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Global Existence and Stability of Periodic Solution for Two Kinds of BAM Neural Networks with Diffusion

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Abstract: This paper deals with the problem of global existence and asymptotic stability of periodic solution for reaction-diffusion BAM neural networks with discrete time-varying delays or distributed delays. Based on the Lyapunov method and coincidence degree, some sufficient conditions for the neural networks with reaction-diffusion terms are derived. Since the activation functions do not need to satisfy the boundedness conditions, the criteria are less conservative than existing ones reported in the literature for delayed neural networks.

1. INTRODUCTION

A class of two-layer interassociative network called bidirectional associative memory (BAM) neural network first introduced by Kosto [1-2] is an important model with the ability of information memory and information association, which is crucial for application in pattern recognition, solving optimization problems and automatic control engineering. In such applications, the dynamical characteristics of networks play an important role. There have been many analytical results for BAM neural networks with and without axonal signal transmission delays [4-15].

Recently, periodic solution for BAM neural networks with delays has been studied, for example, see [3-8] and references therein. In [6-8], some sufficient conditions ensuring the existence, uniqueness and global exponential stability of periodic solution were given for BAM neural networks with constant delays. In [3-5], under the hypothesis for the boundedness or monotonicity on the activation functions, the authors gave several sufficient conditions ensuring the existence and global exponential stability of periodic solution for BAM neural networks with time-varying delays or distributed delays. However, in some applications, one requires to use unbounded activation functions [7]. For example, when neural networks are designed for solving optimization problems in the presence of constraints (linear, quadratic, or more general programming problems), unbounded activations modelled by diode-like exponential-type functions are needed to impose constraints satisfaction. The extension of the quoted results to the unbounded case is not straightforward.

Moreover, so we must consider that the activations vary in space as well as in time. Refs. [5,9-11] have considered the stability of neural networks with diffusion terms, which are expressed by partial differential equations. Recently there has been a somewhat a new category of BAM neural networks, which contains distributed delays. Interested readers may refer to [11-14]. Therefore, it's necessary to consider both diffusion effects and delay effect on the periodic solution and stability of neural networks. To the best of our knowledge, few authors have considered periodic oscillatory solutions for large-scale networks with delays and reaction-diffusion terms due to the difficulty of these complicated networks' analysis [5,15].

On the other hand, the existing literature on artificial neural networks is predominantly concerned with autonomous systems containing temporal uniform networks parameters and input stimuli. Motivated by the

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above discussions, we will make contributions on the issues of existence and global asymptotic stability of the periodic oscillatory solutions of reaction-diffusion BAM neural networks with time-varying delays or distributed delays and the hypothesis for boundedness and monotonicity on the activation functions and differentiability on time-varying delays are removed. Specifically, we consider a class of neural networks with reaction-diffusion terms of the form:

$$\begin{cases} \frac{\partial x_i(t,v)}{\partial t} = \sum_{k=1}^{l} \left(D_{ik} \frac{\partial x_i}{\partial v_k} \right) - a_i(t) x_i(t) \\ \qquad + \sum_{j=1}^{p} a_{ij}(t) f_j(t, y_j(t - \tau_{ij}(t))) + I_i(t), i = 1, 2, \cdots, n, \\ \frac{\partial y_j(t,v)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial v_k} \left(D^*_{jk} \frac{\partial y_j}{\partial v_k} \right) - b_j(t) y_j(t) \\ \qquad + \sum_{i=1}^{n} b_{ji}(t) g_i(t, x_i(t - \sigma_{ji}(t))) + J_j(t), j = 1, 2, \cdots, p \end{cases} \tag{1.1}$$

or

$$\begin{cases} \frac{\partial x_i(t,v)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial v_k} (D_{ik} \frac{\partial x_i}{\partial v_k}) - a_i(t) x_i(t) \\ \qquad + \sum_{j=1}^{p} a_{ij}(t) f_j \left(\int_0^{\infty} h_{ij}(s) y_j(t-s) ds \right) + I_i(t), i = 1, 2, \cdots, n, \\ \frac{\partial y_j(t,v)}{\partial t} = \sum_{k=1}^{l} \frac{\partial}{\partial v_k} \left(D^*_{jk} \frac{\partial y_j}{\partial v_k} \right) - b_j(t) y_j(t) \\ \qquad + \sum_{i=1}^{n} b_{ji}(t) g_i \left(\int_0^{\infty} k_{ji}(s) x_i(t-s) ds \right) + J_j(t), j = 1, 2, \cdots, p. \end{cases} \tag{1.2}$$

Every h_{ij} and $k_{ji} : [0, \infty) \to [0, \infty)$ are pieces of the continuous integral functions, and satisfy that

$$(\mathrm{H}_1)\ \int_0^{\infty} h_{ij}(s) ds = 1, \int_0^{\infty} k_{ji}(s) ds = 1.$$

The boundary conditions of the BAM (1.1) or (1.2) are

$$\begin{cases} \frac{\partial x_i}{\partial n} := \left(\frac{\partial x_i}{\partial v_1}, \frac{\partial x_i}{\partial v_2}, \cdots \frac{\partial x_i}{\partial v_l} \right)^T = 0, \\ \frac{\partial y_j}{\partial n} := \left(\frac{\partial y_j}{\partial v_1}, \frac{\partial y_j}{\partial v_2}, \cdots \frac{\partial y_j}{\partial v_l} \right)^T = 0, \end{cases}$$

where $v = (v_1, v_2, \ldots, v_l)^T \in \Omega \subset R^l$; Ω is a compact set with smooth boundary $\partial\Omega$ and mes $\Omega > 0$ in space R^l; $x = (x_1, x_2, \cdots, x_n)^T \in R^n$; $y = (y_1, y_2, \cdots, y_p)^T \in R^p$. And $y_j(t, v)$ are the state of the ith neurons and the jth neurons at time t and in space v, respectively; $I_i(t)$ and $J_j(t)$ denote the external inputs on the ith neurons and the j th neurons at time t, respectively; the time delays $\tau_{ij}(t)$ and $\sigma_{ji}(t)$ correspond to the finite speed of the axonal transmission of signal. $a_{ij}(t)$ and $b_{ji}(t)$ represent the strengths of synaptical connections.

