Journal of Mathematical Control Science and Applications
Vol7 No. 2 (July-December, 2021)

Submitted: 03rd August 2021 *Revised: 15th September 2021* *Accepted: 12th October 2021*

LOCATING CONNECTED DOMINATING SETS IN GRAPHS

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Abstract. Consider a finite, undirected, simple connected graph. A non – empty set $S^* \subseteq V^*$ of a graph G^* is a dominating set, if every vertex in $V^* - S^*$ is adjacent to atleast one vertex in S^*. A dominating set $S^* \subseteq V^*$ is called a locating dominating set, if for any two vertices v, w ∈ $V^* - S^*$, $N(v) \cap S^* \neq N(w) \cap S^*$. In this paper, a new dominating set namely, locating connected dominating set is introduced, its sharp bounds are studied, relation with other domination parameters are confined and certain classes of graph with equal domination number are also characterized. An algorithm for finding located dominating set for any connected graph G* is confined.

2020 Mathematics Subject Classification: 05C69
KEYWORDS: Dominating Set, Connected Dominating Set, Locating-Dominating Set, Locating-Connected dominates Set.

1. Introduction

1.1 Defining the Problem

Motivated by the safeguards analysis problem the ideas of connected dominating sets and locating were merged. For such applications a room, hallway, stairwell is considered as a vertex. Two areas that are physically adjacent are taken as an edge. **Locating Connected dominating sets** plays an eminent role in the problem of placing monitoring devices. A system consists of multiple sites, in which the monitor is placed in each site to locate the faults. Also, the monitors are adjacent to each other, so that if one monitor goes down, then the adjacent monitors still protect the system. The cost of purchasing the monitors is too high. So, our aim is to place minimum monitor motivates the concept of Locating connected dominating sets (abbreviated as LCD – sets).

1.2 Literature Survey

The main interesting area in graph theory is Dominating on graphs. Claude Berge and Ore are related with developing the study of domination and dominating set into a veritable field of study with the advent of their books 'Theory of Graphs and its Applications' and 'Theory of Graphs' respectively. In their book titled 'Fundamentals of Domination in Graphs', Haynes, Hedetniemi and Slater (1998) have the following to say about the history of domination. A new concept namely dominating set on graphs was first introduced in the year 1958 by Claude Berge. Now we are familiar with the term dominating set, domination number of graphs, it was first combined by Oystein Ore in the year 1962 and the symbol for dominating number was first introduced as $d(G^*)$. The notion $\gamma(G^*)$ the latest notion was introduced by Cakayne and Hedetniemi for the domination number of graphs.

1.3 Notions

The various notations used throughout this thesis are given below.

$G^* = (V^*, E^*)$	finite connected simple undirected graph
$V^* (G^*)$	set of vertices in G^*
$E^* (G^*)$	set of edge in G^*
p^*	cardinality of the vertex set $\|V^*(G^*)\|$
q^*	cardinality of the edge set $\|E^*(G^*)\|$
$deg_G^* (a)$	degree of the vertex a in G^*
$\delta(G^*)$	minimum degree of G^*
$\Delta(G^*)$	maximum degree of G^*
$d_G^* (a, b)$	shortest path between the vertices a and b in G^*
N(a)	open neighborhood of 'a'
N[a]	closed neighborhood of 'a'
$\bar{G}^*$	complement of G^*
P_p	Path on the number of vertices to be p^*
C_p	Cycle on the number of vertices to be p^*
K_p	Complete graph on the number of vertices to be p^*
$K_{m,n}$	Complete bipartite graph on the number of vertices to m + n
$K_{1,p-1}$	Star on p^* vertices
$S^* (r, t)$	Double star on r+ t + 2 vertices
B_k	Binomial tree of order k
$G^* - b$	an vertex b, is removed from G^*
$G^* - e$	an edge e is removed from G^*
$G_1^* \circ G_2^*$	graph obtained by the operations for corona
$<S^*>$	subgraph induced by a set $S^* \subseteq V^*$
$\gamma_D(G^*)$	domination number of the graph G^*
$\gamma_{LD}(G^*)$	locating domination number of G^*
$\gamma_{CD}(G^*)$	connected domination number of G^*

The graph considered are having the property that both the graphs G^* and $\bar{G}^*$, both are connected (that is, G^* is a doubly – connected graph).

1.4 Prior Results

Theorem 1.4.1. If G^* is a graph without isolated vertices, then the complement $V^* - S^*$ of every minimal dominating set S^* is a dominating set.

