International Journal of Computational Vision and Biomechanics
Vol.3 No. 1 (January-June, 2017)

COMPUTATIONAL MODELING OF THE LUMBAR SPINE COMPONENTS: A REVIEW

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This paper reviews the use of the finite element method for modeling of the human lumbar spine. The review covers material from the beginning of finite element modeling in the 1950's up to the latest papers in the field, concentrating on research performed after 1980 when finite element modeling of the lumbar spine became relatively widespread. The general approaches to model development are summarized with a discussion of the pros and cons of each technique. Each component of the lumbar spine is then discussed, starting with an explanation of the biomechanical properties and behavior, followed with a description of the models used and the advantages and disadvantages of each. The finite element models used in the reviewed papers for each component are summarized in tables listing the element type and material properties. Finally, recommendations are suggested for further validation of lumbar spine models and regarding additional required knowledge of material properties and loading.

Keywords: *lumbar spine, finite element modeling, material properties*

1. INTRODUCTION

Finite element analysis (FEA) is a numerical method by which the solution to complex mathematical equations can be approximated to any desired degree of accuracy. In the field of solid mechanics, the quantities to be determined are usually stresses, strains, and displacements. Although meshless methods exist, a given structure is typically divided into a number of elements of small but finite size (mesh), for which the relationship between stress and strain can be derived. Essentially, the solution is approximated within each element using relatively simple equations, but when combined, the overall behavior can be quite complex. The stress-strain relationship is defined through material properties, and the structure loads and constraints determine the boundary conditions. This computational method of analysis allows the solution of many problems that are excessively complicated for a "closed form" approach, due to irregular geometry, variability of the material properties, or both.

Over the last two decades, the Finite Element Modeling (FEM) of lumbar motion segments (vertebra-intervertebral disc-vertebra) has taken multiple directions. Although mathematical models of the spine have been in use since the 1950's, substantial numbers of studies began to appear in the 1970's. These early studies used relatively simple representations of the geometry and material properties. Belytschko *et al.* (1974), Spilker (1980), Spilker *et al.* (1984), and Kurowski *et al.* (1986) assumed an axisymmetric geometry, allowing a single two-dimensional slice to represent the entire motion segment. The annulus fibers and ground substance were combined into a single linear elastic element, and an incompressible nucleus was assumed. Another approach to modeling the nucleus was to use hydrostatic pressure rather than a physical element (Kurowski *et al.*, 1986). A more complex, but still linear, model was developed by Shirazi-Adl *et al.* (1984). A full three-dimensional representation of the motion segment was developed, and the annulus was divided into separate elements for the fibers and ground substance. Further developing the model, Shirazi-Adl *et al.* (1986a, b) added facet joints and the spinous process, and used a nonlinear model for the collagen fibers in the annulus.

Subsequent papers extended the basic models to include various material properties. Kim *et al.* (1991) added nonlinear ligaments, and the loss of fluid in the disc was studied by Shirazi-Adl (1992) by changing the disc volume. The time-dependent characteristics of the nucleus were modeled with poroelastic elements by Argoubi (1996), Shirazi-Adl (1996a, 1996b), and Fagan (2002) and with viscoelastic elements by Lu et al. (1996a, 1996b). Kong *et al.* (1998) studied the effect of the thorax and the attached muscles.

Attempts to model the complete lumbar spine using relatively small numbers of elements, so-called "simple" models (Fagan *et al.*, 2002), have been used since the late 50's (Latham, 1957; Orne and Liu, 1970; Roberts and Chen, 1970; Prasad and King, 1974; Sundaram and Feng, 1977; Belytschko *et al.*, 1978; Dietrich *et al.*, 1991). Although often termed simple because of the use of fewer elements, the individual element models are actually more complex than typical solid finite elements, with higher orders of displacement and stress approximation, and allow for investigation of the entire spine including motion and the effect of material properties, posture, etc. These models offer a tradeoff between prediction of detailed local behavior and overall global spinous structure behavior. Recent examples of whole-spine models were developed by Pankoke *et al.* (1998) and Ezquerro *et al.* (2004), concentrating on small displacement vibration, or static loads.

