Journal of Mathematical Control Science and Applications
Vol. 6 No. 2 (July-December 2020)

Submitted: 16th September 2020 *Revised: 13th October 2020* *Accepted: 17th October 2020*

STUDY OF NUMERICAL SOLUTION OF NONLINEAR INTEGRAL EQUATIONS BY USING ADOMIAN DECOMPOSITION METHOD & HE'S POLYNOMIAL

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Abstract. In this paper we introduce new approaches for numerical solution of nonlinear integral equation of second kind. This numerical solution based on Adomian decomposition method and by using He's polynomial. All calculation performed by MATLAB 13 versions. Many examples are given for comparative study of numerical solution of integral equations with estimated error. This comparative study of numerical solution of nonlinear integral equation is more effective & better accuracy other than existing method.

1. INTRODUCTION

Integral equation is described in many branches of Mathematics, Physics, Biology, Chemistry & Engineering problems are used in initial and boundary value problems are transfered in Volterra & Fredholm integral equation e.g Potential theory, Dirichlet problems of astrophysics, Diffusion problems, Conformal mapping, Mathematical physics models, Water waves are useful in integral equations[1]. Nonlinear integral equations are used in many fields of the Research i.e Quantum mechanics, Fluid dynamics, Numerical Analysis, Queuing theory, Mathematical economics, Vector analysis, Geophysics, Chemical kinetics etc[2-4] are also used in numerical solution by method of Galerkin method, Decomposition method, Quadrature method, Cubic splin method in certain polynomials[5-6]. One of the important and useful method which has received much concern is the Adomian decomposition method. In this method more attention devoted in search of reliable and efficient solution method in various field of science and engineering[7-8]. But "Fredholm integral equation of second kind" has better accuracy then other integral equations and used in Adomian decomposition method of the He's polynomials [9-10].

A nonlinear integral equation of the form

$$v\left(x\right)u\left(x\right)=f\left(x\right)+\lambda\int_{a}^{b}k\left(x,t\right)G\left(u\left(t\right)\right)dt \tag{a}$$

Where a, b are constant λ is non-zero real or complex parameter $f(x)$, $v(x)$, $k(x,t)$ are known functions and $u(x)$ is unknown functions, this equation is called

2000 *Mathematics Subject Classification.* Primary 65R20; Secondary 65R99.
Key words and phrases. Nonlinear integral equations, Adomian decomposition method, He's polynomial.

Fredhom integral equation of third kind.If F is nonlinear function of u we put $v(x)$ =1 in (1) and this equation is known as integral equation of second kind.

$$u\left(x\right)=f\left(x\right)+\lambda\int_{a}^{b}K\left(x,t\right)G\left(u\left(t\right)\right)dt,\ a\leq x\leq b$$

Expand Adomian Decomposition Method

Let us consider that a "non-linear Fredholm integral equation of second kind" is expaned in this form

$$u\left(x\right)=f\left(x\right)+\lambda\int_{a}^{b}K\left(x,t\right)G\left(u\left(t\right)\right)dt,\qquad a\leq x\leq b \tag{1.1}$$

If we suppose $F\left(u\left(t\right)\right)$ is a non-linear function of $u(x)$ and $K\left(x,t\right)$ is a kernel of the of "non-linear Fredholm integral equation". If $u(x)$ is unknown function and $f(x)$ & $K\left(x,t\right)$ are known function. A function $f(x)$ and $K\left(x,t\right)$ is to be determined. In this way we may write solution equation (1.1)

$$u=\sum_{r=1}^{\infty}q^{r}u_{r}\left(x\right)=u_{0}+\ q^{1}u_{1}+\ q^{2}u_{2}+\ q^{3}u_{3}+\ \ldots \tag{1.2}$$

After comparing various order of the power q in equation (1.2) for solving and the best approximation should be

$$u=\lim_{q\to1}\sum_{r=0}^{\infty}q^{r}u_{r}\ \ (x)$$

$$u=\ u_{0}+\ u_{1}+\ u_{2}\ +\ \ldots$$

If $G(u(t))$ is also nonlinear terms may be expaned in "Adomian polynomials" [10].

