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FUZZY MEAN SQUARE SUM SOFT TRI-PARTITE GRAPHS AND ITS COMPLEMENT

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Abstract. Some new Fuzzy Mean SquareSumSoft Tri-partite Graphs (FMSSTG) and complement of Fuzzy Mean Square SumSoft Tri-partite Graphs (FMSSTG) concepts in this work (CFMSSTG). At some of their properties anda few results that are relevant to these notions. Some fundamental theorems and their applications will be examined.

KEYWORDS: Fuzzy Mean Square Sum Soft Tri-partite Graphs (FMSSTG), Complement of Fuzzy Mean Square Sum Soft Tri-partite Graphs (CFMSSTG).

1. Introduction

Zadeh [31] [33] [34] [35] introduced the notion of fuzzy sets in 1965, and it is the most widely used strategy for dealing with uncertainty. However, setting the membership function in each situation is inherently complicated. Kauffman [15] proposed the first concept of fuzzy graphs in 1973. Rosenfeld developed fuzzy graph theory in 1975[24]. At the same time, Yeh *et al.* proposed numerous concepts in connectedness with fuzzy graphs. Feng *et al.* later combined soft set with fuzzy set and rough set. Bhattacharya[6] looked at fuzzy graphs in 1987 and came up with some interesting findings. In fuzzy graphs, a large number of academics have created various sustainable and noteworthy notions. In 1994, Moderson established the notion of the complement of fuzzy graphs. Molodtsov [20] introduced the soft set concept in 1999, which can be thought of as a new mathematical theory for dealing with uncertainty. The soft set theory has been successfully applied to a wide range of applications. In 2001, Maji [16] worked on a theoretical analysis of soft sets. The algebraic structure of soft set theory that deals with uncertainty has also been investigated further. The development of fuzzy soft set was then discussed by B.Ahmadet.al.[5] in 2009. The next year, in 2010, P. Majumdar *et al.*[17] proposed some generalised fuzzy soft set notions, and Xu.W *et al.*[30] introduced some fuzzy soft set ideas. Neog TJ *et al.*[21] discussed the complement of fuzzy soft sets in 2012. Thumbakara *et al.*[29] expanded the fuzzy soft sets to fuzzy soft graphs in 2014. Sumit Mohinda *et al.* [19] introduced the concept of fuzzy soft graphs, and Muhammed Akram *et al.* [1-3] introduced different types of fuzzy soft graphs and their properties in 2015 and 2016. New concepts and relationships among fuzzy soft graphs are introduced by Masarwah AA *et al.*[18] also Alcantud *et al.*[4] explained the same concepts. In 2018, K.Hayat *et al.*[20-22] introduced type 2 fuzzy soft sets and their characterizations .Kalaiarasi & Geethanjali investigated and Kalaiarasi & Gopinath introduced arc sequences in various graphs, as well as explained the join product in mixed split IFGs. In 2020 Priyadharshini *et al.*

explained some ideas in fuzzy MCDM approach for measuring the business .S.Shashikala *et al.* [26] discussed the concepts of fuzzy soft cycles in fuzzy soft graphs in 2019. M.A.Rashid *et al.*[23] developed fuzzy graphs to total uniform fuzzy soft graphs in 2020.In the same year Shanmugavadivu and Gopinath, gave some numerical ideas degree of equations and Mapreduce Methodology for Elliptical Curve Discrete Logarithmic Problems [32].

The remainder of this document is organised as follows: The principles of soft tripartite graphs are discussed in Section 2. The proposed fuzzy mean square sum soft tri-partite graphs for this study are described in Section 3.Section 4 contains a detailed experimental setup as well as a complement of fuzzy mean square sum soft tri-partite graphs, and we examined some fundamental theorems with examples. Section 5 describes the applications, and Section 6 concludes by discussing the article's future scope.