A recent review by Fagan et al. (2002) concentrated on the overall design and application of finite element models to lumbar spine research. Similarly, Prendergast (1997) described the research up to that point that used finite element modeling in orthopaedics, reviewing the applications to which modeling had been applied. Natarajan *et al.* (2004) looked at the applications of a single model type, a poroelastic intervertebral disc model, to different applications including modeling of disc degeneration. Jones and Wilcox (2008) reviewed finite element modeling of the spine with an emphasis on methods of model validation and verification. Their review includes a discussion of the effect of geometry on the model results along with brief descriptions of the material properties. Conversely, this review focuses on lumbar spine structural component modeling with an emphasis on finite element analysis. The discussion of each component includes an overview of the following:

- its physiological and anatomical importance
- mechanical role
- material properties
- modeling procedures and approaches

A summary of the literature relating to the finite element modeling of each component follows the discussion. Although element geometry is extremely important in achieving accurate results with finite element models of the spine, the topic was well covered recently in Jones and Wilcox (2008) and is not addressed in this paper. Emphasis is placed on the type of elements used, the range of material properties, the relationship to the physical behavior, and the limitations of the models.

2. MODEL COMPONENTS

2.1. Vertebral Body

The vertebral body is a cylindrical element of the spinal column, which consists of cancellous bone (in the middle), surrounded with a thin shell of cortical bone (White, 1990). The design of the vertebral body is ideal for sustaining, externally and internally, longitudinally applied loads (Bogduk, 1997). There are a total of 5 lumbar vertebrae in the human spine (L5-L1). Determination of the compressive strength of the lumbar vertebrae has been the subject of biomechanical research for many years (Farfan, 1973; Lin *et al.*, 1978; Hansson *et al.*, 1980; McBroom, 1985) and the variation with level exists primarily due to differences in the size of the vertebrae themselves (White and Panjabi, 1990).

In general, cortical bone is stiffer than trabecular bone, but when the tensile strain *in-vivo* exceeds 2% of the original length, cortical bone fractures, whereas trabecular bone can withstand greater strains (Nordin, 2001). It has been found that under large compression forces the cancellous bone fails first (Brinckmann et al, 1988; 1989; Yingling and McGill, 1996), making it the determinant of the failure tolerance of the spine. A functional interpretation of spine anatomy suggests the presence of a good shock absorbing and load bearing system, as suggested by Farfan (1973). The mechanism of failure in a human vertebral body (cancellous bone) has been described by the architecture of the cancellous bone itself (Fyhrie *et al.*, 1994; Kopperdahl *et al.*, 1999). In addition, with age, vertebrae decrease in strength due to bone loss (White, 1990).

Vertebrae have generally been modeled as an elastic material, often with the cancellous and cortical bone having different properties, using an eight-node solid element. Several studies were performed looking at the effect of variations in bone properties. Goel, *et al.* (1995b) varied bone modulus and showed that bone has the ability to adapt to the loading. Dar *et al.* (2002) performed a sensitivity study of spinal behavior to a wide variety of input parameter changes, including bone material properties, and Imai *et al.* (2008) investigated the sensitivity of fracture load predictions to the model complexity. Liebschner *et al.* (2003) used bone mineral density to determine the elastic modulus of the trabecular bone and varied the cortical bone model to predict vertebral fracture. Simplified models, representing the bone by rigid elements, thus concentrating the deformations in the connecting tissue, were implemented by Pankoke (1998) and Shirazi-Adl and Parnianpour (1993, 1996a, 1999, 2000). Small changes in deformation can lead to large changes in stress in the discs. A model that ignores deformations in the vertebral bodies will overestimate the deformation in the discs, even if only by a small amount, and therefore overestimate the disc stress.