Throughout this paper, we always assume that $a_i(t)$, $b_j(t)$, $a_{ij}(t)$, $b_{ji}(t)$, $\tau_{ij}(t)$, $\sigma_{ji}(t)$, $I_i(t)$ and $J_j(t)$ are continuously periodic functions defined on $t \in [0, \infty)$ with common period $\omega > 0$. Moreover, $a_i(t)$, $b_j(t)$, $\tau_{ij}(t)$, $\sigma_{ji}(t)$, are positive everywhere, $f_j(t, x)$ and $g_i(t, x)$ are continuous and ω–periodic with respect to t. Smooth function $D_{ik} = D_{ik}(t, x, v) \geq 0$ and $D^*_{jk} = D^*_{jk}(t, x, v) \geq 0$ correspond to the transmission diffusion operator along the ith neurons and the j th neurons, respectively.

Suppose

$$\tau = \max\{\tau_{ij}(t);\ 1 \leq i \leq n,\ 1 \leq j \leq p,\ t \in [0, \infty)\},$$
$$\sigma = \max\{\sigma_{ji}(t);\ 1 \leq i \leq n,\ 1 \leq j \leq p,\ t \in [0, \infty)\}.$$

Then we let $K = [-\sigma, 0] \times [-\tau, 0]$ and use $C(K) = \{\phi : K \to R^{n+p}$ is continuous$\}$ with the supernorm as the Banach space for system (1.1) and we will always tacitly use the identification

$$C(K) = C([-\sigma, 0]; R^n) \times C([-\tau, 0]; R^p).$$

For each given initial value $\phi = (\varphi, \psi)^T \in C(K)$ with $\varphi \in v([-\sigma, 0]; R^p)$ and $\psi \in C([-\tau, 0]; R^p)$, one can solve system (1.1) by method of steps to obtain a unique pair of continuous maps $x : [-\sigma,\infty) \to R^p$ and $y : [-\tau, \infty) \to R^n$ such that $(x, y)^T : (0, \infty) \to R^{n+p}$ is continuously differentiable and satisfies (1.1) for $t > 0$, $x|_{[-\sigma, 0]} = \varphi$ and $y|_{[-\tau, 0]} = \psi$.

In this paper, by means of some spectral theorems, Lyapunov functional, inequality analysis and a continuation theorem based on coincidence degree, we obtain some new sufficient conditions ensuring the existence, uniqueness of the periodic solution.

2. GLOBAL EXISTENCE OF PERIODIC SOLUTIONS

For convenience, we use the following notations:

$$\overline{f} = \frac{1}{\omega}\int_0^\omega f(t)dt,\ [f(t)]^+ = \max_{t\in[0,\omega]}\{|f(t)|\},$$

where f is a continuous periodic ω–periodic function. For matrix $A = (a_{ij})_{n\times n}$, let $\rho(A)$ denote the spectral radius of A. A matrix or a vector $A \geq 0$ means that all the entries of A are greater than or equal to zero, similarly define $A > 0$.

Theorem 1: Assume that $a_i(t) > 0$ and $b_j(t) > 0$ $i = 1, 2, \cdots, n$; $j = 1, 2, \cdots, p$ for $t \geq 0$. Moreover,

(A_1) there exist non-negative constants p_j , q_i, α_j and β_i such that

$$|f_j(t, u)| \leq p_j\, |u| + \alpha_j\ ,\ |g_i(t, u)| \leq q_i|u| + \beta_i \tag{2.1}$$

for any $t, u \in R$, $= 1, 2, \cdots, n$, $j = 1, 2, \cdots, p$;

(A_2) $\rho(M) < 1$, where $M = (m_{ij})_{(n+p)\times(n+p)}$ and

$$m_{ij} = \begin{cases} 0, & if\ 1 \leq i, j \leq n\ or\ n+1 \leq i, j \leq n+p. \\ \{|a_{i,j-n}(t)|p_{j-n}/a_i(t)\}^+, & if\ 1 \leq i \leq n\ and\ n+1 \leq j \leq n+p. \\ \{|b_{i-n,j}(t)|q_j/b_{i-n}(t)\}^+, & if\ n+1 \leq i \leq n+p\ and\ 1 \leq j \leq n. \end{cases}$$

Then (1.1) has at least one ω–periodic solution.

Proof: In order to use the continuation theorem for (1.1), we denote by Z (respectively, X) as the set of all continuously respectively, differentiable ω–periodic functions $u(t) = (x_1(t), \cdots, x_n(t), y_1(t), \cdots, y_p(t))^T$ defined on

R and denote $|u|_0 = \max_{1\le i\le n, 1\le j\le p} \{[x_i(t)^+, y_j(t)^+]\}$. Then X and Z are Banach spaces when they are endowed with the norms $|\cdot|_0$. For $u \in X$ and $z \in Z$, set

$$(Lu)(t) = \frac{\partial u(t,v)}{\partial t},\ Pu = \frac{1}{\omega}\int_0^\omega u(t)dt,\ Qz = \frac{1}{\omega}\int_0^\omega z(t)dt,$$

$$(Nu)_i(t) = \sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right) - a_i(t)x_i(t) + \sum_{j=1}^{p} a_{ij}(t) f_j(t, y_j(t-\tau_{ij}(t))) + I_i(t)$$

for $i = 1, 2, \cdots, n$, and

$$(Nu)_{n+j}(t) = \sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right) - b_j(t)y_j(t) + \sum_{i=1}^{n} b_{ji}(t) g_i(t, x_i(t-\sigma_{ji}(t))) + J_j(t),$$

for $j = 1, 2, \cdots, p$.

It is not difficult to show that $Ker\ L = R^{n+p}$, and that P,Q are continuous projectors such that $Im\ L = \{z \in Z : \int_0^\omega z(t)dt = 0\}$ is closed in Z and that $dim\ Ker\ L = n + p = co\ dim\ ImL$, and that P, Q are continuous projectors such that $Im\ P = Ker\ L$ and $Ker\ Q = Im\ L = Im(I - Q)$. It follows that L is Fredholm mapping of index zero. Furthermore generalized inverse (of L) $Kp : i = Im\ L \to Ker\ P \bigcap Dom\ L$ reads

$$(Kp\ u)_i(t) = \int_0^t u_i(s)ds - \frac{1}{\omega} \int_0^\omega \int_0^t u_i(s)\, dsdt$$

for $u = u(t) \in Z$. Thus, it is easy to see that QN and $Kp(I - Q)N$ are continuous An application of the Arzela-Ascoli theorem to $Kp(I - Q)N$ results in the fact that $Kp(I - Q)N(\bar{\Omega})$ is compact for any open bounded $\Omega \subset X$. Moreover, $QN(\bar{\Omega})$ is clearly bounded. Thus, N is L–compact on $\bar{\Omega}$ with any open bounded set $\Omega \subset X$.