2. Preliminaries

Definition 2.1 [2]A fuzzy graph is an ordered triple $G_F(V_F, \sigma_F, \mu_F)$ where V_F is a set of vertices $\{u_{F_1}, u_{F_2}, ..., u_{F_n}\}$ and σ_F is a fuzzy subset of V_F that is $\sigma_F : V_F \to [0,1]$ and is denoted by $\sigma_F = \{(u_{F_1}, \sigma_F(u_{F_1})), (u_{F_2}, \sigma(u_{F_2})), ..., (u_{F_n}, \sigma(u_{Fn}))\}$ and μ_F is a fuzzy relation on σ_F.

Definition 2.2[12]Let $V = \{x_1, x_2, ...x_n\}$ non empty set. E (parameters set) and $A \subseteq E$ also let

(i) $\rho : A \to F(V)$ (collection of all fuzzy subsets in V) $e \to \rho(e) = \rho_e$ (say)and $\rho_e : V \to [0,1], X :\to \rho_e(X_i)$ (A, ρ) : *Fuzzy soft vertex.*

(ii) $\mu : A \to F(V \times V)$ (collection of all fuzzy subsets in $(V \times V)$ $e \to \mu(e) = \mu_e$ (say)

and $\mu_e : v \times v \to [0,1]$ $(x_i, x_j) \to \mu_e(x_i, x_j)$ (A, μ) : *Fuzzy soft edge*.Then $((A, \rho), (A, \mu))$ is called a *fuzzy soft graph*iff $\mu_e(x_i, x_j) \le \rho_e(x_i) \wedge \rho_e(x_j)$ for all $e \in A$ and for all $i, j = 1,2,....,n$ and this fuzzy soft graph is denoted by $G_{A,V}$.

Definition 2.3[18]A fuzzy soft graph $G_{A,V} = ((A, \rho), (A, \mu))$ is said be a fuzzy soft Bi-partite graph. If the vertex set V is partition into two disjoint vertex pair and $\mu_e(x_i, x_j) = \rho_e(x_i) \wedge \rho_e(x_j)$ for all $x_i \in v_i$ and $y_j \in v_j$

Definition 2.4[18]If a fuzzy soft graph $G_{A,V} = ((A,\rho),(A,\mu))$ is said be a fuzzy soft Bi-partite graph, then Size of Fuzzy soft bipartite graphis

$$S(G_{A,V\cup v_{j_i}}) = \sum_{e\in A}\Big(\sum_{x_i y_j \in V_i \cup v_j}\mu_e(x_i, x_j)\Big)$$

3. Fuzzy Mean Square Sum Soft Tri-Partite Graphs and Its Complement

In this section, we'll examine at certain concepts and their meanings

G_{TF} - Fuzzy Tripartite graph, G_{STF} -Fuzzy Square sum soft tripartite graph, $\wp$ -set of all nodes, $\Im$ -set of all edges

Definition 3.1. A fuzzy soft tripartite graph(FSTG) $G_{TF} = (\wp, \Im)$ is said to be a fuzzy mean square sum soft tripartite graph(FMSSTG) $G_{STF} = (\wp_e, \Im_e)$ if the following conditions

(i) The vertices can be partitioned into 3 disjoint vertex pair and

(ii) $$\Im_e(x_{ii}, y_{jj}) \le \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\}$$

(iii) $$\Im_e(y_{jj}, z_{kk}) \le \frac{1}{2}\left\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\right\}$$

(iv) $$\Im_e(z_{kk}, x_{ii}) \le \frac{1}{2}\left\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\right\}$$

Where $\wp_e : \wp \to [0,1]$ and $\Im_e : \wp \times \wp \to [0,1]$

Definition 3.2. A FMSSTG $G_{STF} = (\wp, \Im)$ is said to be a SFMSSTGif

(i) $$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\left[\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right]$$

(ii) $$\Im_e(y_{jj}, z_{kk}) = \frac{1}{2}\left[\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\right]$$

(iii) $$\Im_e(z_{kk}, x_{ii}) = \frac{1}{2}\left[\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\right] \quad \forall e \in A$$

Where $\wp_e : \wp \to [0,1]$ and $\Im_e : \wp \times \wp \to [0,1]$

Also a FMSSTG G_{STF} is complete if

(i) $$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\left[\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right]$$