$$G\left(u\right)=\sum_{r=0}^{\infty}q^{r}H_{r}\left(u_{0},\ u_{1},u_{2},\ \ldots u_{r}\right)\quad\ldots \tag{1.3}$$

$$=H_{0}\left(u_{0}\right)+\ q^{1}H_{1}\left(u_{0},\ u_{1}\right)+\ q^{2}H_{2}\left(u_{0},\ u_{1}\right)+\ \ \cdots+\ q^{r}H_{r}(u_{0,},\ u_{1},\ \ldots\ u_{r})$$

Here H_r' S are called Adomian polynomials and measured by the formula [11-12]

$$H_{r}\left(u_{0},\ u_{1},\ \ldots u_{r}\right)=\ \frac{1}{r!}\frac{d^{r}}{dq^{r}}\left[G\left\langle\sum_{i=0}^{r}q^{i}u_{i}\right\rangle\right],\ r=0,1,2,\ \ldots. \tag{1.4}$$

Victimization equation (1.2), (1.3) and (1.4) in (1.1) then

$$\sum_{i=0}^{\infty}q^{i}u_{i}\left(x\right)=f\left(x\right)+\ \lambda\int_{a}^{b}k(x,t)\sum_{J=0}^{\infty}\left(q^{j}u_{j}\right)dt\quad\ldots \tag{1.5}$$

The above equation (1.5) equating identical power of q

$$q^{0}\ :\ u_{0}\left(x\right)=f(x)$$

$$q^{1}\ :\ u_{1}\ =\ \lambda\int_{a}^{b}k\left(x,t\right)H_{0}\left(t\right)dt$$

Similar explain in genraly

$$u_{j+1} = \lambda \int_a^b k(x,t)\, H_j(t)\, dt, \quad j = 0, 1, 2, 3, \ldots. \tag{1.6}$$

The n- term approximation series solution by using the recursive scheme (1.6) is obtained as follows

$$\phi_n(x) = \sum_{j=0}^{n} u_j(x) \tag{1.7}$$

2. Illustrative Example

In this section, we intend to compare "Adomian decomposition methods" which are calculated in non-linear integral equation with the help of numerical solution in Fredhom type. Compute the result in exact solution and numerical solution technique and comparative study with the help of example.

Example 2.1. Suppose the "nonlinear integral of the second kind"

$$u(x) = 1 - \frac{5}{12}x + \int_0^1 xtu^2(t)\, dt \tag{2.1}$$

In this equation the exact solution is $u(x) = 1 + \frac{1}{3}x$. In this equation we explain Adomian polynomials of the non-linear terms $u^2(t)$ in equation (2.1) are form of non-linear integral equation& putting r = 0 in equation (1.4) becomes

$$H_0 = \frac{1}{0!}\frac{d^0}{dq^0}\left[G\left(\sum_{i=0}^{8} q^i u_i\right)\right] q = 0$$

$$H_0 = \left[G(q^0 u_0 + q^1 u_1 + q^2 u_2 + \ldots \qquad)^2\right] q = 0$$

$$H_0 = u_0^2$$

$$H_1 = 2u_0 u_1$$

If again calculation are explain in MATLAB 13 versions software in few terms are given bellow

$$H_2 = 2(u_0 u_2) + u_1^2$$

$$H_3 = 2(u_0 u_3 + u_1 u_2)$$

$$H_4 = 2(u_1 u_3 + u_0 u_4) + u_2^2$$

$$H_5 = 2(u_2 u_3 + u_1 u_4 + u_0 u_5)$$

$$H_6 = 2(u_2 u_4 + u_1 u_5 + u_0 u_6) + u_3^2$$

$$H_7 = 2(u_3 u_4 + u_2 u_5 + u_1 u_6 + u_0 u_7)$$

$$H_8 = 2(u_3 u_5 + u_2 u_6 + u_1 u_7 + u_0 u_8) + u_4^2$$

$$H_9 = 2(u_4 u_5 + u_3 u_6 + u_2 u_7 + u_1 u_8 + u_0 u_9)$$

In this polynomials we applying in this technique of equation (1.6) then we have

$$p^0 \ : \ u_0(x) = 1 - \frac{5}{12}x$$

$$p^1 \ : \ u_1(x) = \int_0^1 xtH_0 dt = x\int_0^1 tu_0^2(x) = x\int_0^1 t(1 - 0.4166x)^2 dt = 0.2657x$$

