state vector, $A, B, D \in \mathscr{R}^{n\times n}$, $b, c \in \mathscr{R}^n$, $\tau(t)$ is time-varying continuous function that satisfies $0 \le \tau(t) \le \bar{\tau}$, $0 \le \dot{\tau}(t) \le u$, in which $u, \bar{\tau}, h$ are constants. $\phi(\cdot)$ is a continuous vector valued initial function. The nonlinearity $f(\sigma)$ satisfies

$$f(\sigma) \in K[0, \infty] = \{f(\sigma) \mid f(0) = 0, 0 < f(\sigma) < \infty, \sigma \neq 0\} \tag{2}$$

Lemma 1 [20]: For any constant symmetric matrix Q_{11}, Q_{12}, $Q_{22} \in \mathscr{R}^{n\times n}$, $Q_{11} = Q_{11}^T > 0$, $Q_{22} = Q_{22}^T > 0$, $\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} > 0$, scalar $\bar{\tau} > 0$, and vector function $\dot{x}(t)$: $[-\bar{\tau}, 0] \to \mathscr{R}^n$, such that the integrations in the following are well defined, then

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \dot{x}(t) \end{bmatrix} dt \le -\begin{bmatrix} \int_{t-\bar{\tau}}^{t} x(s)ds \\ \int_{t-\bar{\tau}}^{t} \dot{x}(s)ds \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} \int_{t-\bar{\tau}}^{t} x(s)ds \\ \int_{t-\bar{\tau}}^{t} \dot{x}(s)ds \end{bmatrix} \tag{3}$$

Lemma 2 [21]: For given matrices A_{11}, A_{12}, A_{22} with appropriate dimensions, $\begin{bmatrix} A_{11} & A_{12} \\ A_{12}^T & A_{22} \end{bmatrix} < 0$, holds if and only if $A_{22} < 0$, $A_{11} - A_{12}A_{22}^{-1}A_{12}^T < 0$.

Lemma 3 [22]: For given matrices $Q = Q^T$, H, E and $R = R^T > 0$ of appropriate dimension, then

$$Q + HFE + E^T F^T H^T < 0$$

For all F satisfies $F^T F \le R$, if and only if there exist a positive number $\mathcal{E} > 0$, such that

$$Q + \mathcal{E}^{-1} HH^T + \mathcal{E}E^T RE < 0$$

Based on these lemmas, the following section will show our main results.

3. MAIN RESULTS

Rewrite system (1) in the following descriptor system:

$$\begin{cases} \dot{x}(t) = y(t) \\ y(t) = Ax(t) + Bx(t-\tau(t)) + bf(\sigma(t)) + Dy(t-h) \\ \sigma(t) = c^T x(t) \end{cases} \tag{4}$$

To derive discrete-delay-dependent stability conditions, which include the information of the time delay $\tau(t)$, one usually uses the fact

$$x(t-\tau(t)) = x(t) - \int_{t-\tau(t)}^{t} \dot{x}(s)ds = x(t) - \int_{t-\tau(t)}^{t} y(s)\,ds \tag{5}$$

to transform the original system to a system with distributed delays. In order to improve the bound of the discrete-delay $\tau(t)$, let us decompose the matrix B as $B = B_1 + B_2$, where B_1, B_2 are constant matrices. Then the original system (4) can be represented in the form of a descriptor system with discrete and distributed delays.

$$\begin{cases} \dot{x}(t) = y(t) \\ 0 = -y(t) + (A - B_1)x(t) + B_2 x(t-\tau(t)) - B_1 \int_{t-\tau(t)}^{t} y(s)\,ds + bf(\sigma(t)) + Dy(t-h) \\ \sigma(t) = c^T x(t) \end{cases} \tag{6}$$

For the absolute stability of system described by (1) and (2), we have the following result.