(ii) $$\Im_e(y_{jj}, z_{kk}) = \frac{1}{2}\left[\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\right]$$

(iii) $\Im_e\left(z_{kk}, x_{ii}\right) = \frac{1}{2}\left[\wp_e^2\left(z_{kk}\right) + \wp_e^2\left(x_{ii}\right)\right]$

Definition 3.3. A FSTG $G_{TF} = (\wp, \Im)$ is said to be a FMSSTG, then size of FMSSTGis, $S(G_{STF}) = \sum_{e \in A} \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \Im_e(a_i, b_j, c_k)$ where

$$S[F_{STF}(e_1)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \Im_{e_1}(a_i, b_j, c_k)$$

$$S[F_{STF}(e_2)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \Im_{e_2}(a_i, b_j, c_k)$$

$$S[F_{STF}(e_3)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \Im_{e_3}(a_i, b_j, c_k)$$

Definition 3.4. If a FSTG $G_{TF} = (\wp, \Im)$ is said to be a FMSSTG, then degree of FMSSTGis,

$$d(G_{STF}) = \sum_{e_1 \in A} \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \Im_{e_1}(a_i, b_j, c_k)$$ Where

$$i = 1,2,3,.....$$

Definition 3.5. AFSTG $G_{TF} = (\wp, \Im)$ is said to be a FMSSTG, then order of FMSSTGis,

$$O(G_{STF}) = \sum_{e_1 \in A} \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \wp_e(a_i, b_j, c_k)$$

where

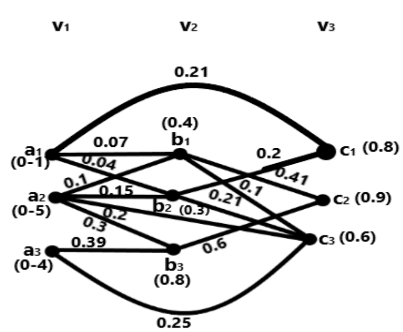
$$O[F_{STF}(e_1)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \wp_{e_1}(a_i, b_j, c_k)$$

$$O[F_{STF}(e_2)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \wp_{e_2}(a_i, b_j, c_k)$$

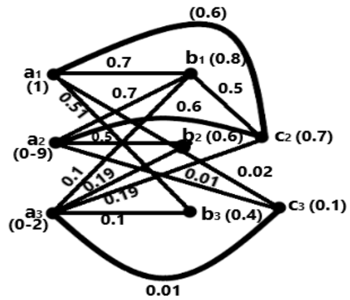
$$O[F_{STF}(e_3)] = \sum_{a_i b_j c_k \in \wp_i \cup \wp_j \cup \wp_k} \wp_{e_3}(a_i, b_j, c_k)$$

Example 3.1

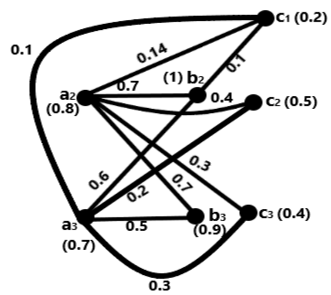
$F(e_1)$:



$F(e_2)$:



$F(e_3)$



e	a_1	a_2	a_3	b_1	b_2	b_3	c_1	c_2	c_3
e_1	0.1	0.5	0.4	0.4	0.3	0.8	0.8	0.9	0.6
e_2	1	0.9	0.2	0.8	0.6	0.4	0	0.7	0.1
e_3	0	0.8	0.7	0	1	0.9	0.2	0.5	0.4

μ	e_1	e_2	e_3
$a_1\, b_1$	0.07	0.7	0
$a_1\, b_2$	0.04	0.61	0
$a_1\, b_3$	0	0.51	0
$a_1\, c_1$	0.21	0	0
$a_1\, c_2$	0	0.6	0
$a_1\, c_3$	0	0	0
$a_2\, b_1$	0.1	0.7	0
$a_2\, b_2$	0.15	0.5	0.7
$a_2\, b_3$	0.3	0	0.7
$a_2\, c_1$	0	0	0.14