Theorem 1.4.2. A dominating set S^* is the necessary and sufficient condition for the graph G^* with domination set S^* to be minimal is that for each vertex $a \in S^*$, both the conditions holds:

(i) a is an isolated vertex in S^*;

(ii) For every vertex $b \in V^* - S^*$ there exists a vertex $a \in S^*$ such that $N(b) \cap S^* = \{a\}$

Theorem 1.4.3. For any graph G^*, $\gamma_D(G^*) \leq p^* - \Delta(G^*)$.

Theorem 1.4.4. If G^* is a graph without isolated vertices, then $\gamma_D(G^*) \leq \frac{p^*}{2}$.

Theorem 1.4.5. For any tree T^*, $\gamma_D(T^*) = p^* - \Delta(G^*) \Leftrightarrow T^*$ is a wounded spider.

Theorem 1.4.6. For any graph G^*, $\left\lceil \frac{p^*}{1+\Delta(G^*)} \right\rceil \leq \gamma_D(G^*)$.

Theorem 1.4.7. For any graph G^*, $p^* - q^* \leq \gamma_D(G^*)$.

Furthermore, $\gamma_D(G^*) = p^* - q^*$ if and only if each component of G^* is a star.

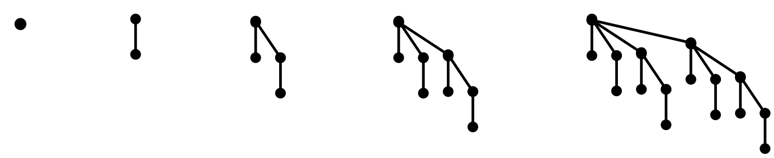
Theorem 1.4.8. Let S^* be a Locating dominating set of G^*. If $|S^*|=k$, then $|V^* - S^*| \leq 2^k - 1$.

Definition 1.4.9. Consider a rooted tree B_k, with $k>0$ then B_k is Binomial if

(i) If $k = 0$, then $B_k = B_0 = \{R\}$.

(ii) If $k \geq 0$ then $B_k = \bigcup_{i=0}^{k-1} \{B_i\}$. That is, the binomial tree B_k of order $k \geq 0$ comprises the root and the $k - 1$ binomial subtrees.

The first five binomial subtrees are shown in Figure 1.4.10.



B_0 B_1 B_2 B_3 B_4

Figure 1.4.10

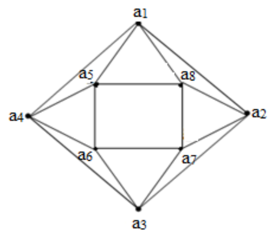
The paper comprises of 5 vertices. In Section 2.1., locating – connected domination number is defined. A bound for locating- connected domination number is obtained in Section 2.2. Further in Section 2.3., relation with other domination parameters are established. In Section 2.4., some results on trees have been discussed. In Section 2.5., an algorithm for finding locating dominating set for the connected graph G^* are confined.

2. Main Results

We introduce a new concept called Locating connected domination number.

2.1 Locating Domination Number

Definition 2.1.1. A connected dominating set of a graph G^* is called a locating-connected dominating set abbreviated LCD − set, in G^* with the property that distinct



G_1

Figure 2.1.3.

vertices in V^*-S^* are co … y distinct subset of S^*. The minimum cardinality of a locating … g set of the graph G^* is called the locating connected domi… …d by $\gamma_{LCD}(G^*)$.

Illustration 2.1.2:

For the graph G_1^* given in Figure 2.1.3.,

$N(a_1) = \{a_2, a_4, a_5, a_8\}$; $N(a_3) = \{a_2, a_4, a_6, a_7\}$

$S1^*=\{a_1, a_3\}$ is a minimum dominating set ,Since $N[S1^*]=V^*(G^*)$

Therefore, $\gamma_D(G^*)=2$

Also $S2^*=\{a_1, a_2, a_3\}$ is a minimum connected dominating set.

Since $N[S2^*] \cong P_3$ is connected. Hence $\gamma_{CD}(G^*) = 3$

Again S2 is a minimum locating connected dominating set.

Since for $V^*-S^*=\{a_4, a_5, a_6, a_7, a_8\}$

$N(a_4) \cap S=\{a_1, a_3\}$; $N(a_5) \cap S=\{a_1\}$; $N(a_7) \cap S=\{a_2, a_3\}$;

$N(a_6) \cap S=\{ a_3\}$; $N(a_8) \cap S=\{ a_1, a_2\}$.

