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HARARY ENERGY OF COMPLETE, COMPLETE BIPARTITE AND COMPLETE TRIPARTITE GRAPHS

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Abstract. The harary energy of the graphs are found on the basic of the energy concepts in graphs. The condition of the harary energy is the matrix of a graph, H(G) (with no isolated vertices) is defined as $\frac{1}{d(v_i,v_j)}$ $,if\ i \neq j$ and zero otherwise, where $d(v_i, v_j)$ stands for the distance between the vertices. Here the distance energy of the complete with bipartite and tripartite graphs are explained.

KEYWORDS: Harary energy , complete with bipartite and tripartite graphs.

1. Introduction

Zadeh [28] have initiated fuzzy sets [30] [31] [32]. Parvathi and Karunambigai[13] have initiated the idea of Intuitionistic Fuzzy Graphs (IFGs). Gani and Begum [5] talked about the extension of fuzzy graphs. Products in IFGs were discussed by Sahoo & Pal [17]. Sahoo and Pal [18,19] studied some types of fuzzy graphs. Sahoo et al [21] initatied new ideas in intuitionistic fuzzy graphs. Kalaiarasi and Mahalakshmi have also expressed fuzzy strong graphs [8]. Shanmugavadivu and Gopinath, suggested non homogeneous ternary five degrees equation [24]. Shanmugavadivu and Gopinath, have also expressed on the homogeneous five degree equation [25], Bozhenyuk et al[2] has talked about dominating set and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [29].

Ore and Berge introduced the concept of domination in 1962. Cockayne and Hedetniemi have further studied about domination in graphs [6]. Somasundaram and Somasundaram have initiated domination in fuzzy graphs by making use of effective edges [23]. Xavior et al. [27] has talked about domination in fuzzy graphs but differently. Dharmalingam and Nithya have also expressed domination parameters for fuzzy graphs [3]. Equitable domination number for fuzzy graphs was introduced by Revathi and Harinarayaman in [16]. Sarala and Kavitha have also expressed (1,2)-domination for fuzzy graphs [22]. Gani and Chandrasekaran have talked about strong arcs[12]. Sunitha and Manjusha have also expressed strong domination [26]. Kalaiarasi and Mahalakshmi have also expressedfuzzy inventory EOQ optimization mathematical model [9]. Kalaiarasi and Gopinath suggested fuzzy inventory order EOQ model with machine learning [10]. Fuzzy Incidence Graphs (FIGS) discussed by Dinesh [4]. Mordeson talked about incidence cuts in FIGS [11]. Priyadharshini et al.[18] have also expressed a fuzzy MCDM approach for measuring the business impact of employee selection [15].

The concept of energy originated in chemistry. The energy of G was first defined by Gutman in 1978 as the sum of the absolute values of the eigen values of A[G]. According to R.B. Bapat and Sakanta Pati, if the energy of a graph is rational then it must be an even integer. In the same year Shanmugavadivu & R.Gopinath gave some numerical ideas degree of equations. Kalaiarasi & R.Gopinath analysed and introduced Fuzzy inventory and arc sequences in different graphs and explained the join product in mixed split IFGs. In 2020 Priyadharshini & R.Gopinath explained some ideas in fuzzy MCDM approach for measuring the business. An important result is that the energy of the graph is greater than the number of vertices of the graph. Here the harary energy of complete graphs with bipartite and tripartite are explained.

2. Preliminaries

Definition:2.1

A *linear graph* or *a graph* G=(V,E) consists of a set of objects V={ $v_1,v_2,\ldots v_n$} called vertices and another set E={ $e_1,e_2,\ldots e_n$} whose elements are called edges, such that each e_k, is identified with an unordered pair (v_i,v_j) of vertices.

Definition:2.2

A graph that has neither self loops nor parallel edges is called *a simple graph.*

Definition:2.3

A graph is said to be *connected* if there is atleast one path between every pair of vertices in G.

Definition:2.4

A complete graph is a simple undirected graph in which every pair of distinct vertices is connected by a unique edge.

Definition:2.5

A complete bipartite graph is a bipartite graph((i.e.,) a set of graph vertices decomposed into two disjoint sets such that no two graph vertices within the same set are adjacent) such that every pair of graph vertices in the two sets are adjacent.

Definition:2.6

A complete tripartite graph is a tripartite graph((i.e.,) a set of graph vertices decomposed into three disjoint sets such that no two graph vertices within the same set are adjacent) such that every vertex of each set of graph vertices is adjacent to every vertex in the other two sets.

3. Harary Energy of the Complete, Complete Bipartite and Complete Tripartite Graphs

Definition 3.1

The harary matrix of a graph(with no isolated vertices),is defined as $H_{ij} = \begin{cases} \frac{1}{d(v_i,v_j)}, if\ i \neq j \\ 0, otherwise \end{cases}$ where $d(v_i, v_j)$ stands for the distance between the vertices.

Definition:3.2

Let $\lambda_1,\lambda_2,\ldots\lambda_n$ be the eigen values of H(G) then its energy HE(G) is defined as the sum of the absolute eigen values

4. Main Results

Theorem 4.1

The harary energy of the complete graphs K_n of order n≥3 is n.

Proof

Let K_n be a complete graph of order n≥3.

To Prove: The harary energy of K_n is n.

To calculate the harary energy, we have to construct the harary matrix.

By the definition,

The harary matrix of a graph(with no isolated vertices),is defined as $H_{ij} = \begin{cases} \frac{1}{d(v_i,v_j)}, if\ i \neq j \\ 0, otherwise \end{cases}$ where $d(v_i, v_j)$ stands for the distance between the vertices.