$a_2\ c_2$	0	0.6	0.4
$a_2\ c_3$	0.2	0.1	0.3
$a_3\ b_1$	0	0.1	0
$a_3\ b_2$	0	0.19	0.6
$a_3\ b_3$	0.39	0.1	0.5
$a_3\ c_1$	0	0	0.1
$a_3\ c_2$	0	0.19	0.2
$a_3\ c_3$	0.21	0.01	0.3
$b_1\ c_1$	0	0	0
$b_1\ c_2$	0.41	0.5	0
$b_1\ c_3$	0.1	0	0
$b_2\ c_1$	0.2	0	0.1
$b_2\ c_2$	0	0	0
$b_2\ c_3$	0.21	0.02	0
$b_3\ c_1$	0	0	0
$b_3\ c_2$	0.6	0	0
$b_3\ c_3$	0	0	0

Size of FMSSTG

$$S[F_{STF}(e_1)] = 3.19 \qquad S[F_{STF}(e_2)] = 5.53$$

$$S[F_{STF}(e_3)] = 4.04$$

$$\therefore\ S[G_{STF}] = 12.76$$

Order of FMSSTG

$$O[F_{STF}(e_1)] = 4.8\ O[F_{STF}(e_2)] = 4.7 \qquad O[F_{STF}(e_3)] = 4.5$$

$$\therefore\ O[G_{STF}] = 14$$

Degree of FMSSTG

$$d_{STF}(a_1) = 2.74\ d_{STF}(a_2) = 4.8\ d_{STF}(a_3) = 2.93\ d_{STF}(b_1) = 2.68$$

$$d_{STF}(b_2) = 3.32\ d_{STF}(b_3) = 2.5\ d_{STF}(c_1) = 0.75\ d_{STF}(c_2) = 3.5$$

$$d_{STF}(c_3) = 1.8$$

4. Complement of Fuzzy Mean Square Sum Tripartite Graph (CFMSSTG)

Definition 4.1. Let G_{STF} be a FMSSTG. Then complement of G_{STF} is defined as $\overline{G}_{STF}$ where

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj}))$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Where $\wp_e : \wp \to [0.1]$ and $\Im_e : \wp \times \wp \to [0,1]$

Definition 4.2. A CFMSSTG is called a complement of SFMSSTG if

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\}$$

and is complement of complete FMSSTGif

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\}$$

Preposition 4.1.

$$S(\overline{G}_{STF}) + S(G_{STF}) < \sum_{e \in A} \sum_{x_{ii} \neq y_{jj}} \wp_e^2(x_{ii}) + \wp_e^2(y_{jj})$$

$$S(\overline{G}_{STF}) + S(G_{STF}) < \sum_{e \in A} \sum_{y_{jj} \neq z_{kk}} \wp_e^2(y_{jj}) + \wp_e^2(z_{kk})$$

$$S(\overline{G}_{STF}) + S(G_{STF}) < \sum_{e \in A} \sum_{z_{kk} \neq x_{ii}} \wp_e^2(z_{kk}) + \wp_e^2(x_{ii})$$

Proof. Since

$$\Im_e(x_{ii}, y_{jj}) \leq \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} \to (1)$$

consider $\Im_e(x_{ii}, y_{jj}) < \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

Also

$$\overline{\Im}_e(x_{ii}, y_{jj}) < \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} \rightarrow (2)$$

Adding (1) and (2)

$$\Im_e(x_{ii}, y_{jj}) + \overline{\Im_e}(x_{ii}, y_{jj}) < 2\left\{\frac{1}{2}\left[\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right]\right\}$$

$$\sum_{e\in A}\sum_{x_{ii}\neq y_{jj}} \Im_e(x_{ii}, y_{jj}) + \sum_{e\in A}\sum_{x_{ii}\neq y_{jj}} \overline{\Im}_e(x_{ii}, y_{jj}) < \sum_{e\in A}\sum_{x_{ii}\neq y_{jj}} \wp_e^2(x_{ii}) + \wp_e^2(y_{jj})$$

$$S(G_{STF}) + S(\overline{G}_{STF}) < \sum_{e\in A}\sum_{x_{ii}\neq y_{jj}} \wp_e^2(x_{ii}) + \wp_e^2(y_{jj})$$