From this it follows that, each vertex in V^*-S^* has distinct neighbors.

Therefore $\gamma_{LCD}(G^*)=3$.

2.2 Bounds on Locating connected Domination number

Proposition 2. 2.1.

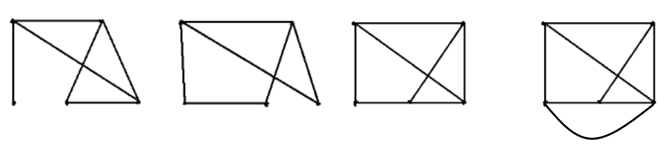
For a connected graph G^*, $1\leq \gamma_{LCD}(G^*) \leq p^* - 1$

Upper bound is satisfied for $G^*\cong K_1$. Lower bound is satisfied for $G^* \cong K_2$.

Theorem 2.2.2.

For a connected graph G^*, $\gamma_{LCD}(G^*) = 2$ if and only if G^* is any one of the graphs given graphs C_3, P_4, C_4, $C_3 + e$, $K_4 - e$, Bull graph or

$G_2^* \cong$



(a) (b) (c) (d)

Figure 2.2.3

Proof.

If G^* is any one of the graph mentioned in Theorem 2.2.2., then it is obvious that

$\gamma_{LCD}(G^*) = 2$

For the sufficient part,

Let $\gamma_{LCD}(G^*) = 2$, then to prove that G^* is isomorphic to any one of the graphs.

Let $S^* = \{b_1, b_2\}$ be the minimum locating connected dominating set of G^*.

Then $S^* \cong K_2$,

By Theorem1.4.8., $|V^*-S^*| \leq 2^2 - 1 = 3$,

Case 1: $|V^*-S^*| = 1$

Let $<V^*-S^*> =1$

Let a_1 be adjacent to b_1

Then $G^* \cong P_3$

If a is adjacent to both b_1 and b_2 then $G^* \cong C_3$

Case 2: $|V^*-S^*| = 2$

Let $V^*-S^* = \{a_1, a_2\}$

If a_1 is adjacent to b_1 and

b_2 is adjacent to b_3, then $G^* \cong P_4$

In addition, if a_1 and a_2 are adjacent then $G^* \cong C_4$.

If a_1 is adjacent to b_1 and

a_2 is adjacent to both b_1 and b_2, then $G^* \cong C_3+e$

In addition, if a_1 and a_2 are adjacent, then $G^* \cong K_4 - e$

Case 3: $|V^*-S^*| = 3$

Let $V^*-S^* = \{a_1, a_2, a_3\}$

If a_1 is adjacent to b_1

a_2 is adjacent to b_2

a_3 is adjacent to both b_1 and b_2

Then, $G^* \cong$ Bull graph

In addition, if a_1 and a_2 are adjacent, then G^* is isomorphic to the graph given in Figure2.2.3. (b)

If $< V^*-S^*> \cong P_3$, then G^* is isomorphic to the graph given in Figure 2.2.3. (c)

If $< V^*-S^* > \cong C_3$, then G^* is isomorphic to the graph given in Figure 2.2.3. (d)

If a_2 and a_3 are adjacent, then G^* is isomorphic to the graph given in Figure 2.2.3. (a)

Observation 2.2.4.

i. For a path, $\gamma_{LCD}(P_p) = p^* - 2;\ p^* \geq 2$
ii. For a cycle, $\gamma_{LCD}(C_p) = p^* - 2;\ p^* \geq 4$
iii. For a complete graph, $\gamma_{LCD}(K_p) = p^* - 1,\ p^* \geq 4$
iv. For a Bipartite graph, $\gamma_{LCD}(K_{m,n}) = m + n - 2,\ m, n \geq 2$
v. For a double star, $\gamma_{LCD}(S^*_{r,t}) = r + t;\ r + t \geq 2$.

2.3 Relation with other parameters

Theorem 2.3.1. For any connected simple graph G^*, $\gamma_D(G^*) \leq \gamma_{CD}(G^*) \leq \gamma_{LCD}(G^*)$

Proof. The result follows from the fact that

i. Every connected Dominating set is a Dominating set.

ii. Every Locating connected Dominating set is a connected Dominating set.