$$p^2 : u_2(x) = \int_0^1 xtH_1 dt = 2x\int_0^1 tu_0u_1 dt = 2x\int_0^1 t(1-0.4166t)(0.2657t)\,dt$$
$$= 0.1218x$$

In the same way at the tenth step we stop iteration by writing
$u(x) = 1-0.4166x+0.2657x+0.1218x+0.0735x+0.0499x+0.0431x+0.0309x+0.0243x+0.197x+0.0157x+0.0135x = 1+0.2415x$

Example 2.2.

$$u(x) = \frac{3}{4}x + \int_0^1 xtu^2(t)\,dt$$

And applying the above procedure, we have

$$p^0 \ : \ u_0(x) = 0.7500x$$
$$p^1 \ : \ u_1(x) = \int_0^1 xtu_0^2\ (t)\,dt = 0.14060x$$

In next ten step iteration we are written as
$u(x) = 0.7500x+0.1406x+0.0527x+0.0247x+0.0130x+0.0073x+0.0043x+0.0026x+0.0016x+0.0010x+0.0006566x = 0.9984566x$

Exact solution in this equation $u = x$.

Example 2.3.

$$u(x) = x + \int_0^1 xtu^2(t)\,dt$$

Exact solution in this equation $u = 2x$

Example 2.4.

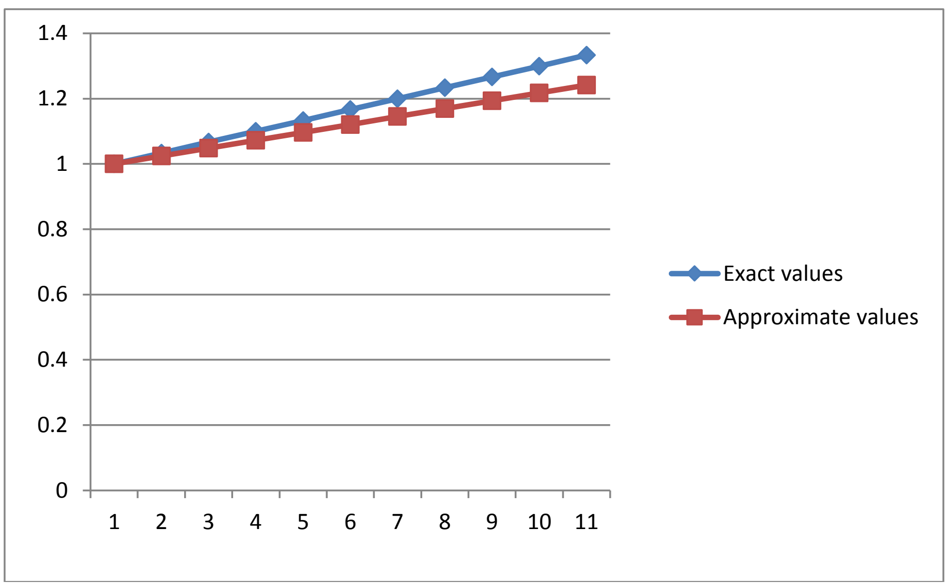
$$u(x) = 1+0.6625x+\frac{x}{20}\int_0^1 xu^2(t)\,dt$$