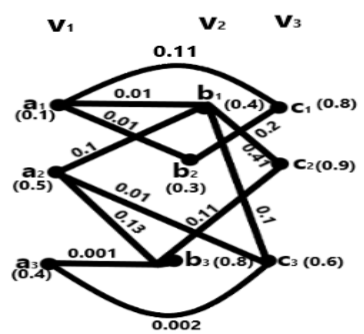
Similarly,

$$S(\overline{G}_{STF}) + S(G_{STF}) < \sum_{e\in A}\sum_{y_{jj}\neq z_{kk}} \wp_e^2(y_{jj}) + \wp_e^2(z_{kk})$$

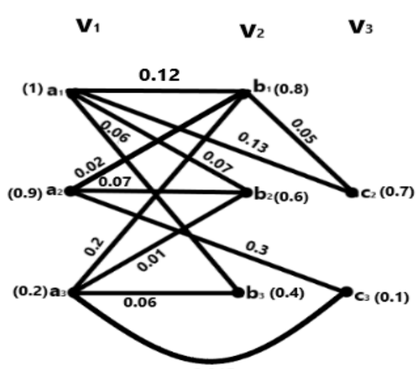
$$S(\overline{G}_{STF}) + S(G_{STF}) < \sum_{e\in A}\sum_{z_{kk}\neq x_{ii}} \wp_e^2(z_{kk}) + \wp_e^2(x_{ii})$$

Example 4.1

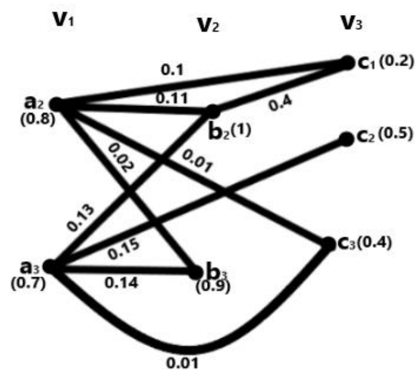
$F(\bar{e}_1)$



$F(\bar{e}_2)$



$F\left(\bar{e}_3\right)$



$$S\left(\overline{G}_{v_{ii}\cup v_{jj}}\right)=1.261 \qquad S\left(\overline{G}_{v_{jj}\cup v_{kk}}\right)=1.27$$
$$S\left(\overline{G}_{v_{kk}\cup v_{ii}}\right)=0.827 \qquad S\left(\overline{G}\right)=3.358$$
$$S\left(G_{v_{ii}\cup v_{jj}}\right)=1.0500 \qquad S\left(G_{v_{jj}\cup v_{kk}}\right)=1.5200$$
$$S\left(G_{v_{kk}\cup v_{ii}}\right)=0.6600 \quad S(G)=3.2300$$
$$\wp_e^2(x_{ii})+\wp_e^2(y_{jj})=7.64 \qquad \wp_e^2(y_{jj})+\wp_e^2(z_{kk})=3.33$$
$$\wp_e^2(z_{kk})+\wp_e^2(x_{ii})=5.635$$
$$S(G_{STF})+S\left(\overline{G}_{STF}\right)<\sum_{e\in A}\sum_{x_{ii}\neq y_{jj}}\wp_e^2(x_{ii})+\wp_e^2(y_{jj})$$

Properties of CFMSSTG:

1. The order of $\overline{G}_{STF}$ is equal to the order of G_{STF}
2. The number of elements in the edge set of $\overline{G}_{STF}$ is less than or equal to the number of elements in the edge set of G_{STF}.
3. Node set of $\overline{G}_{STF}$ is same as the node set of G_{STF}.
4. $$S\left(\overline{G}_{STF}\right)+S(G_{STF})<\sum_{e\in A}\sum_{x_{ii}\neq y_{jj}}\wp_e^2(x_{ii})+\wp_e^2(y_{jj})$$

$$S\left(\overline{G}_{STF}\right)+S(G_{STF})<\sum_{e\in A}\sum_{y_{jj}\neq z_{kk}}\wp_e^2(y_{jj})+\wp_e^2(z_{kk})$$
$$S\left(\overline{G}_{STF}\right)+S(G_{STF})<\sum_{e\in A}\sum_{z_{kk}\neq x_{ii}}\wp_e^2(z_{kk})+\wp_e^2(x_{ii})$$

Theorem 4.1

The complement of a SFMSSTGis also SFMSSTG **.**

Proof. Let G_{STF} be a SFMSSTG.

$$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$\Im_e(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\}$$

$$\Im_e(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\}$$

By the definition of complement $\overline{G}_{STF}$ is defined as,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Now consider,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$= \begin{cases} 0 & , \Im_e(x_{ii}, y_{jj}) > 0 \\ \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}, & \Im_e(x_{ii}, y_{jj}) = 0 \end{cases}$$

$$\therefore \overline{\Im_e}(x_{ii}, y_{jj}) = 0,$$

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

Similarly,

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \begin{cases} 0 \\ \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\} \end{cases}$$ and

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \begin{cases} 0 \\ \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\} \end{cases}$$

$\therefore$ The complement of aSFMSSTGis alsoSFMSSTG.

Theorem 4.2

The complement of a complete FMSSTGis also complete FMSSTG.

Proof. Let G_{STF} be a complete FMSSTG

$$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$\Im_e(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\}$$

$$\Im_e(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\}$$

By the definition of complement $\overline{G}_{STF}$ is defined as,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\} - \Im_e(y_{jj}, z_{kk})$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\} - \Im_e(z_{kk}, x_{ii})$$

Now consider,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \Im_e(x_{ii}, y_{jj})$$

$$= \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\} - \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

$$= \begin{cases} 0 & , \Im_e(x_{ii}, y_{jj}) > 0 \\ \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}, & \Im_e(x_{ii}, y_{jj}) = 0 \end{cases}$$

$$\therefore \overline{\Im_e}(x_{ii}, y_{jj}) = 0,$$

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\}$$

Similarly,

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \begin{cases} 0 \\ \frac{1}{2}\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\} \end{cases}$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \begin{cases} 0 \\ \frac{1}{2}\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\} \end{cases}$$

$\therefore$ The complement of a complete FMSSTGis also complete FMSSTG.

Theorem 4.3

If G_{STF} is an IFMSSTGiff CFMSSTG is a complete FMSSTG **.**

Proof. Given G_{STF} is an IFMSSTG.Now consider G_{STF} is anIFMSSTG.

Then $\Im_e(x_{ii}, y_{jj}) = 0 \rightarrow (1)$

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \Im_e(x_{ii}, y_{jj}) \rightarrow (2)$$

Substitute (1) in (2)

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\}$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\left\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\right\} \ \&$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\left\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\right\}$$

Hence the CFMSSTGis a FMSSTG .

Conversely,

Given the CFMSSTGis a complete FMSSTG.

(i.e) $$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} \rightarrow (3)$$

To prove,

G_{STF} is an IFMSSTG .

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \Im_e(x_{ii}, y_{jj})$$

$$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \overline{\Im_e}(x_{ii}, y_{jj})$$

$$= \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} \quad By\,(3)$$

$$\Im_e(x_{ii}, y_{jj}) = 0$$

Similarly

$$\Im_e(y_{jj}, z_{kk}) = 0 \ \& \ \Im_e(z_{kk}, x_{ii}) = 0$$

Hence G_{STF} is an IFMSSTG.s

Theorem 4.4

If G_{STF} is an IFMSSTG iff CFMSSTG is aSFMSSTG.

Proof. Given G_{STF} is anIFMSSTG.Now consider G_{STF} is an IFMSSTG.