Definition 2.3.2. A new graph called thorn graph was introduced by Gutman, for any graph connected graph G^* a new graph G^{**} is obtained by attaching the pendant edge $p_i>0$ to the vertices of G^*.

Theorem 2.3.3. There exists a graph G^*, for which

$\gamma_D(G^*) = \gamma_{CD}(G^*) = \gamma_{LCD}(G^*)=p^*$, for any positive integer p^*, with $p^*\geq 2$.

Proof. Consider any connected graph G^* with $p^* \geq 2$ vertices.

Let G_1^{**} be the thorn graph with exactly one pendent edge at each vertex.

Let S^* be a dominating set of G_1^{**}, then S^* will contain all the support vertices that is the vertices of the graph G^* to dominate the pendent vertices.

Therefore, $\gamma_D(G_1^*) = |S^*| = |V^*(G^*)| = p^*$

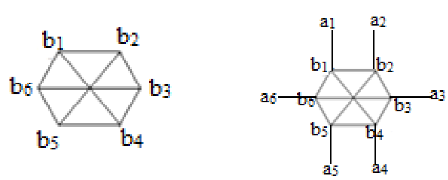
Also S^* is a connected dominating set of G^*, Since G^* is a connected Graph and S^* contain all the vertices of G. Hence $\gamma_D(G_1^{**}) = \gamma_{CD}(G_1^{**}) = p^*$

All the pendant vertices of G_1^{**} will have distinct neighbours in S^*. Therefore, S^* is also a LCD- set of G_1^{**}.

Thus $\gamma(G_1^{**}) = \gamma_{CD}(G_1^{**}) =\gamma_{LCD}(G_1^{**})=p^*$; $p^* \geq 2$.

Illustration 2.3.4.

Consider the cubic graph G_3^*,



G_3 G_3^{**}

Figure 2.3.5.

For the thorn graph G_3^{**} given in Figure 2.3.5.,

$S^* = \{b_1, b_2, b_3, b_4, b_5, b_6\}$ be a γ_D – set.

S^* is also a minimum LCD-set of G_3^{**}

Therefore $\gamma_D(G_3^{**})= \gamma_{CD}(G_3^{**})= \gamma_{LCD}(G_3^{**}) = |V^*(G_3^*)|=6$

Corollary 2.3.6.

For any two integers $a \geq 2$, $b \geq 3$ there exists a connected graph G^* which satisfy

$a = \gamma_D(G^*) = \gamma_{CD}(G^*) <\gamma_{LCD}(G^*) < b$

Proof.

Let G_1^{**} be the thorn graph of a connected graph G in which atleast one pendant vertex.

According to the Theorem 2.3.3,

$\gamma_D(G^*) = \gamma_{CD}(G^*) = |V^*(G^*)| = a$ (consider)

For a LD – set S^*, if a vertex is attached to 't' pendant vertices then (t-1) pendant vertices must be include in S^*. Hence $\gamma_{LCD}(G^*) >\gamma_{CD}(G^*)$

This completes the proof.

Theorem 2.3.7. There exists graph, G^* with the property that $\gamma_D(G^*) < \gamma_{LD}(G^*) < \gamma_{CD}(G^*)$ where G^* is a connected graph.

Proof.

Consider a connected graph G^{**} with vertex set $V^*(K_p) = \{b_1, b_2, \ldots\ldots b_p\}$

Let G^* be a graph obtained by attaching a path of length two to each vertex of G^{**}

Let $V^*(G) = \{b_1, b_2, \ldots, b_p, a_1, a_2, \ldots, a_p, c_1, c_2, \ldots, c_p\}$

Here $b_i a_i c_i$ is a path of length two for each $1 \le i \le p$.

c_i's are the pendant vertices.

a_i's are vertices of degree two.

The set $S^* = \bigcup_{i=1}^{p*} \{a_i\}$ will form a dominating set, Thus $\gamma_D(G^*) = |S^*| = p^*$.

Also $N(b_i) \cap S^* = N(c_i) \cap S^* = \{a_i\}$, for all $1 \le i \le n$.

Therefore the set $S1 = S^* \cup \{b_1\}$ will form a Locating dominating set.

Since $N(b_i) \cap S^* = \{b_i, a_i\}$; $1 \le i \le n$

$N(c_i) \cap S^* = \{a_i\}$; $1 \le i \le n$

Therefore $\gamma_{LD}(G^*) = |S1^*| = p^* + 1$

$S1^*$ is not connected, it contains a path $P_1(b_1, a_1)$ and $p^* - 2$ is isolated vertices $\{a_2, a_3, \ldots, a_{p-1}\}$.