Now we reach the position to search for an appropriate open bounded subset Ω for the application of the continuation theorem Corresponding to the operator equation $Lu = \lambda Nu$, $\lambda \in (0, 1)$, we have

$$\begin{cases} \frac{\partial x_i(t,v)}{\partial t} = \lambda\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right) - \lambda a_i(t)x_i(t) + \lambda\sum_{j=1}^{p} a_{ij}(t) f_j(t, y_j(t-\tau_{ij}(t))) + \lambda I_i(t), \\ \frac{\partial y_j(t,v)}{\partial t} = \lambda\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right) - \lambda b_j(t)y_j(t) + \lambda\sum_{i=1}^{n} b_{ji}(t) g_i(t, x_i(t-\sigma_{ji}(t))) + \lambda J_j(t), \end{cases} \tag{2.2}$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, p$.

Assume that $u = u(t) \in X$ is a solution of (2.2) for a certain $\lambda \in (0, 1)$. Then $x_i(t)$ and $y_j(t)$, as the components of $u(t)$, are all continuously differentiable. Thus, there exist t_i, $t'_j \in [0, \omega]$ such that $|x_i(t_j)| = [x_i(t)]^+$ and $|y_j(t'_j)| = [y_j(t)]^+$.

Hence, $\dot{x}_i(t_i) = \dot{y}_j(t_j) = 0$. This implies that

$$\begin{cases} a_i(t_i)x_i(t) = \sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right) + \sum_{j=1}^{p} a_{ij}(t) f_j(t, y_j(t - \tau_{ij}(t))) + I_i(t), \\ b_j(t')y_j(t') = \sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right) + \sum_{i=1}^{n} b_{ji}(t) g_i(t, x_i(t - \sigma_{ji}(t))) + J_j(t), \end{cases} \tag{2.3}$$

It follows from the first equation of (2.3) that for $i = 1, 2, \cdots, n$, we have

$$|x_i(t_i)| = \left|\sum_{j=1}^{p} \frac{a_{ij}(t_i)}{a_i(t_i)} f_j(t_j, y_j(t_i - \tau_{ij}(t_i))) + \frac{I_i(t_i)}{a_i(t_i)} + \frac{1}{a_i(t_i)}\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right)\right|$$

$$\le \sum_{j=1}^{p} m_{i,n+j}\left|y_j(t'_j)\right| + D_i, \tag{2.4}$$

where

$$D_i = \sum_{j=1}^{p}[a_{ij}(t)\alpha_j/a_i(t)]^+ + [I_i(t)/a_i(t)]^+\left[\frac{1}{a_i(t)}\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right)\right]^+.$$

Similarly, one can obtain

$$\left|y_j(t'_j)\right| \le \sum_{i=1}^{n} m_{n+j,i}\left|x_i(t_i)\right| + D_{j+n},\ j = 1, 2, \cdots, p, \tag{2.5}$$

where

$$D_{n+j} = \sum_{i=1}^{n}[b_{ji}(t)\beta_i/b_j(t)]^+ + [J_j(t)/b_j(t)]^+\left[\frac{1}{b_j(t)}\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right)\right]^+.$$

In view of $\rho(M) < 1$, it follows that

$$(E - M)^T \ge 0 \text{ and } h = (E - M)^{-1}D \ge 0$$

(see [16] for more details), where $D = (D_1, D_2, \cdots, D^T_{n+p}$. It follows from (2.4) and (2.5) that

$$[x_i(t)]^+ \le h_i \text{ and } [y_j(t)]^+ \le h_{n+j}, \text{ for } i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, \tag{2.6}$$

where h_i is the ith component of vector h. Clearly, h_i ($i = 1, 2, \cdots, n + p$) are independent of λ.

Moreover, it follows from the first equation of (2.2) that

$$\left[\frac{\partial x_i(t,v)}{\partial t}\right]^+ \le \max_{t\in[0,\omega]}\left[\lambda\left|a_i(t)\right|\left|x_i(t)\right| + \lambda\sum_{j=1}^{p}\left|a_{ij}(t)\right|\left|f_j(t, y_j(t - \tau_{ij}(t)))\right| + \lambda\left|I_i(t)\right|\right.$$

$$\left. +\lambda\left|\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right)\right|\right] \le \max_{t\in[0,\omega]}\left|a_i(t)\right|\left[h_i + \sum_{j=1}^{p} m_{i,n+j}h_{n+j} + D_i\right] = 2h_i[a_i(t)]^+.$$

Namely,

$$\left[\frac{\partial x_i(t,v)}{\partial t}\right]^+ \le 2h_i[a_i(t)]^+,\ i=1,2,\cdots,n. \tag{2.7}$$

Similarly, we have

$$\left[\frac{\partial y_j(t,v)}{\partial t}\right]^+ \le 2h_{n+j}[b_j(t)]^+,\ j=1,2,\cdots,p. \tag{2.8}$$

Let $A = \max\limits_{1\le i\le n, 1\le j\le p} \{h_i(1+2[a_i(t)]^+), h_{n+j}(1+2[b_j(t)]^+)\}$. Then there exists some $d > 1$ such that $dh_i > A$ for all $i = 1, 2, \cdots, n + p$. We take $\Omega = \{u \in X; -dh < u(t) < dh \text{ for all } t\}$. If $u = (x_1, x_2, \cdots, x_n, y_1, y_2, \cdots, y_p)^T \in \partial\Omega \cap Ker\ L = \partial\Omega \cap R^{n+p}$, then u is a constant vector in R^{p+n} with $|x_i| = dh_i$ and $|y_j| = dh_{n+j}$ for $i = 1, 2, \cdots, n$, $j = 1, 2, \cdots, p$. It follows that

$$(QN\,u)_i = \frac{1}{\omega}\int_0^\omega [-a_i(t)x_i + \sum_{j=1}^{p} a_{ij}(t)f_j(t,y_j) + I_i(t)]\,dt,$$

and

$$(QN\,u)_{n+j} = \frac{1}{\omega}\int_0^\omega \left[-b_j(t)y_j + \sum_{i=1}^{n} b_{ji}(t)g_i(t,x_i) + J_j(t)\right]dt.$$