Then $\Im_e(x_{ii}, y_{jj}) = 0 \to (1)$

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \Im_e(x_{ii}, y_{jj}) \to (2)$$

Substitute (1) in (2)

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\}$$

Similarly

$$\overline{\Im_e}(y_{jj}, z_{kk}) = \frac{1}{2}\left\{\wp_e^2(y_{jj}) + \wp_e^2(z_{kk})\right\} \ \&$$

$$\overline{\Im_e}(z_{kk}, x_{ii}) = \frac{1}{2}\left\{\wp_e^2(z_{kk}) + \wp_e^2(x_{ii})\right\}$$

Hence the CFMSSTGis a SFMSSTG.

Conversely,

Given the CFMSSTGis a SFMSSTG.

(i.e) $\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} \to (3)$

To prove,

G_{STF} is an IFMSSTG.

We know that,

$$\overline{\Im_e}(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \Im_e(x_{ii}, y_{jj})$$

$$\Im_e(x_{ii}, y_{jj}) = \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \overline{\Im_e}(x_{ii}, y_{jj})$$

$$= \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} - \frac{1}{2}\left\{\wp_e^2(x_{ii}) + \wp_e^2(y_{jj})\right\} \quad By\,(3)$$

$$\Im_e(x_{ii}, y_{jj}) = 0$$

Similarly, $\Im_e(y_{jj}, z_{kk}) = 0 \ \& \ \Im_e(z_{kk}, x_{ii}) = 0$

Hence G_{STF} is an IFMSSTG.

5. Application

The important information of a student study and career opportunity includes: (i) Student (ii) Department (iii) career. The relationship among them are straight forward. A student can like or choose any departments and the departments might have some career opportunity. As students do directly perform actions on the career of the others. We can abstract the relationship among students, Departments and career of the other in to a fuzzy mean square sum tripartite graph. For example to model students data as a FMSSTG. The FMSSTG is composed of three types of nodes: (i) Students nodes (ii) Department nodes (iii) Career nodes.In the FMSSTG, the students nodes connect with the department nodes and the career nodes. While the department nodes connect with the career nodes. Let $V = V_i \cup V_j \cup V_k = \{V_i;(a_1,a_2,a_3)\}\{V_j;(b_1,b_2,b_3)\}\{V_k;(c_1,c_2,c_3)\}$ are set of all three disjoint vertices and $A = \{e_1,e_2,e_3,e_4\}$ are parameter set.

Identified qualities of students are given below

$S_{11} =$ Good study and Responsible

$S_{22} =$ Self discipline and Good study

$S_{33} =$ Extra curricular activities and Responsible

Identified combinations of the Departments are given below

$D_{11} =$ Physics and Maths

$D_{22} =$ Tamil and Physics

$D_{33} =$ Computer science and Maths

Identified combinations of the career opportunity

$C_{11} =$ IT company and School

$C_{22} =$ School and Self-employment

$C_{33} =$ BPO's and IT company

and the parameters are,

$e_1 =$ {Students-Good study; Department-Maths;Career-School}

$e_2 =$ {Students-Responsible; Department-Physics;Career-BPO'S}

$e_3 =$ {Students-Extra curricularactivities;Department-Tamil; Career-Self employment}

$e_4 =$ {Students-Self discipline; Department- Computer science; Career-IT company}

In Example 4.1, We calculate size of FMSSTGand determine which department gave good career in the students. Most of the best self discipline students hired the computer science department and the computer science department students preferred IT company. According to the above discussion, "the most favourable matching occurs between self discipline in student, they choose computer science department and preferred IT company.As a result the concept has been successfully applied to the student studies and career opportunity.

6. Conclusion

FMSSTG is a new fuzzy graph idea. FMSSTG and its variants are a relatively new phenomenon based on the use of fuzzy graphs. The FMSSTG and its counterparts have been defined, as well as numerous of their fundamental features and theorems. With relevant examples, the order, size, and degree of FMSSTG and its complements have been specified. This FMSSTG has been used in a real-world application. This research could be expanded to include interval-valued FMSSTG, as well as bi-polar and m-polar FMSSTG. Furthermore, many real-world examples can be found. Furthermore, many real-world applications can be investigated.

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