The set $S2^* = \bigcup_{i=1}^{p*} \{a_i\} \cup \bigcup_{i=1}^{p*} \{b_i\}$ will for a connected dominating set of G^*.

Hence $\gamma_{CD}(G^*) = |S2^*| = p^* + p^* = 2p^*$.

This implies that, $\gamma_D(G^*) < \gamma_{LD}(G^*) < \gamma_{CD}(G^*)$ for the graph G^*.

Corollary 2.3.8.

For any tree T^* with each support has exactly one pendant vertex, $\gamma_{LD}(T^*) \le \gamma_{CD}(T^*)$

Theorem 2.3.9.

There exists a graph G^*, for which $\gamma_{CD}(G^*) \le \gamma_{LD}(G^*)$

Proof.

Consider any connected graph G^{**} with vertex set $V(G^{**}) = \{b_1, b_2, \ldots\ldots b_k\}$

Let $G^* \cong G^{**} \circ K_2$, then

$V^*(G^*) = V^*(G^{**}) \cup \{a_1, a_2, \ldots. a_k\} \cup \{c_1, c_2, \ldots\ldots c_k\}$ where each $\{c_i a_i\}$ is a path, for $1 \le i \le k$.

The set $S^* = V^*(G^{**})$ is a dominating set of G^*.

Since G^{**} is connected, it is also a connected dominating set of G^*.

Therefore $\gamma_D(G^*) = \gamma_{CD}(G^*) = |S^*| = |V(G^{**})| = k$

$V^* - S^* = \{a_1, a_2, \ldots., a_k, c_1, c_2, \ldots, c_k\}$

$N(a_i) \cap S^* = N(c_i) \cap S^* = \{b_i\}$, $\forall\ 1 \le i \le k$

Hence S^* is not a LD – set of G^*.

Any one of a_i's or c_i's should be include to form a LD – set.

Let $S1^* = S^* \cup \bigcup_{i=1}^{k} \{a_i\}$. Then $V^* - S1^* = \{c_1, c_2, \ldots, c_k\}$ and $N(c_i) \cap S1^* = \{a_i, b_i\}$ for $1 \le i \le k$.

Thus $S1^*$ is a LD – set. Hence $\gamma_{CD}(G^*) < \gamma_{LCD}(G^*)$.

2.3 Results on Acyclic Graphs

Theorem 2.4.1.

There exists a tree T^*, with $\gamma_{LCD}(T^*) = p^* - l^*$ where $p^* = |V^*(T)|$ and l^* denote the number of leaves.

Proof:

Let T^* be a connected tree, with each support has exactly one leaf and there are no intermediate vertices, then $\gamma_{LCD}(T^*) = p^* - l^*$.

Here $|V^*(T^*)| = p^*$; s^* is the number of supports and l^* = number of leaves

But then $p^* = s^* + l^*$.

To dominate the leaves all the support are included in the dominating set.

Since there are no intermediate vertices, the dominating set will also be a CD – set. In addition, each leaf has distinct supports.

Hence the dominating set will also be a locating dominating set.

Thus the support vertices will form a minimum LCD – set.

Thus implies, $\gamma_{LCD}(T^*) = p^* - l^*$.

Example 2.4.2. For a Binomial tree B_3 given in Figure 3.1.10,

$\gamma_{LCD}(T^*) = p^* - l^*$

$= 8 - 4$

$\gamma_{LCD}(T^*) = 4$.

Corollary 2.4.3.

For a caterpillar T^*, with exactly one leaf for each support, $\gamma_{LCD}(T^*) = p^* - l^*$

Proof.

For the tree T^*, mentioned in corollary can leave intermediate vertices, those vertices are also be include in the γ_{LCD} – set to be a connected domination set.

Therefore $\gamma_{LCD}(T^*) = p^* - l^*$.

Example 2.4.4.

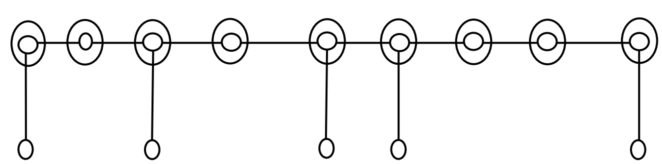


Figure 2.4.5.

Here $p^*=14$, $l^*=5$.