We can claim that

$$|(QN\,u)_i| > 0 \tag{2.9}$$

for $i = 1, 2, \cdots, n + p$. By a way of contrary, suppose that there exists some $i \in \{1, 2, \cdots, n\}$ such that

$$|(QN\,u)_i| = 0,\ i.e.,\ \frac{1}{\omega}\int_0^\omega \left[-a_i(t)x_i + \sum_{j=1}^{p} a_{ij}(t)f_j(t,y_j) + I_i(t)\right]dt = 0.$$

Then there exists some $t^* \in [0, \omega]$, such that $-a_i(t^*)x_i + \sum\limits_{j=1}^{p} a_{ij}(t^*)f_j(t^*,y_j) + I_i(t^*) = 0.$ Thus,

$$dh_j = |x_i| \le \sum_{j=1}^{p} \frac{|a_{ij}(t^*)|}{a_i(t^*}|f_j(t^*,y_j)| + \frac{|I_i(t^*)|}{a_i(t^*)} \le \sum_{j=1}^{p} m_{i,\,n+p}dh_{n+j} + D_i.$$

In view of $d > 1$ and $dh = d(Mh + D) > Mdh + D$, we have $dh_i > \sum\limits_{j=1}^{n+p} m_{ij}dh_j + D_i$ for $i = 1, 2, \cdots, n$. It follows from the above inequality that $dh_i < dh_i$, which is a contradiction.

Thus we have $|(QN\,u)_i| > 0$ for all $i \in \{1, 2, \cdots, n\}$. Similarly, $|(QN\,u)_i| > 0$ for all $i \in \{n+1, n+2, \cdots, n+p\}$. Therefore, (2.9) holds and

$$QN\,u \neq 0, \quad u \in \partial\,\Omega \cap Ker\,L.$$

Define

$$\psi : \Omega \cap Ker\,L \times [0, 1] \to X = Im\,L = X^c by,$$

$$\psi(u, \mu) = \mu\, diag(-\bar{a}_1, \cdots, -\bar{a}_n, -\bar{b}_1, \cdots, -\bar{b}_p)\,u + (1-\mu)QNu,$$

for all $u = (x_1, \cdots, x_n, y_1, \cdots, y_p)^T \in \Omega \cap R^{n+p}$ and $\mu \in [0, 1]$.

When $u \in \partial\Omega \cap Ker\,L$ and $\mu \in [0, 1]$, $u = (x_1, \cdots, x_n, y_1, \cdots, y_p)^T$ is a constant vector in R^{n+p} with $|x_i| = dh_i (i = 1, 2, \cdots, n)$ and $|y_j| = dh_{n+j} (j = 1, 2, \cdots, p)$. Thus,

$$\max_{1\le i\le n, 1\le j\le p}\left\{\left|-\bar{a}_j x_i + (1-\mu)\left[\frac{1}{\omega}\sum_{j=1}^{p}\int_0^{\omega} a_{ij}(t) f_j(t, y_j)\,dt \;+ \bar{I}_i + \sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right)\right]\right|,\right.$$

$$\left.\left|-\bar{b}_j y_j + (1-\mu)\left[\frac{1}{\omega}\sum_{i=1}^{n}\int_0^{\omega} b_{ji}(t) g_i(t, x_i)dt \text{ - } + \bar{J}_j + \sum_{k=1}^{l} d\frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right)\right]\right|\right\}.$$

So,

$$|\psi(u, \mu)|_0 > 0. \tag{2.10}$$

Now suppose that $|\psi(u, \mu)|_0 = 0$, then we have

$$\begin{cases} -\bar{a}_j x_i + \frac{(1-\mu)}{\omega}\int_0^{\omega}\sum_{j=1}^{p} a_{ij}(t) f_j(t, y_j)dt + (1-\mu)\bar{I}_i + (1-\mu)\overline{\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i}{\partial v_k}\right)} = 0, \\ -\bar{b}_j y_j + \frac{(1-\mu)}{\omega}\int_0^{\omega}\sum_{i=1}^{n} b_{ji}(t) g_i(t, x_i)dt + (1-\mu)\bar{J}_j + (1-\mu)\overline{\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D^*_{jk}\frac{\partial y_j}{\partial v_k}\right)} = 0 \end{cases} \tag{2.11}$$

for all $i = 1, 2, \cdots, n, j = 1, 2, \cdots, p$. It follows from the first equality that there exists some $t^* \in [0, \omega]$ such that

$$-a_i(t^*)x_i + (1-\mu)\sum_{j=1}^{p} a_{ij}(t^*) f_j(t^*, y_j) + (1-\mu)I_i(t^*) + (1-\mu)\overline{\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i(t^*)}{\partial v_k}\right)} = 0.$$

Thus,

$$dh_i = =|x_i| \le (1-\mu)\sum_{j=1}^{p}\frac{|a_{ij}(t^*)|}{a_i(t^*)}|f_j(t^*, y_j)| + (1-\mu)\frac{|I_i(t^*)|}{a_i(t^*)}$$

$$+\frac{1}{a_i(t^*)}(1-\mu)\overline{\sum_{k=1}^{l}\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial x_i(t^*)}{\partial v_k}\right)} \le \sum_{j=1}^{p} m_{i,n+j}\, dh_{n+j} + D_i,$$

which contradicts that $dh_i > \sum_{j=1}^{n+p} m_{ij} dh_j + D_i$ for $i = 1, 2, \cdots, n$. Thus, (2.10) holds. Therefore, $\psi(u, \mu) \neq 0$, for any $u \in \partial\Omega \cap Ker\,L$.

Using the property of topological degree and taking J to be the identity mapping $I : Im\, Q \to Ker\, L$, we have

$$deg(JQN, \Omega \cap Ker\, L, 0) = deg(\psi(\cdot, 0), \Omega \cap Ker\, L, 0) = deg(\psi(\cdot, 1), \Omega \cap Ker\, L, 0),$$
$$= deg(diag(-\bar{a}_1, \cdots, -\bar{a}_n, -\bar{b}_1, \cdots, -\bar{b}_p), \Omega \cap Ker\, L, 0) = 1.$$

Therefore, according to the continuation theorem of Gaines and Mawhin [17], system (1.1) has at least one ω–periodic solution. The proof is completed.

Remark 1: In this section we has proved the existence of periodic solution of (1.1) and avoided finding a Lyapunov functional for the convenience of proof.