Since in almost all cases the vertebrae were modeled with linear, elastic, solid elements, two parameters, elastic modulus (E) and Poisson's ratio (ν), completely define the material properties. Only a small number of papers used anisotropic material properties for the vertebral body, and then only for the cortical bone (Crawford *et al.* 2003, Liebschner *et al.*, 2003, Tchirhart *et al.*, 2004). Cortical bone elastic modulus varied from 1580 MPa to 400,000 MPa with Poisson's ratio between 0.2 and 0.3. The elastic modulus used for cancellous bone varied from 87 MPa to 347 MPa with a Poisson's ratio between 0.2

and 0.36. A comparison of the values used in different publications is shown in Table 1. The most commonly used values, found in at least one-half of the published studies, were E = 12,000 MPa and ν = 0.2 (cortical) and E = 100 MPa and ν = 0.2 (cancellous). Some studies (Belytschko *et al.*, 1974; Spilker, 1980; Spilker *et al.*, 1984) used values of cortical bone modulus far in excess of reported test data, up to 400,000 MPa, and are clearly not accurately representing the material properties. However, even the more commonly used values, 10,000 and 12,000 MPa are based on a small number of material tests (Brown *et al.*, 1981; Carter *et al.*, 1981; Evans, 1973).

2.2. Intervertebral Disc

The intervertebral disc is the largest avascular structure in the spine. The main function of the intervertebral disc is anchoring adjacent vertebral bodies to each other; this gives the spine flexibility and at the same time, it absorbs and distributes the loads applied to the spine (Bogduk, 1976; Buckwalter, 1995). In order to perform these functions, it must transmit compression, bending, torsion, and shear forces between the vertebrae, and even resist tensile stresses under some loading and movement conditions. Furthermore, the intervertebral disc is recognized as being innervated in the outer surface, because of the presence of nerve endings, and therefore may be responsible for back pain (Hampton *et al.*, 1989; Coopes *et al.*, 1997; Cavanaugh *et al.*, 1997; Salminen *et al.*, 1999; Schwarzer *et al.*, 1994).

The disc consists of two portions: the nucleus pulposus (85% water content), and surrounding it, the annulus fibrosus (70-75% water content) (Repanti *et al.*, 1998; Ogata and Whiteside, 1981). The nucleus pulposus forms the central region of the disc. It is essentially a gel with fibrous strands and is nearly incompressible. The annulus fibrosus is made up of multiple layers of fibrous band that wrap around the nucleus in a helical fashion.

Table 1
Description of the Material Properties and Element Type used for Modeling of Cortical and Cancellous Bone of the Lumbar Vertebrae. A '?' Indicates that the Information was not Clear and a '-' Indicates that the Information was not Provided

Study	*Level*	*VERTEBRAE*					
		Cortical Bone			*Cancellous Bone*		
		E (MPa)	*ν*	*Element Type*	*E (MPa)*	*ν*	*Element Type*
Belytschko *et al.*, 1974	L2-L3	~75000	0.25	plane strain triangle	~347	0.25	plane strain triangle
Spilker, 1980	?	1580 - 15800	0.25	axisymmetric quad	-	-	combined element
Spilker *et al.*, 1984	?	56000 - 400000	0.25	axisymmetric quad	-	-	combined element
Shirazi-Adl, 1984, 1986a, b, 1989, 1991	L2-L3	12000	0.3	8 node solid	100	0.2	8 node solid
Kurowski *et al.*, 1986	?	15800	0.36	plane strain triangle	324	0.36	plane strain triangle
Kim *et al.*, 1991	L3-L5	12000	0.3	8 node solid/shell	100	0.2	8 node solid
Lavaste *et al.*, 1992	L1-L5	12000	0.3	8 node solid	100	0.3	8 node solid
Goel *et al.*, 1994	S1-L4	12000	0.3	8 node solid	100	0.2	8 node solid
Natarajan *et al.*, 1994	L4-L4	12000	-	20 node solid	100	-	20 node solid
Goel *et al.*, 1995a, 1995b.	L3-L5	12000	0.3	8 node solid	100	0.2	8 node solid
Argoubi *et al.*, 1996	L2-L3	10000	0.3	20 node poroelastic	100	0.2	20 node poroelastic
Lu *et al.*, 1996a, 1996b, 1998	L2-L3	22000	0.203	8 node solid	200	0.315	8 node solid
Wang *et al.*, 1997, 2000	L2-L3	12000	0.3	20 node solid	100	0.2	20 node solid
Kong *et al.*, 1998, 2003	S1-L1	12000	0.3	solid	100	0.2	solid
Natarajan *et al.*, 1999	L3-L4	12000	0.3	20 node solid	100	0.2	20 node solid
Lee *et al.*, 2000	L3-L4	10000	0.25	8 node shell	100	0.25	20 node solid
Cao *et al.*, 2001	L2-L4	11032	0.3	4 node shell	87.44	0.3	8 node solid
Goto *et al.*, 2002	L4-L5	12000	0.3	8 node solid	100	0.2	8 node solid
Ezquerro *et al.*, 2004	S1- L1	12000	0.3	8 node solid	100	0.2	8 node solid
Chosa *et al.*, 2004	L4-L5	12000	0.3	8 node solid	100	0.2	8 node solid
Imai *et al.*, 2008	L1	10000	0.4	3 node plate	-	-	4 node solid
Liebschner *et al.*, 2003	T12-L3	457	0.3	20 node solid	-	-	20 node solid