Theorem 1: Given a scalar $\bar{\tau} > 0$, the Lurie control system described by (1) and (2) is absolutely stable if there exist symmetric positive matrices P_1, S_{22}, Q_{11}, Q_{22}, R, M, N, Z, any scalar $\alpha > 0$, $\beta > 0$, and any matrices S_{12}, Q_{12}, $P_i (i = 2, 3, \cdots, 14)$ with appropriate dimensions, such that the following LMIs hold:

$$\begin{bmatrix} P_1 & S_{12} \\ * & S_{22} \end{bmatrix} \geq 0, \text{ with } P_1 > 0 \tag{7}$$

$$\begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \geq 0 \tag{8}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & \Sigma_{14} & \Sigma_{15} & \Sigma_{16} & \Sigma_{17} & uS_{12} & 0 \\ * & \Sigma_{22} & \Sigma_{23} & -P_5 + S_{12} & \Sigma_{25} & P_3^T b & P_3^T D - P_7 & 0 & 0 \\ * & * & \Sigma_{33} & B_2^T P_5 - P_{11} & \Sigma_{35} & \Sigma_{36} & \Sigma_{37} & 0 & 0 \\ * & * & * & -Q_{11} & \Sigma_{45} & P_5^T b & P_5^T D & 0 & uS_{22} \\ * & * & * & * & \Sigma_{55} & \Sigma_{56} & \Sigma_{57} & 0 & 0 \\ * & * & * & * & * & \Sigma_{66} & \Sigma_{67} & 0 & 0 \\ * & * & * & * & * & * & \Sigma_{77} & 0 & 0 \\ * & * & * & * & * & * & * & -uM & 0 \\ * & * & * & * & * & * & * & * & -uZ \end{bmatrix} < 0 \tag{9}$$

where

$$\begin{aligned} \Sigma_{11} &= P_2^T (A + B_1) + (A + B_1)^T P_2 + P_8 + P_8^T + \bar{\tau}^2 Q_{11} + R \\ \Sigma_{12} &= P_1^T - P_2^T + (A + B_1)^T P_3 + P_9 + \bar{\tau}^2 Q_{12} \\ \Sigma_{13} &= P_2^T B_2 - P_8^T + (A + B_1)^T P_4 + P_{10} \end{aligned}$$

$$\Sigma_{14} = (A + B1)TP5 + P11$$
$$\Sigma_{15} = -P_2^T B_1 - P_8^T + (A + B_1)^T P_6 + P_{12} + S_{12}$$
$$\Sigma_{16} = P_2^T B + P_{13} + (A + B_1)^T c + \alpha c$$
$$\Sigma_{17} = P_2^T D + (A + B_1)^T P_7 + P_{14}$$
$$\Sigma_{22} = -P_3^T - P_3 + \bar{\tau}_2 Q_{22} + N$$
$$\Sigma_{23} = P_3^T B_2 - P_9^T - P_4$$
$$\Sigma_{25} = -P_3^T B_1 - P_9^T - P_6$$
$$\Sigma_{33} = P_4^T B_2 + B_2^T P_4 - P_{10}^T - P_{10} - (1 - u)R + u(M + Z)$$
$$\Sigma_{35} = -P_4^T B_1 - P_{10}^T + B_2^T P_6 - P_{12}$$
$$\Sigma_{36} = P_4^T B - P_{13} + \beta B_2^T \mathrm{c}$$
$$\Sigma_{37} = P_4^T D + B_2^T P_7 - P_{14}$$
$$\Sigma_{45} = -P_5^T B_1 - P_{11}^T + S_{22} - Q_{12}$$
$$\Sigma_{55} = -P_6^T B_1 - B_1^T P_6 - P_{12}^T - P_{12} - Q_{22}$$
$$\Sigma_{56} = P_6^T B - P_{13} - \beta B_1^T c$$
$$\Sigma_{57} = P_6^T D - P_{14} - B_1^T P_7$$
$$\Sigma_{57} = P_2^T B_2 - P_8^T + (A + B_1)^T P_4 + P_{14}$$
$$\Sigma_{66} = 2\beta c^T B$$
$$\Sigma_{67} = b^T P_7 + c^T D$$
$$\Sigma_{77} = P_7^T D + D^T P_7 - N$$