The harary matrix of K_n is a n×n identity matrix

For this matrix, the eigen values are n times of one.

By the definition

The harary energy is defined as

K_3	3
K_4	4
K_5	5
K_6	6
K_7	7
K_8	8

Therefore,

HE(G)=$|1 + 1 + \cdots . +1(n\ times)|$

=$|n|$

HE(G)=n

Hence it is proved.

5. Energy Relations

Theorem 5.1

The harary energy of the complete bipartite graphs $K_{n,n}$, n≥2 is 2n.

Proof

Let $K_{n,n}$ be a complete bipartite graph of order n≥2.

To Prove: The harary energy of $K_{n,n}$ is 2n.

To calculate the harary energy, we have to construct the harary matrix.

By the definition,

The harary matrix of a graph(with no isolated vertices),is defined as $H_{ij} = \begin{cases} \frac{1}{d(v_i,v_j)}, if\ i \neq j \\ 0, otherwise \end{cases}$ where $d(v_i, v_j)$ stands for the distance between the vertices.

The harary matrix of $K_{n,n}$ is a 2n×2n identity matrix

For this matrix, the eigen values are 2n times of one.

By the definition

The harary energy is defined as

$HE(G) = \sum_{i=1}^{n} |\lambda_i|$

Therefore,

HE(G)=$|1 + 1 + \cdots . +1(2n\ times)|$

=$|2n|$

HE(G)=2n

Hence it is proved.

6. Energy Relations

The edge energy relation shows that there is only 14% of the interaction and relationship between the two clinics, and due to financial issues, there is 86% on the conflict between them. energy relation ruling arrangements of the energy relation graph is the arrangement of clinics which give the crisis treatment autonomously. Along these lines, we can save the time of energy relation and conquer the long going of patients by giving the couple of offices to the remainder of energy relation.

Name of the graph	Harary Energy of $K_{n,n}$
$K_{2,2}$	4
$K_{3,3}$	6
$K_{4,4}$	8
$K_{5,5}$	10
$K_{6,6}$	12

Theorem 6.1:

The harary energy of the complete tripartite graphs $K_{n,n,n}$, n≥2 is 3n.

Proof:

Let $K_{n,n,n}$ be the complete tripartite graph with n≥2.

To Prove: The harary energy of $K_{n,n,n}$ is 2n.

To calculate the harary energy, we have to construct the harary matrix.

By the definition,

The harary matrix of a graph(with no isolated vertices),is defined as $H_{ij} = \begin{cases} \frac{1}{d(v_i,v_j)}, if\ i \neq j \\ 0, otherwise \end{cases}$ where $d(v_i, v_j)$ stands for the distance between the vertices.

The harary matrix of $K_{n,n}$ is a 3n×3n identity matrix

For this matrix, the eigen values are 3n times of one.

By the definition

The harary energy is defined as

$HE(G) = \sum_{i=1}^{n} |\lambda_i|$

Therefore, HE(G) =$|1 + 1 + \cdots . +1(3n\ times)|$

=$|3n|$

HE(G)=3n

Hence it is proved.

6. Energy Relations

The edge $h_{11}h_{22}(0.14,0.86)$ shows that there is only 14% of the interaction and relationship between the two clinics, and due to financial issues, there is 86% on the conflict between them. Energy relation ruling arrangements of the graph is the arrangement of clinics which give the crisis treatment autonomously in energy relation. Along these lines, we can save the time of patients and conquer the long going of patients by giving the couple of offices to the remainder of the clinics.

Name of the graph	Harary Energy of Kn,n
K2,2	4
K3,3	6
K4,4	8
K5,5	10
K6,6	12

Theorem 6.1

The harary energy of the complete tripartite graphs $K_{n,n,n}$, n≥2 is 3n.

Proof

Let $K_{n,n,n}$ be the complete tripartite graph with n≥2.

To Prove: The harary energy of $K_{n,n,n}$ is 2n.

To calculate the harary energy, we have to construct the harary matrix.

By the definition,

The harary matrix of a graph(with no isolated vertices),is defined as $H_{ij} = \begin{cases} \frac{1}{d(v_i,v_j)}, if\ i \neq j \\ 0, otherwise \end{cases}$ where $d(v_i, v_j)$ stands for the distance between the vertices.

The harary matrix of $K_{n,n}$ is a 3n×3n identity matrix

For this matrix, the eigen values are 3n times of one.

By the definition

The harary energy is defined as

$HE(G) = \sum_{i=1}^{n} |\lambda_i|$

Therefore, HE(G) $= |1 + 1 + \cdots . +1(3n\ times)|$

$= |3n|$

HE(G)=3n

Hence it is proved.

7. Energy Relations

The edge $h_{11}h_{22}(0.14,0.86)$ shows that there is only 14% of the interaction and relationship between the two clinics, and due to financial issues, there is 86% on the conflict between them. Energy relation ruling arrangements of the graph is the arrangement of clinics which give the crisis treatment autonomously in energy relation. Along these lines, we can save the time of patients and conquer the long going of patients by giving the couple of offices to the remainder of the clinics.

Name of the graph	Harary Energy of Kn,n

$K_{2,2,2}$	6
$K_{3,3,3}$	9
$K_{4,4,4}$	12
$K_{5,5,5}$	15
$K_{6,6,6}$	18

9. Conclusion

The idea of domination graphs is imperative from religious just as an applications perspective. In this paper, the possibility of complete domination graphs fuzzy incidence graph, strong and weak intuitionistic fuzzy incidence dominating set and strong and weak intuitionistic fuzzy incidence domination number is talked about. Further work on these thoughts will be accounted for in impending papers.

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