3. GLOBAL ASYMPTOTIC STABILITY OF PERIODIC SOLUTION

In this section, we shall construct a suitable Lyapunov functional to derive a sufficient condition which ensures the global asymptotic stability of periodic solution of system (1.2).

In the sequel, we will use the following notations:

$$a_i^- = \min_{t \in [0,\omega]} a_i(t),\ b_j^- = \min_{t \in [0,\omega]} b_j(t),\ a_{ij}^+ = \max_{t \in [0,\omega]} |a_{ij}(t)|,\ b_{ji}^+ = \max_{t \in [0,\omega]} |b_{ji}(t)|,$$

where $i = 1, 2, \cdots, n, j = 1, 2, \cdots, p$.

Assumption 1: For system (1.2), suppose that the activation functions $f_i(\cdot)$, $gi(\cdot)$ are bounded and satisfy that (H_2) there exists a positive number L_i such that

$$|f_i(x) - f_i(y)| \cdot L_i |x - y|, |g_i(x) - g_i(y)| \cdot L_i |x - y|$$

for all $x, y \in R$, $i = 1, 2, \cdots, \max\{p, n\}$.

Theorem 2: Assume that (H_1)–(H_2) hold. If

$$B_1 : 2a_i^- - \sum_{j=1}^{p} \left(a_{ij}^+ + L_i^2 b_{ji}^+ \int_0^\infty k_{ji}^2(s) ds \right) > 0,$$

$$B_2 : 2b_j^- - \sum_{i=1}^{n} \left(b_{ji}^+ + L_i^2 a_{ij}^+ \int_0^\infty h_{ij}^2(s) ds \right) > 0,$$

for $i = 1, 2, \cdots, n, j = 1, 2, \cdots, p$, then the system (1.2) has a unique ω-periodic solution and all solutions of (1.2) converge to its unique ω–periodic solution.

Proof: Let

$$x(t) = \{x_1(t), x_2(t), \cdots, x_n(t), y_1(t), y_2(t), \cdots, y_p(t)\}$$

be an arbitrary solution of (1.2) and

$$x^*(t) = \{x_1^*(t), x_2^*(t), \cdots, x_n^*(t), y_1^*(t), y_2^*(t), \cdots, y_p^*(t)\},$$

be an ω–periodic solution of (1.2). First, from (1.2) we have

$$\begin{cases} \frac{\partial \alpha_i}{\partial t} = -a_i(t)\alpha_i(t) + \sum\limits_{j=1}^{p} a_{ij}(t) A_j(\beta_j(t-s)) + \sum\limits_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik} \frac{\partial x_i}{\partial v_k} \right), i = 1, 2, \cdots, n, \\ \frac{\partial \beta_j}{\partial t} = -b_j(t)\beta_j(t) + \sum\limits_{i=1}^{n} b_{ji}(t) A_i(\alpha_i(t-s)) + \sum\limits_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{jk}^* \frac{\partial y_j}{\partial v_k} \right), j = 1, 2, \cdots, p, \end{cases} \tag{3.1}$$

where

$$\alpha_i(t) = x_i(t) - x_i^*(t),\ i = 1, 2, \cdots, n;\ \beta_j(t) = y_j(t) - y_j^*(t), j = 1, 2, \cdots, p,$$
$$A_j(\beta_j(t-s)) = f_j(\int_0^\infty h_{ij}(s)y_j(t-s)ds) - f_j(\int_0^\infty h_{ij}(s)y_j^*(t-s)ds), j = 1, 2, \cdots, p,$$
$$A_i(\alpha_i(t-s)) = g_i(\int_0^\infty k_{ji}(s)x_i t-s)ds) - g_i(\int_0^\infty k_{ji}(s)\ x_i^* t-s)ds), i = 1, 2, \cdots, n.$$

Consider the Lyapunov functional $V(t) = V_1(t) + V_2(t)$, where

$$V_1(t) = \int_\Omega \sum_{i=1}^n \left|\alpha_i(t)\right|^2 + \sum_{j=1}^p \left|\beta_j(t)\right|^2 dv,$$

$$V_2(t) = \int_\Omega \sum_{i=1}^n \sum_{j=1}^p L_j^2 a_{ij}^+ \int_0^\infty \int_{t-s}^t h_{ji}^2(s)\beta_j^2(\eta)d(\eta)\,ds\,dv$$

$$+\int_\Omega \sum_{j=1}^m \sum_{i=1}^n L_i^2 b_{ji}^+ \int_0^\infty \int_{t-s}^t K_{ji}^2(s)\alpha_i^2(\eta)d\eta ds dv, \tag{3.2}$$

where $\int_\Omega$ denotes the integral for v in Ω.

Calculating the derivative $D^+V_1(t)$ and $D^+V_2(t)$ along the solution of (3.1), we derive that

$$D^+V_1(t) = \int_\Omega \left[2\sum_{i=1}^n \left|\alpha_i(t)\right| \tfrac{\partial\alpha_i}{\partial t} + 2\sum_{j=1}^p \left|\beta_j(t)\right| \tfrac{\partial\beta_j}{\partial t} \right] dv$$

$$= \int_\Omega [2\sum_{i=1}^n \{\alpha_i(t)[-a_i(t)\alpha_i(t) + \sum_{j=1}^p a_{ij}(t)A_j(\beta_j(t-s)) + \sum_{k=1}^l \tfrac{\partial}{\partial v_k}\left(D_{ik}\tfrac{\partial\alpha_i}{\partial v_k}\right)]\}dv$$

$$+\int_\Omega [2\sum_{j=1}^p \{\beta_j(t)[-b_j(t)\beta_j(t) + \sum_{i=1}^n b_{ji}(t)A_i(\alpha_i(t-s)) + \sum_{k=1}^l \frac{\partial}{\partial v_k}(D_{jk}^* \frac{\partial\beta_j}{\partial v_k})]\}dv$$

$$\le \int_\Omega \sum_{i=1}^n \{-2a_i(t)\alpha_i^2(t) + \sum_{j=1}^p |a_{ij}(t)|[\alpha_i^2(t) + L_j^2\int_0^\infty h_{ij}^2(s)\beta_j^2(t-s)ds]\}$$

$$+\int_\Omega \sum_{j=1}^p \{-2b_j(t)\beta_j^2(t) + \sum_{i=1}^n |b_{ji}(t)|[\beta_j^2(t) + L_i^2\int_0^\infty k_{ji}^2(s)\alpha_i^2(t-s)ds]\}$$