Proof: Choose a new class of Lyapunov-Krasovskii functional candidate as follow:

$$V(t) = V_1(t) + V_2(t) + V_3(t) + V_4(t) + V_5(t)$$

where

$$V_1(t) = X^T(t)EPX(t) + \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)\, ds \end{bmatrix}$$

$$V_2(t) = \bar{\tau}\int_{-\bar{\tau}}^{0}\int_{t+\theta}^{t} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix} \begin{bmatrix} x(s) \\ y(s) \end{bmatrix} ds d\theta$$

$$V_3(t) = \int_{t-\tau(t)}^{t} x^T(s)Rx(s)ds$$

$$V_4(t) = 2\beta\int_0^{\sigma} f(\sigma)d\sigma$$

$$V_5(t) = \int_{t-h}^{\tau} y^T(s)Ny(s)ds$$

$$X(t) = \left[x^T(t)\ y^T(t)\ x^T(t-\tau(t))\ \left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T \left(\int_{t-\tau(t)}^{t} y(s)ds\right)^T f^T(\sigma(t))\ \ y^T(t-h)) \right]^T$$

$$E=\begin{bmatrix} I & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}^T \quad P=\begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 & 0 & P_7 \\ P_8 & P_9 & P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \end{bmatrix}$$

The time derivative of $V(t)$ along the trajectory of system (6) is given by:

$$\dot{V}(t) = \dot{V}_1(t) + \dot{V}_2(t) + \dot{V}_3(t) + \dot{V}_4(t) + \dot{V}_5(t)$$

since

$$\dot{V}_1(t) \;= 2X^T(t)EP\,\dot{X}(t) + 2\begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} y(t) \\ \int_{t-\tau(t)}^{t} y(s)ds + \dot{\tau}(t)x(t-\tau(t)) \end{bmatrix} \tag{10}$$

Employing the descriptor system (6) and using the $x(t) - x(t-\tau(t)) - \int_{t-\tau(t)}^{t} y(s)ds = 0$, allows to compute the first summing term in (10) explicitly as follows:

$$2X^T(t)EP\dot{X}(t) = 2X^T(t)P^T\begin{bmatrix} y(t) \\ 0 \\ 0 \end{bmatrix}$$

$$= 2X^T(t)P^T\begin{bmatrix} y(t) \\ -y(t)+(A+B_1)x(t)+B_2x(t-\tau(t))-B_1\int_{t-\tau(t)}^{t} y(s)ds+bf(\sigma(t))+Dy(t-h) \\ x(t)-x(t-\tau(t))-\int_{t-\tau(t)}^{t} y(s)ds \end{bmatrix}$$

$$= X^T(t)(\Phi_1^T + 1)X(t) \tag{11}$$

where

$$\Phi_1=\begin{bmatrix} P_1 & 0 & 0 & 0 & 0 & 0 & 0 \\ P_2 & P_3 & P_4 & P_5 & P_6 & 0 & P_7 \\ P_8 & P_9 & P_{10} & P_{11} & P_{12} & P_{13} & P_{14} \end{bmatrix}^T \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ A+B_1 & -I & B_2 & 0 & -B_1 & b & D \\ I & 0 & -I & 0 & -I & 0 & 0 \end{bmatrix}$$

$$2\begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t} x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 & S_{12} \\ * & S_{22} \end{bmatrix} \begin{bmatrix} y(t) \\ \int_{t-\tau(t)}^{t} y(s)ds + \dot{\tau}(t)x(t-\tau(t)) \end{bmatrix}$$

$$= 2\begin{bmatrix} x^T(t) & \left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T \end{bmatrix} \begin{bmatrix} S_{12}\int_{t-\tau(t)}^{t} y(s)ds + \dot{\tau}(t)S_{12}x(t-\tau(t)) \\ S_{12}^T y(t) + S_{22}\int_{t-\tau(t)}^{t} y(s)ds + \dot{\tau}(t)S_{22}x(t-\tau(t)) \end{bmatrix}$$

$$= x^T(t)\,S_{12}\int_{t-\tau(t)}^{t} y(s)ds + 2\dot{\tau}(t)x^T(t)S_{12}x(t-\tau(t)) + 2\left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T S_{12}^T y(t)$$

$$+2\left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T S_{22}\int_{t-\tau(t)}^{t} y(s)ds + 2\dot{\tau}(t)\left(\int_{t-\tau(t)}^{t} x(s)ds\right)^T S_{22}x(t-\tau(t)) \tag{12}$$