The bands are made up of collagen fibers suspended in a matrix, with the fibers making up about 20% of the band. The fibers are highly directional in orientation, angled approximately 30 degrees from horizontal. The disc height and width are directly related to its stiffness and load carrying capacity and are therefore important for model development.

Intervertebral discs can be subjected to very large forces, even when not carrying extra weight. In a sitting position, the pressure on the disc can be up to three times the trunk weight (Nachemson, 1981), with even larger forces possible during lifting or due to dynamic loading. Loading rate is also important and adds another complexity to understanding disc behavior. The disc is a viscoelastic material, meaning that it exhibits time-dependent behavior, such as creep and relaxation. When a load is applied it will result in an immediate deflection, followed by continued deflection over time if the load is maintained.

With age or under excessive or repetitive stress, the water content in the disc drops (DePalma and Rothman, 1970) and those changes can alter stiffness and may become critical to the load response (Stairmand *et al.*, 1991; Urban and McMullin, 1988; Osti *et al.*, 1990; Gruber and Hanley, 1998; 2002). Excessive stresses applied to the disc may also cause tearing and bulging of the annulus fibers, and even cause disc herniation or prolapse (Yasuma *et al.*, 1986; 1993; Duncan and Ahmed, 1991; Gordon *et al.*, 1991). In addition, it has been found that the disc structure has varying tolerance levels to compression and shear (Hutton, 1982; Adams, 2000), and its strength decreases with repetitive load (Brinckmann *et al.*, 1988; 1989; Adams and Dolan, 1996).

The intervertebral discs are often assumed to be the critical link in the spinal system. As such, a great deal of effort has gone into their modeling. A summary of the materials, geometry, and element types used to model the nucleus and annulus are found in Tables 2 and 3 respectively. A starting point for this discussion is a simple model that considers both the annulus fibrosus and nucleus pulposus to be elastic solids (Yoganandan *et al.*, 1996a; Kumaresan *et al.*, 1999a; Teo and Ng, 2001; Ng, 2001; Duncan and Ahmed, 1991). This model, while simple to implement, does not realistically capture the disc behavior. The interaction of the nucleus, ground substance, and annular fibers plays a critical role in determining the behavior, stresses, and mode of failure of the disc. Lumping the ground substance and fibers together ignores this interaction and makes it impossible to determine the relative contribution of each component. An alternative model, developed by Kurowski (1986), used elastic solid elements for the annulus and modeled the effect of the nucleus by applying an internal pressure. Natarajan *et al.* (2008) combined the internal pressure model with a poroelastic model to account for the flow of fluid in the disc. Using internal pressure was somewhat more realistic as it captured the essentially fluid-like behavior of the disc. However, this method is impractical since the pressure must be assumed apriori or determined through iteration, neither of which is possible in a dynamic model. In general, more complex models have generally been used for disc analysis.