$$+\sum_{i=1}^n \sum_{k=1}^l 2\alpha_i(t)\left(\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial\alpha_i}{\partial v_k}\right)\right)dv + \sum_{j=1}^p \sum_{k=1}^l 2\beta_j(t)\left(\frac{\partial}{\partial v_k}\left(D_{jk}^*\frac{\partial\beta_j}{\partial v_k}\right)\right)dv$$

$$\le \int_\Omega \sum_{i=1}^n [-2a_i^- + \sum_{j=1}^p a_{ij}^+(t)]\alpha_i^2(t) + \sum_{i=1}^n \sum_{j=1}^p a_{ij}^+ L_j^2 \int_0^\infty h_{ij}^2(s)\beta_j^2(t-s)ds$$

$$+\sum_{j=1}^{p}\sum_{i=1}^{n} b_{ji}^{+}L_i^2\int_0^{\infty} k_{ji}^2(s)\alpha_i^2(t-s)ds+\sum_{i=1}^{n}\sum_{k=1}^{l}2\alpha_i(t)\left(\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial\alpha_i}{\partial v_k}\right)\right)$$

$$+\sum_{j=1}^{p}[-2b_j^{-}+\sum_{i=1}^{n} b_{ji}^{+}]\beta_j^2(t)+\sum_{j=1}^{p}\sum_{k=1}^{l}2\beta_j(t)\left(\frac{\partial}{\partial v_k}\left(D_{jk}^{*}\frac{\partial\beta_j}{\partial v_k}\right)\right)dv, \tag{3.3}$$

and

$$D^{+}V_2(t)=\int_{\Omega}\sum_{i=1}^{n}\sum_{j=1}^{p} L_j^2 a_{ij}^{+}[\beta_j^2(t)\int_0^{\infty} h_{ij}^2(s)ds-\int_0^{\infty} h_{ij}^2(s)\beta_j^2(t-s)\,ds]$$

$$+\sum_{j=1}^{p}\sum_{i=1}^{n} L_i^2 b_{ji}^{+}[\alpha_i^2(t)\int_0^{\infty} k_{ji}^2(s)ds-\int_0^{\infty} k_{ji}^2(s)\alpha_i^2(t-s)ds]dv. \tag{3.4}$$

It follows from (3.3)-(3.4) and Q_i, Q_j that

$$D^{+}V(t)\le\int_{\Omega}\sum_{i=1}^{n}[-2a_i^{-}+\sum_{j=1}^{p} a_{ij}^{+}+L_i^2\sum_{j=1}^{p} b_{ji}^{+}\int_0^{\infty} k_{ji}^2(s)ds]\alpha_i^2(t)+\sum_{i=1}^{n}\sum_{k=1}^{l}2\alpha_i(t)\left(\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial\alpha_i}{\partial v_k}\right)\right)$$

$$+\sum_{j=1}^{p}[-2b_j^{-}+\sum_{i=1}^{n} b_{ji}^{+}+L_j^2\sum_{i=1}^{n} a_{ij}^{+}\int_0^{\infty} h_{ij}^2(s)ds]\,\beta_j^2(t)+\sum_{j=1}^{p}\sum_{k=1}^{l}2\beta_j(t)\left(\frac{\partial}{\partial v_k}\left(D_{jk}^{*}\frac{\partial\beta_j}{\partial v_k})\right)\right)$$

$$=\int_{\Omega}-\sum_{i=1}^{n} Q_i\alpha_i^2(t)-\sum_{j=1}^{p} Q_j\beta_j^2(t)+\sum_{i=1}^{n}\sum_{k=1}^{l}2\alpha_i(t)\left(\frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial\alpha_i}{\partial v_k}\right)\right)$$

$$+\sum_{j=1}^{p}\sum_{k=1}^{l}2\beta_j(t)\left(\frac{\partial}{\partial v_k}\left(D_{jk}^{*}\frac{\partial\beta_j}{\partial v_k}\right)\right)dv. \tag{3.5}$$

From the boundary condition (2), we get

$$\int_{\Omega}\sum_{i=1}^{n}2\alpha_i\sum_{k=1}^{l}\left(\tfrac{\partial}{\partial v_k}\left(D_{ik}\tfrac{\partial\alpha_i}{\partial v_k}\right)\right)dv=\sum_{i=1}^{n}\int_{\Omega}2\alpha_i\nabla\cdot\left(D_{ik}\tfrac{\partial\alpha_i}{\partial v_k}\right)_{k=1}^{l}dv$$

$$=\sum_{i=1}^{n}2\int_{\Omega}\nabla\cdot\left(\alpha_i D_{ik}\tfrac{\partial\alpha_i}{\partial v_k}\right)_{k=1}^{l}dv-\sum_{i=1}^{n}2\int_{\Omega}\left(D_{ik}\tfrac{\partial\alpha_i}{\partial v_k}\right)_{k=1}^{l}\nabla\cdot\alpha_i\,dv$$

$$=\sum_{i=1}^{n}2\int_{\partial\Omega}\left(\alpha_i D_{ik}\frac{\partial\alpha_i}{\partial v_k}\right)_{k=1}^{l}dv-\sum_{i=1}^{n}2\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial\alpha_i}{\partial v_k}\right)^2dv$$

$$=-\sum_{i=1}^{n}2\sum_{k=1}^{l}\int_{\Omega}D_{ik}\left(\frac{\partial\alpha_i}{\partial v_k}\right)^2dv, \tag{3.6}$$

where $r = \nabla = \left(\frac{\partial}{\partial v_1}, \cdots, \frac{\partial}{\partial v_l}\right)$ is the gradient operator, and

$$\left(D_{ik}\frac{\partial \alpha_i}{\partial v_k}\right)_{k=1}^{l} = \left(D_{i1}\frac{\partial \alpha_i}{\partial v_1}, \cdots, D_{il}\frac{\partial \alpha_i}{\partial v_l}\right).$$

Similarly, by using of the same way we get

$$\int_\Omega \sum_{j=1}^{p} 2\beta_j \sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{jk}^* \frac{\partial \beta_j}{\partial v_k}\right) dv = -\sum_{j=1}^{p} 2\sum_{k=1}^{l} \int_\Omega D_{jk}^* \left(\frac{\partial \beta_j}{\partial v_k}\right)^2 dv. \tag{3.7}$$

Since $D_{ik} \geq 0$, D_{jk}^* ¸ 0 ($i = 1, 2, \cdots, n, j = 1, 2, \cdots, p, k = 1, 2, \cdots, l$), from (3.4), (4.6) and (3.7) we get $D^+ V(t) \geq 0$, and so $V(t) \leq V(0)$, $t \geq 0$.