For some symmetric and positive definite matrices $M > 0$, $Z > 0$, the following inequalities hold:

$$2\dot{\tau}(t)x^T(t)S_{12}x(t-\tau(t)) \le ux^T(t)S_{12}M^{-1}S_{12}^Tx(t) + ux^T(t-\tau(t))Mx(t-\tau(t)) \tag{13}$$

$$2\dot{\tau}(t)\left(\int_{t-\tau(t)}^{t}x(s)ds\right)^T S_{22}x(t-\tau(t)) \le u\left(\int_{t-\tau(t)}^{t}x(s)ds\right)^T S_{22}\int_{t-\tau(t)}^{t}x(s)ds + ux^T(t-\tau(t))Zx(t-\tau(t)) \tag{14}$$

Then, we obtain:

$$2\begin{bmatrix} x(t) \\ \int_{t-\tau(t)}^{t}x(s)ds \end{bmatrix}^T \begin{bmatrix} 0 & S_{12} \\ * & S_{22} \end{bmatrix}\begin{bmatrix} y(t) \\ \int_{t-\tau(t)}^{t}y(s)ds + \dot{\tau}(t)x(t-\tau(t)) \end{bmatrix} \le X^T(t)\Phi_2X(t) \tag{15}$$

where

$$\Phi_2 = \begin{bmatrix} uS_{12}M^{-1}S_{12}^T & 0 & 0 & 0 & S_{12} & 0 & 0 \\ * & 0 & 0 & S_{12} & 0 & 0 & 0 \\ * & * & u(M+Z) & 0 & 0 & 0 & 0 \\ * & * & * & uS_{22}Z^{-1}S_{22} & S_{22} & 0 & 0 \\ * & * & * & * & 0 & 0 & 0 \\ * & * & * & * & * & 0 & 0 \\ * & * & * & * & * & * & 0 \end{bmatrix}$$

$$\dot{V}_2(t) = \bar{\tau}^2\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} - \bar{\tau}\int_{t-\bar{\tau}}^{t}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}ds$$

Using Lemma 1 to obtain

$$-\bar{\tau}\int_{t-\bar{\tau}}^{t}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}ds$$

$$\le -\tau(t)\int_{t-\tau(t)}^{t}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}ds$$

$$\le -\tau(t)\begin{bmatrix} \int_{t-\tau(t)}^{t}x(s)ds \\ \int_{t-\tau(t)}^{t}y(s)ds \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} \int_{t-\tau(t)}^{t}x(s)ds \\ \int_{t-\tau(t)}^{t}y(s)ds \end{bmatrix}$$

Hence, we obtain:

$$\dot{V}_2(t) \le \bar{\tau}^2\begin{bmatrix} x(t) \\ y(t) \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} x(t) \\ y(t) \end{bmatrix} - \begin{bmatrix} \int_{t-\tau(t)}^{t}x(s)ds \\ \int_{t-\tau(t)}^{t}y(s)ds \end{bmatrix}^T \begin{bmatrix} Q_{11} & Q_{12} \\ * & Q_{22} \end{bmatrix}\begin{bmatrix} \int_{t-\tau(t)}^{t}x(s)ds \\ \int_{t-\tau(t)}^{t}y(s)ds \end{bmatrix} \tag{16}$$