$\gamma_{LCD}(T^*) = p^* - l^* = 14 - 5 = 9$.

2.4 Algorithm for finding the LCD – set of G^*

Let S^* be LCD – set of G^*

Step 1: Set $S^* = \phi$

Step 2: There exists a diametral path in G^* and let A^* = {vertices in the diametral path} set

$S^* = S^* \cup A^*$

Step 3: Find the induced subgraph of $[S^*]$.

Step 4: If any two vertices in $V^* - S^*$ have same neighbour in S^* namely a and b then take an arbitrary vertex a include to S^*. That is, $S^* = S^* \cup \{a\}$.

Step 5: If all the vertices in $V^* - S^*$ have distinct neighbours in S^* and $[N(S^*)] \cong G^*$, then S^* will form a LCD – set of G^*. Go to Step 8.

Step 6: If $[N(S^*)] \ncong G^*$, then there exists a vertex $a \in V^* - S^*$ and $b \in S^*$ such that $ab \in E^*(G^*)$.

Include a in S^*.

That is set, $S^* = S^* \cup \{a\}$. Go to Step 4.

Step 7: If $ab \in E^*(G)$ with $a,b \in V^* - S^*$ then there are two cases

Case i) $N(a) \in S^*$

Include a in S^* and Go to Step 4.

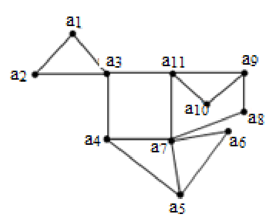
Case ii) $N(a) \notin S^*$

Then there exist a vertex $c \in V^*(G^*)$ such that $ac \in E^*(G)$, since G^* is connected.

Include c in S^* and Go to Step 4.

Step 8: Stop.

Illustration 2.5.1.



$G^* \cong$

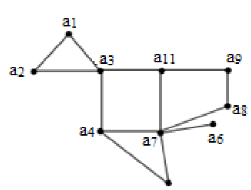
Figure 2.5.2.

Step 1: $S^* = \phi$

Step 2: $\{a_1, a_3, a_4, a_7, a_8\}$ is one of the diametral path.

Set $S^* = \{a_{1.}, a_3, a_4, a_7, a_8\}$.

Step 3:



$[S^*] \cong$

Figure 2.5.3.

Step 4:

$N(a_2) \cap S^* = \{a_1, a_3\}$

$N(a_5) \cap S^* = \{a_4, a_7\}$

$N(a_6) \cap S^* = \{a_7\}$

$N(a_9) \cap S^* = \{a_8\}$

$N(a_{11}) \cap S^* = \{a_3\}$

Thus the vertices $\{a_2, a_5, a_6, a_9, a_{11}\}$ have distinct neighbours.

Step 5: But $[N(S^*)] \ncong G^*$.

Step 6: $S^* = S^* \cup \{a_9\} = \{a_1, a_3, a_4, a_7, a_8, a_9\}$

Step 4: $N(a_{10}) \cap S^* = \{a_9\}$

$N(a_{11}) \cap S^* = \{a_{10}\}$

Thus the vertices in complement of S^* has distinct neighbours.

Step 5: Also $N[S^*] \cong G^*$.

Thus $S^* = \{a_1, a_3, a_{4,} {}_{a7}, a_8, a_9\}$ will form a LCD – set of G^*.

For the graph G^* shown Figure 2.5.2., $\gamma_{LCD}(G^*) = 6$.

Theorem 2.5.4.

The set S^* found using the Algorithm is a LCD – set of G^*.

Proof.

From Step 2 and Step 4, it is understand that S^* is a connected set

Step 4, assure that S^* is a locating set.

From Step 6, it follows that S^* is a dominating set.

Thus the set S^* constructed using the Algorithm will form a LCD – set of G^*.

Remark 2.5.5.

The set S^* constructed using the Algorithm is not necessarily a minimum locating connected dominating set. By choosing the Arbitrary vertex in Step 5, will generate different LCD – sets. By trial-and-error method we can obtain the minimum LCD – set.

3. Conclusion

A new domination parameter called Locating Connected domination number is defined on connected graphs and its bounds are also studied. Further certain classes of graphs satisfying the relation with other domination parameters are classified. Exact bounds for $\gamma_{LCD}(G^*)$ relating the maximum and minimum degree of the graph is an open problem. Studying the properties of Locating connected domination number on cubic graphs and unicyclic graphs are considered to be the future work.

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