Exact solution in this equation $u = 1+x$

3. RESULTS AND DISCUSSION

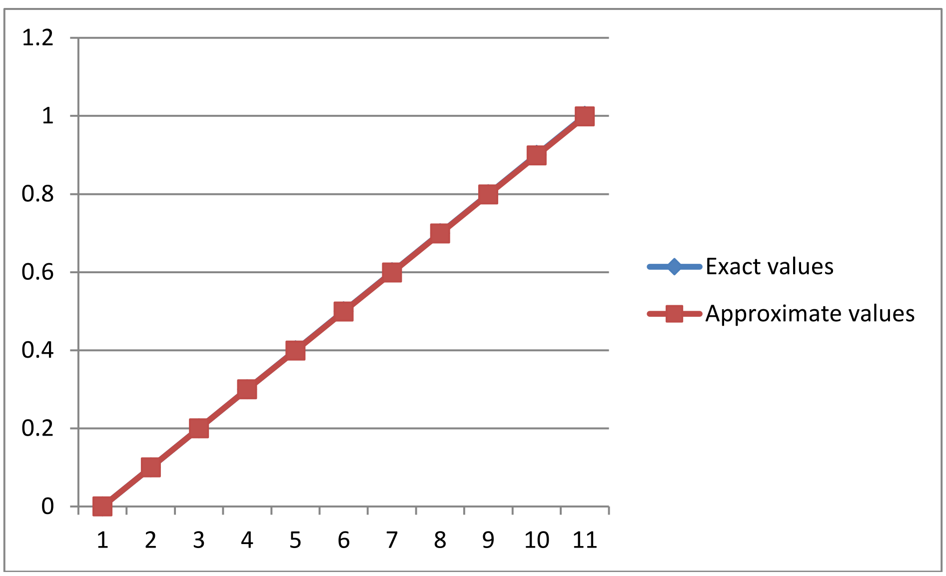
Example 3.1

Nodes(x)	Exact solutions	Approximate solutions	Absolute error
0.0	1.0000	1.00000	0.00000
0.1	1.0333	1.02415	0.00920
0.2	1.0667	1.04830	0.01840
0.3	1.0999	1.07250	0.02740
0.4	1.1332	1.09660	0.03660
0.5	1.1665	1.12070	0.04580
0.6	1.1998	1.14490	0.05490
0.7	1.2331	1.16900	0.06410
0.8	1.2664	1.19320	0.07320
0.9	1.2994	1.21730	0.08210
1.0	1.3333	1.24150	0.09180



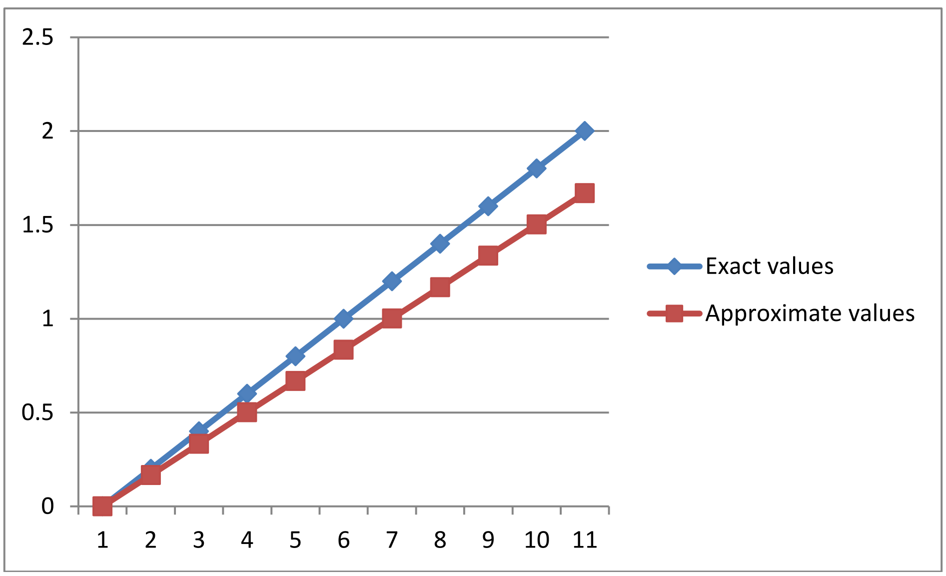
Example 3.2

Nodes(x)	Exact solutions	Approximate solutions	Absolute error
0.0	0.0	0.0000000	0.0000000
0.1	0.1	0.0998456	0.0001544
0.2	0.2	0.1996912	0.0003088
0.3	0.3	0.2995368	0.0004632
0.4	0.4	0.3993824	0.0006176
0.5	0.5	0.4992280	0.0007720
0.6	0.6	0.5990736	0.0009264
0.7	0.7	0.6989192	0.0010808
0.8	0.8	0.7987648	0.0012352
0.9	0.9	0.8986104	0.0013896
1.0	1.0	0.9984560	0.0015440