$\dot{V}_3(t)$ and $\dot{V}_4(t)$ are computed as follows:

$$\begin{aligned}\dot{V}_3(t) &= x^T(t)Rx(t) - (1 - \dot{\tau}(t))x^T(t - \tau(t))Rx(t - \tau(t)) \\ &\leq x^T(t)Rx(t) - (1 - u)x^T(t - \tau(t))Rx(t - \tau(t))\end{aligned} \tag{17}$$

$$\dot{V}_4(t) \leq 2\beta[(A + B_1)x(t) + B_2x(t - \tau(t)) - B_1\int_{t-\tau(t)}^{t} y(s)ds + bf(\sigma(t)) + Dy(t - h)]^T cf(\sigma(t)) + 2\alpha x^T(t)cf(\sigma(t)) \tag{18}$$

$$\dot{V}_5(t) = y^T(t)Ny(t) - yT(t - h)\,Ny(t - h) \tag{19}$$

Hence, according to (11)-(19), we obtain

$$\dot{V}(t) \leq X^T(t)\Phi X(t) \tag{20}$$

where

$$\Phi = \Phi_1^T + \Phi_1 + \Phi_2 + \Phi_3$$

$$\Phi_3 = \begin{bmatrix} R + \bar{\tau}^2 Q_{11} & \bar{\tau}Q_{12} & 0 & 0 & 0 & \beta(A + B_1)^T c + \alpha c & 0 \\ * & \bar{\tau}Q_{22} + N & 0 & 0 & 0 & 0 & 0 \\ * & * & -(1-u)R & 0 & 0 & \beta B_2^T c & 0 \\ * & * & * & -Q_{11} & -Q_{12} & 0 & 0 \\ * & * & * & * & -Q_{22} & -\beta B_1^T c & 0 \\ * & * & * & * & * & 2\beta b^T c & \beta c^T D \\ * & * & * & * & * & * & -N \end{bmatrix}$$

If $\Phi < 0$, there exist a scalar $\lambda > 0$, such that $\dot{V}(t) \leq -\lambda\|x(t)\|^2$, thus according to Ref.[23], system (1) is absolutely stable. By Lemma 2, $\Phi < 0$ is equivalent to $\Sigma < 0$, hence, this completes the proof.

Remark 1: If the time-delay is time invariant, that is $\tau(t) \equiv \tau > 0$, $\dot{\tau}(t) = 0$, According to the Proof of Theorem 1, we can obtain the following Corollary.

Corollary 1: Consider the system (1) with $D = 0$, $\tau(t) \equiv \tau > 0$, $\dot{\tau}(t) = 0$, then given a scalar $\bar{\tau} > 0$, this system is absolutely stable if there exist symmetric positive matrices P_1, S_{22}, Q_{11}, Q_{22}, R, any scalar $\alpha > 0$, $\beta > 0$, and any matrices S_{12}, Q_{12}, $P_i (i = 2, 3, \cdots, 13)$ with appropriate dimensions, such that (7), (8) and the following LMI holds:

$$\Omega = \begin{bmatrix} \Sigma_{11} & \Sigma_{12} & \Sigma_{13} & (A + B_1)^T P_5 + P_{11} & \Sigma_{15} & \Sigma_{16} \\ * & \Sigma_{22} & \Sigma_{23} & -P_5 + S_{12} & \Sigma_{25} & P_3^T b \\ * & * & \Omega_{33} & B_2^T P_5 - P_{11} & \Sigma_{35} & \Sigma_{36} \\ * & * & * & -Q_{11} & \Sigma_{45} & P_5^T b \\ * & * & * & * & \Sigma_{55} & \Sigma_{56} \\ * & * & * & * & * & \Sigma_{66} \end{bmatrix} \leq 0 \tag{21}$$

where

Σ_{ij}, $i, j = 1, \cdots, 6$, $i = j \neq 3$ are the same as defined in the Theorem 1.

Table 1

	[12]	[13]	[14]	[15]	[16]	[17]	[18]	This paper
τ_{max}	0.3053	0.3230	0.9278	2.055	3.7272	5.1587	7.4527	13.0730

and $\Omega_{33} = P_4^T B_2 + B_2^T P_4 - P_{10}^T - P_{10} - R$

Proof: Choose a new class of Lyapunov-Krasovskii functional candidate the same as the Theorem 1 with $Q = 0$. Setting $u = 0$ in $\Phi < 0$ which is defined following (20), hence we obtain the LMI (21) easily.