Integrating both sides of (3.5) from 0 to t, we get

$$V(t) + \int_\Omega \int_0^t \sum_{i=1}^{n} Q_i \alpha_i^2(s)\, ds + \int_0^t \sum_{j=1}^{p} Q_j \beta_j^2(s) ds$$

$$-\sum_{i=1}^{n}\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial \alpha_i}{\partial v_k}\right) - \sum_{j=1}^{p}\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{jk}^*\frac{\partial \beta_j}{\partial v_k}\right) dv \leq V(0),$$

which implies $\alpha_i(t), \beta_j(t) \in L_1[0, \infty)$. It follows from (1.2) that

$$-\Delta_i \leq \frac{\partial x_i}{\partial t} + a_i(t) x_i(t) \leq \Delta_i,\ i = 1, 2, \cdots, n,$$

and

$$-\Lambda_j \leq \frac{\partial y_j}{\partial t} + b_j(t) y_j(t) \leq \Lambda_j,\ j = 1, 2, \cdots, p,$$

where

$$\Delta_i = \sum_{j=1}^{p} (a_{ij}^+ \sup_{x\in R} |f_j(x)| + I_i^+ \left[\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{ik}\frac{\partial \alpha_i}{\partial v_k}\right)\right]^+,$$

$$\Lambda_j = \sum_{i=1}^{n} b_{ji}^+ \sup_{x\in R} |g_i(x)| + J_j^+ \left[\sum_{k=1}^{l} \frac{\partial}{\partial v_k}\left(D_{jk}^*\frac{\partial \beta_j}{\partial v_k}\right)\right]^+.$$

So one can easily see that all the solutions of (1.2) are bounded on $[0,\infty)$. By (2.1) we know that $\frac{\partial \alpha_i}{\partial t}$ and $\frac{\partial \beta_j}{\partial t}$ are bounded on $[0, \infty)$. Hence $\alpha_i(t)$ and $\beta_j(t)$ are uniformly continuous on $[0, \infty)$. Therefore, by Barbalatt's Lemma [18], we have $\lim_{t\to\infty} u_i = 0, \lim_{t\to\infty} v_j = 0$, $i = 1, 2, \cdots, n, j = 1, 2, \cdots, p$. This completes the proof.

4. AN ILLUSTRATIVE EXAMPLE

Example 1: Consider the following two-dimension BAM neural network with distributed delays and reaction-diffusion terms:

$$\begin{cases} \frac{\partial x_i(t,v)}{\partial t} = \frac{\partial}{\partial v_1}\left(D_{i1}\frac{\partial x_i}{\partial v_1}\right) - a_i(t)x_i(t) \\ \qquad + \sum_{j=1}^{2} a_{ij}(t) f_j\left(\int_0^\infty h_{ij}(s) y_j(t-s)ds\right) + I_i(t), i = 1,2, \\ \frac{\partial y_j(t,v)}{\partial t} = \frac{\partial}{\partial v_1}\left(D_{j1}^*\frac{\partial y_j}{\partial v_1}\right) - b_j(t)y_j(t) \\ \qquad + \sum_{i=1}^{2} b_{ji}(t) g_i\left(\int_0^\infty k_{ji}(s) x_i(t-s)ds\right) + J_j(t), j = 1,2. \end{cases} \tag{4.1}$$

Take

$$g_1(u) = g_2(u) = f_1(u) = f_2(u) = \tanh(u),$$
$$D_{11} = D_{11}^* = t^6x^2,\ D_{21} = D_{21}^* = t^4x^4,\ h_{ij}(s) = k_{ji}(s) = e^{-s},$$
$$a_1(t) = 0.9,\ a_2(t) = 0.5,\ b_1(t) = 1.2,\ b_2(t) = 0.8,$$
$$a_{11}(t) = 0.6,\ a_{12}(t) = 0.2,\ a_{21}(t) = 0.7,\ a_{22}(t) = 0.5,$$
$$b_{11}(t) = -0.5,\ b_{12}(t) = 0.3,\ b_{21}(t) = 0.3,\ b_{22}(t) = 0.25,$$
$$I_1(t) = I_2(t) = 8\sin(t),\ J_1(t) = J_2(t) = -5\cos(t).$$

Obviously, assumptions (H_1) and (H_2) hold and $L_1 = L_2 = 1$. It can be easily checked that the following conditions hold:

$$2a_i^- - \sum_{j=1}^{2}\left(a_{ij}^+ + L_i^2 b_{ji}^+ \int_0^\infty k_{ji}^2(s)ds\right) > 0, \quad 2b_j^- - \sum_{i=1}^{2}\left(b_{ji}^+ + L_i^2 a_{ij}^+ \int_0^\infty h_{ij}^2(s)ds\right) > 0.$$

Therefore, it follows from Theorem 2 that the system (4.1) has a unique ω-periodic solution and all solutions of (4.1) converge to its unique ω–periodic solution.

The dynamical behaviors are shown in Fig.1-Fig.4 with the initial $x_1(0, v) = -0.5$, $x_2(0, v) = 0.8$, $y_1(0, v) = 0.6$, $y_2(0, v) = 1$ and the boundary conditions $\frac{\partial x_1}{\partial v}(t, 0) = \frac{\partial y_1}{\partial v}(t, 0) = 0$, $\frac{\partial x_2}{\partial v}(t, 1) = \frac{\partial y_2}{\partial v}(t, 1) = 0$, $x_1(t, 1) = y_1(t, 1) = 1$, $x_2(t, 0) = y_2(t, 0) = 0$. And the space surface plots are shown in Fig. 5 and Fig. 6. As we can see that the neural states converge towards a unique ω–periodic solution.