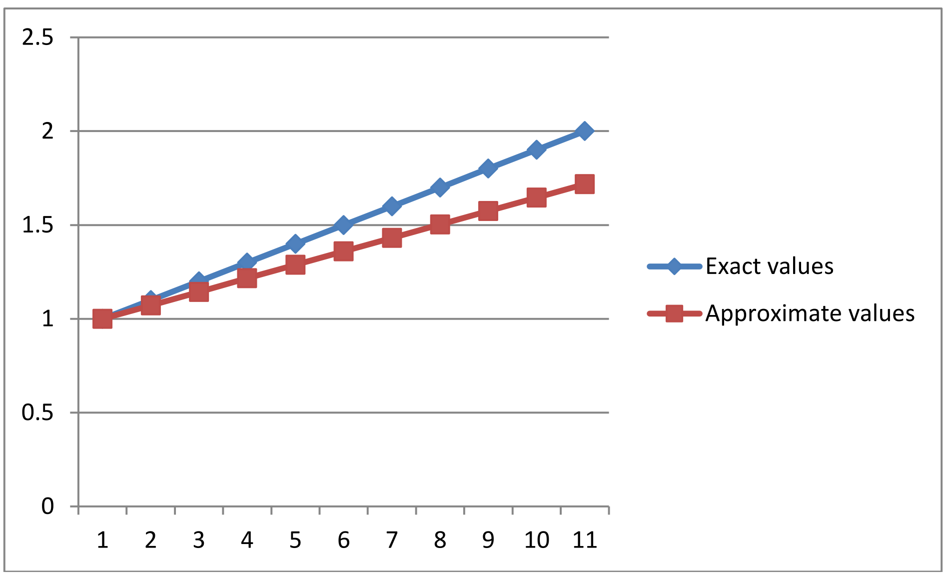
Example 3.3

Nodes	Exact solutions	Approximate solutions	Absolute error
0.0	0.0	0.00000	0.00000
0.1	0.2	0.16695	0.03305
0.2	0.4	0.33390	0.06610
0.3	0.6	0.50085	0.09915
0.4	0.8	0.66780	0.13220
0.5	1.0	0.83475	0.16525
0.6	1.2	1.00170	0.19830
0.7	1.4	1.16865	0.23135
0.8	1.6	1.33560	0.26440
0.9	1.8	1.50255	0.29745
1.0	2.0	1.66950	0.33050



Example 3.4

Nodes(x)	Exact Solutions	Approximate Solutions	Absolute Error
0.0	1.0	1.000000000	0.000000000
0.1	1.1	1.071787282	0.028212718
0.2	1.2	1.143574576	0.056425424
0.3	1.3	1.215361845	0.084638155
0.4	1.4	1.287149127	0.112850873
0.5	1.5	1.358936409	0.141063591
0.6	1.6	1.430723691	0.169276309
0.7	1.7	1.502510973	0.197489027
0.8	1.8	1.574298254	0.225701746
0.9	1.9	1.646085536	0.253914464
0.1	2.0	1.717828180	0.282171820



4. CONCLUSIONS

In this way we find that Adomian Decomposiyion method is effective and accuracy of numerical results is proved in estimated solutions with nonlinear Fredhom integral equation. All compared results were found to be more accurate & effective. This research gives new ideas for further research in the field of integral equation.

Acknowledgment. Manuscript communication number (MCN) :IU/R&D/2018-MCN000538 office of research and development integral university, Lucknow.

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