4. A NUMERICAL EXAMPLE

Consider the system (1) with

$$A = \begin{bmatrix} -2 & 0 \\ -1 & -2 \end{bmatrix}, \quad B = \begin{bmatrix} -0.2 & -0.5 \\ 0.5 & -0.2 \end{bmatrix}, \quad b = \begin{bmatrix} -0.2 \\ -0.3 \end{bmatrix}, \quad c = \begin{bmatrix} 0.6 \\ 0.8 \end{bmatrix}$$

No conclusion can be made using criteria in [19]. The existing best of the maximum allowable delay is $\tau_{max} = 7.4527$. Now we use the Corollary 1 in this paper to study the problem, let us decompose matrix B as $B = B_1 + B_2$, where

$$B_1 = \begin{bmatrix} 0.15 & -0.15 \\ -0.01 & -0.10 \end{bmatrix}, \quad B_2 = \begin{bmatrix} -0.35 & -0.35 \\ 0.51 & -0.10 \end{bmatrix}$$

the same as the Ref. [18]. Solving LMI (20), the maximum value of τ_{max} for absolute stability of system (1) is $\tau_{max} = 13.0730$. For a comparison with the results of other papers, we list Table 1. According to the Table 1, this example shows that the absolute stability criterion in this paper gives much less conservative results than those in Refs.[12-19].

5. CONCLUSIONS

The absolute stability of neutral Lurie control systems with time-delays is considered. A new class of Lyapunov-Krasovskii functionals combined with the descriptor model transformation and decomposition technique of coefficient matrix is constructed to derive some novel absolute stability criteria. A numerical example has shown significant improvement over the existing results.

REFERENCES

[1] Z. X. Gan, J. Q. Han, Lyapunov functional of general Lurie systems with nonlinearities, Applied Mathematics Letters , Vol. **16**, pp.119–126, 2003.

[2] K. S. Narendra, J. H. Taylor, Frequency domain criteria for absolute stability, Academic press, New York, 1973.

[3] L. T. Grujic, On absolute stability and Aizerman conjecture, Automatica, Vol. **17**, pp. 335–349, 1981.

[4] V. M. Popov, A. Halanay, About stability of nonlinear controlled systems with delay, Automation and Remote Control, Vol. **22**, pp. 849–851, 1962.

[5] S. X. Zhao, On absolute stability of control systems with several executive elements, Science, Sinica, Series A, Vol. **31**, pp. 395–405, 1988.

[6] P. Park, A revisited Popov criterion for nonlinear Lurie systems with sector restrictions, International Journal of Control, Vol. **68**, pp. 461–469, 1997.

[7] A. Somolines , Stability of Lurie type functional equations. *Journal of Differential Equations*, Vol. **26**, pp. 191–199, 1997.

[8] P. A. Bliman, Extension of Popv absolute stability criterion to non-autonomous systems with delays, *International Journal of Control*, Vol. **73**, pp. 1349–1361, 1997.

[9] P. A. Bliman, Absolute stability criteria with prescribed delay rate for finite-demensional and delay systems, *Automatica*, Vol. **38**, pp. 2015–2019, 2002.

[10] Z. X. Gan, W. G. Ge, Lyapunov function for multiple delay general Lurie control systems with multiple nonlinearities, *Journal of Mathematical Analysis and Applications*, Vol. **259**, pp. 596–608, 2001.

[11] Y. He, M. Mu, Absolute stability for multiple delay general Lurie control systems with multiple nonlinearities, *Journal of Computational and Applied Mathematics*, Vol. **159**, pp. 241–248, 2003.

[12] X. H. Nian, Delay dependent conditions for absolute stability of Lurie type control systems, *Acta Automatica Sinica*, Vol.**25**, pp. 564–556, 1999.

[13] B. J. Xu, X. X. Liao, Absolute stability criteria of delay-dependent for Lurie control systems, *Acta Automatica Sinica*, Vol.**28**, pp. 317–320, 2002.

[14] D. Y. Chen, W. H. Liu, Delay-dependent robust stability for Lurie control systems with multiple time delays, *Control Theory and Applications*, Vol. **22**, pp. 499–502, 2005.