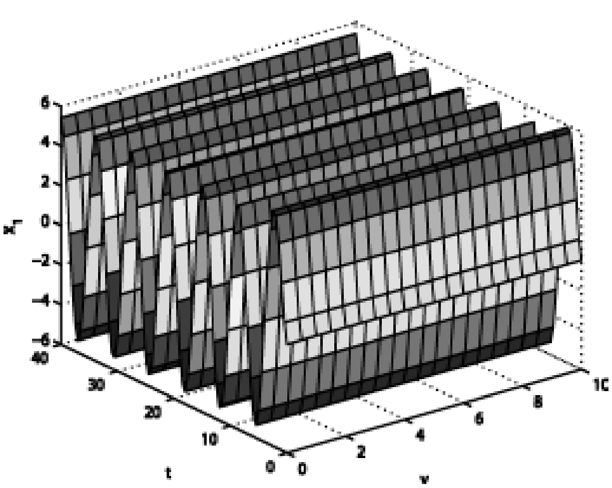

Figure 1: Dynamical Behavior Simulation of t–v –x_1

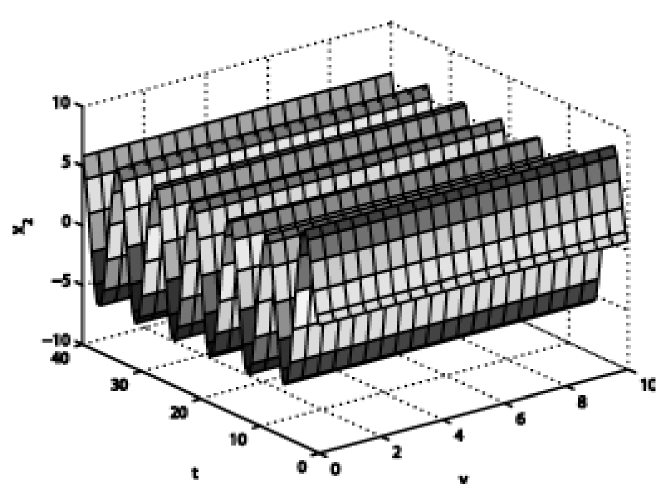

Figure 2: Dynamical Behavior Simulation of t–v –x_2

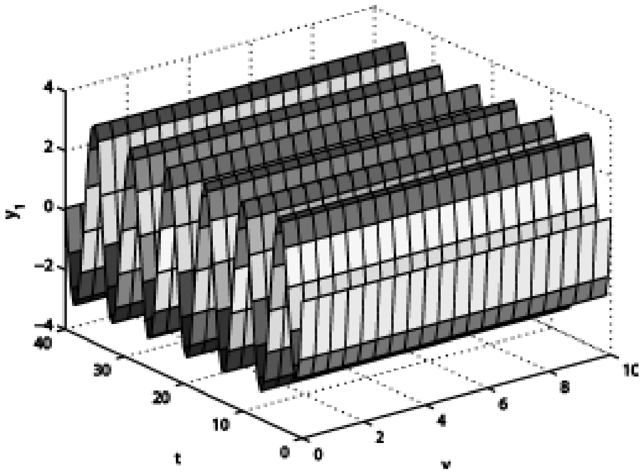

Figure 3: Dynamical Behavior Simulation of $t - v - y_1$

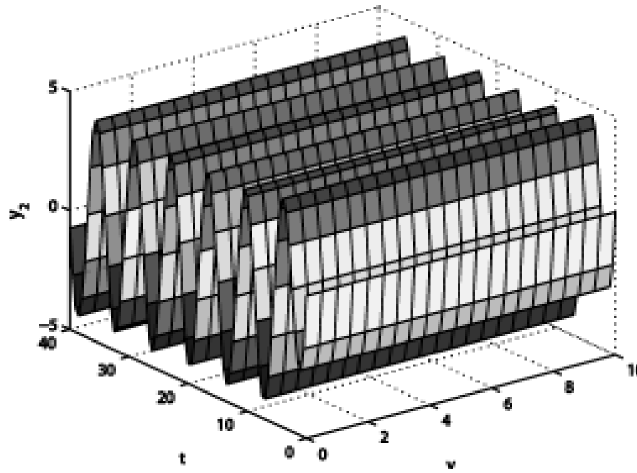

Figure 4: Dynamical Behavior Simulation of $t - v - y_2$

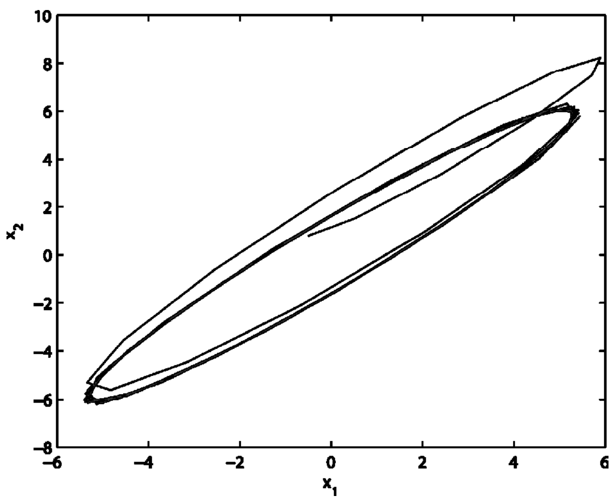

Figure 5: Space Surface Plot of $x_1 - x_2$

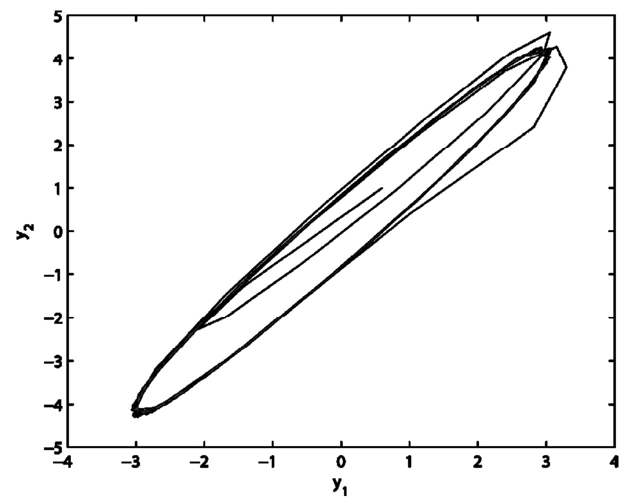

Figure 6: Space Surface Plot of $y_1 - y_2$

5. CONCLUSIONS

In this paper, the existence of periodic solution for reaction-diffusion BAM neural networks with discrete time-varying delays and the asymptotic stability of periodic solution for reaction-diffusion BAM neural networks with distributed delays are studied. By Lyapunov method and coincidence degree, some sufficient conditions for the neural networks with reaction-diffusion terms are derived and the activation functions may not be bounded.

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