[15] Y. Bin, M. Y. Chen, Delay-dependent criterion for absolute stability of Lurie type control systems with time delays, *Control Theory and Applications*, Vol. **18**, pp. 929–931, 2001.

[16] J. K. Tian, S. M. Zhong, L. L. Xiong, Delay-dependent absolute stability of Lurie control systems with multiple time delays, *Applied Mathematics and Computation*, Vol. **188**, pp. 379–384, 2007.

[17] J. W. Cao, S. M. Zhong, New delay-dependent condition for absolute stability of Lurie control systems with multiple time delays, *Applied Mathematics and Computation*, Article in press.

[18] J. W. Cao, S. M. Zhong, New delay-dependent condition for absolute stability of Lurie control systems with multiple time delays, *Journal of Computational and Applied Mathematics*, Accepted Manuscript.

[19] Y. He, M. Mu. J. H. Liu, G. P. Liu, Robust stability for delay Lurie control systems with multiple nonlinearities , *Journal of Computational and Applied Mathematics*, Vol. **176**, pp. 371–380, 2005.

[20] K. Gu, V.L. Kharitonov, J. Chen, Stability of time-delay systems, *Bikhause*r, Basel, 2003.

[21] S. Boyd, L. EL. Ghaoui, E. Feron, V. Balakrishnam, Linear Matrix Inequalities in Systems and Control, *SIMA*, Philadelphia, 1994.

[22] L. Xie, Output feedback H} control of systems with parameter uncertainty, *International Journal of Control*, Vol. **63**, pp. 741–750, 1996.

[23] J. K. Hale, *Theory of Functional Differential Equation*. Springer-Verlag, New York, 1977.

[24] Q. L. Han, Absolute stability of time-delay systems with sector-bounded nonlinearity, *Automatica*, Vol. **41**, pp. 2171– 2176, 2005.

[25] Tanima Choudhur, Status and Role of Women Workers in Informal Sectors, *International Journal of Rural Development and Management Studies*

[26] J Nirmala, WOMEN HEADED FAMILIES, *Indian Journal of Social Development*

[27] V. Lokesha, Shruti R and Sunilkumar M. Hosamani, OPERATIONS ON GEAR GRAPH WITHTOPOLOGICAL INDICES, *Indian Journal of Mathematics and Mathematical Sciences*

[28] Zhi-Hua Zhang, The Weighted Heron Mean in n Variables, *Journal of Analysis and Computation*

[29] Tae Chon Ahn, Intutionistic Fuzzy Lattices, *International Review of Fuzzy Mathematics*

[30] ABID KHAN, Xia-Mu NIU, A Hybrid Approach for Mobile Agent Security using Reversible Watermarking, *International Journal of Computer Sciences and Engineering Systems*

[31] LORELLE YUEN, PROSPECTS FOR ECONOMIC DEVELOPMENT IN BURMA USING THE NEO-CLASSICAL MODEL, *Journal of Business and Economics*

[32] Profulla C. Sarker, Impact of Physical Abuse on the Health of the Working Children in a City of Bangladesh, *International Journal of Cross-Cultural Studies*

[33] Leena N. Shenoy, B. Shanmukha, H. Mariswamy & R. Murali, Hamiltonicity in Distance Graphs of Paths, *International Journal of Combinatorial Graph Theory and Applications*

[34] Raj Pathania, Praveen Kaur, Participation of Tribal Adolescents in Various Activities, *Journal of Social Anthropology*

[35] AKM Shafiul Islam , Nature of Dependency of the Elderly in Bangladesh: A Study of Rajshahi City, *Journal of Social Anthropology*

[36] K. C. Tan, C. Y. Cheong and Y. Peng, A Genetic Approach for Real-Time Identification and Control of a Helicopter System, *International Journal of Computational Intelligence in Control*

[37] Tain-Sou Tsay, *On-Line Computing and Control for Decoupling Multivariable Processes with Gain and Phase Specifications, Journal of Mathematical Control Science and Applications*

[38] M. Osintsev and V. Sobolev, Global Invariant Manifolds in a Problem of Kalman-Bucy Filtering for Gyroscopic Systems, *Global and Stochastic Analysis*

[39] Jin Han Park, Generalized fuzzy semi-closed Sets and Generalizations of Fuzzy Continuous Functions, *International Journal of Fuzzy Systems and Rough Systems*

[40] Ana S.Q. Liberato and Kathleen Ratajczak , The Feminist Debate on Prostitution and Trafficking: Reflections for a Unified and Theory-Driven Approach, *Emerging Sociology Vol. 4 No. 1 (2017)*

[41] P. Natesan., The Risk Disgust of Banks in Budding Credit Markets: with Reference to India, *Journal of Asian Business Management,* Vol. 1, No. 1, pp. 85-91.

[42] Falendra K. Sudan & Jennifer McKay., Institutional Aspects of Groundwater Governance: Experiences From South Australia and Lessons for India, *International Journal of Environment and Development,* Vol. 3, No. 1, pp. 1-32.

[43] Kian-Ping Lim & M. Azali., Episodic Non-linear Behaviour of Bilateral Malaysian Ringgit-U. S. Dollar Spot Rate, *Journal of International Economic Review,* Vol. 1, No. 1, pp. 25-48.

[44] Anastasios V. Semos., Assessing Income Disparities Between Agricultural and Non-agricultural Households, *Journal of Agricultural and Food Economics,* Vol. 1, No. 2, pp. 155-161.

[45] M. Bakhshoodeh & A. Karami., Determinants of Poor Accessibility to Microcredits in Rural Iran, *International Review of Applied Economic Review,* Vol. 3, No. 1-2, pp. 1-9.

[46] Julia M. D'Antonio-Del Rio & Matthew R. Lee., Violent Women? Southern Culture and Gendered Patterns of Argument Homicide, *International Journal of Sociological Research,* Vol. 2, Nos. 1

[47] Masihullah Khan & Mohmmad Afsar Alam , A Study of Intra-district Development in Etah District, Uttar Pradesh, India, *International Review of Comparative Sociology*

[48] D.D. Pandey., Child Budgeting in India: Friedman's Analysis, *Indian Development Review*, Vol. 3, No. 2, pp. 259.

[49] George Pfeffer, *Sahlin's Tribesmen (1968) Reconsidered, Anthropological Insights*

[50] Jung Hur & Donghyun Park., A Note on the Infant Industry Argument, *International Economics and Finance Journal*, Vol. 1, No. 1, pp. 67-71.

[51] Abdus Samad., Comparative Analysis of Domestic and Foreign Bank Operations in Bangladesh, *The Global Journal of Finance and Economics*, Vol. 4, No. 1, 37-46.

[52] Kamal Sarkar, Application of Dimensional Analysis to Predict Poly Ethylene Oxide (PEO) Fiber Diameters from Electrospinning Process. *International Journal of Electrospun Nanofibers and Applications*

[53] Henry Aray., International Monopoly under Uncertainty, *International Journal of Economics*, Vol. 1, No. 2, pp. 45-53.

[54] Gan-Yun Huang & Shou-Wen Yu, The Surface Instability of A Graded Thin Film Induced By Interaction with A Rigid Contactor, *Journal of Mechanics and MEMS*

[55] Cheol Gyu Kim, Sewage Sludge Carbonization for Adsorbents Use, *Journal of Multifunctional Materials and Photoscience*

[56] Hala El-Ramly, Using Panel Data Models to Test for A Unit Root in Inflation Rates, *Global Review of Business and Economic Research*

[57] Sadananda Prusty, Household Saving Behaviour: Role of Financial Literacy and Saving Plans, *Journal of World Economic Review*

[58] Alan Whaley, William E. Gillis and Ermanno Affuso, A Panel Data Analysis of Efficiency and Productivity of the U.S. Hospital Care System, *Review of Applied Economics*

[59] Amira H.S. Halaby, Suggested Marketing Strategy Using Apriori and FP-Growth Algorithms in Retail Sales in Egypt , *International Journal of Data Modelling and Knowledge Management*