The most commonly used disc model combines an incompressible fluid model for the nucleus, along with a composite model for the annulus. A typical approach is to assume that the ground substance in the annulus is an elastic solid, and to model the annulus fibers as cables, trusses, etc. (Kumaresan *et al.*, 1999b, 2001; Yoganandan, 1996b; Natarajan *et al.*, 1994, 1999, 2002; Shirazi-Adl, 1984, 1986a, 1986b, 1989, 1991, 1992, 1993, 1999, 2000; Argoubi, 1996; Lu, 1996a, 1996b, 1998; Goel, 1994; Lim *et al.*, 1994; Kong *et al.*, 1996). Geometric orientation of the fibers provides for anisotropic behavior in the annulus. A variation of this model used the composite approach for the annulus, but modeled the nucleus as an elastic solid (Kim, 2000; Goel, 1995a, 1995b, Kong, 1998). The combined annulus model has the limitation of not differentiating between the contributions of the ground substance and fibers, and fails to capture the confining effect of the fibers. Kim (1991) combined the two models by assuming that the nucleus was an incompressible fluid, but that after damage the behavior changed to elastic with stiffness greater than that of an undamaged disc. Either approach can accurately capture the behavior of the intervertebral disc, but their complexity leads to large computational times, making them inappropriate for modeling repetitive loading conditions. The models additionally differed from one another in the properties that were chosen. The fiber modulus and nonlinear behavior, ground substance modulus, and fiber orientation were all varied by the researchers, as summarized in Table 3. Nucleus elastic modulus ranged from 0.2 MPa to 4.0 MPa for solid models. Nearly incompressible behavior was simulated with bulk modulus (K) values between 1667 MPa and 2255 MPa. These values generally lie within the range reported from experimental studies. Both linear and nonlinear materials were used to model the annulus fibers, with E ranging from 6 MPa to 550 MPa. The fibers are made up primarily of collagen, similar to ligaments, and the upper bound for the modulus seems to be approximately 60-70 MPa, indicating that some studies (Lavaste *et al.*, 1992; Lu *et al.*, 1996a, 1996b, 1998) used values of elastic modulus much higher than is reasonable. The ground substance portion of the annulus fibrosus was modeled as an elastic solid, with E between 0.8 MPa and 8.0 MPa and ν from 0.1 to 0.45.

Table 2

Description of the Geometry, Material Properties and Element Type Used for Modeling of Nucleus of the Lumbar Intervertebral Disc. A '?' Indicates that the Information was not Clear and a '-' Indicates that the Information was not Provided

Study	Level	INTERVERTEBRAL DISC					
		Area (mm²)	*Height (mm)*	Nucleus			
				E (MPa)	*K (MPa)*	*Area (mm²)*	*Element Type*
Belytschko *et al*., 1974	L2-L3	1425	11.4	-	-	712	incompressible fluid
Spilker, 1980	?	706 - 2826	5- 25	-	-	16%–64% of total	incompressible fluid
Spilker *et al*., 1984	?	706 - 2826	5 - 25	-	-	50% of total	incompressible fluid
Shirazi-Adl *et al*., 1984	L2-L3	1371	11.0	-	-	632	incompressible fluid
Shirazi-Adl, 1986a,b, 1989	L2-L3	1360	9.5 - 11.5	-	-	612.1	incompressible fluid
Kim *et al*., 1991	L3-L5	-	-	-	1666.7	1658-1725	incompressible fluid
Lavaste *et al*., 1992	L1-L5	-	-	4.0	-	-	8 node solid
Goel *et al*., 1994	S1-L4	-	-	-	1666.7	-	incompressible fluid
Natarajan *et al*., 1994	L3-L4	-	-	-	2210	-	????
Goel *et al*., 1995a, 1995b.	L3-L5	-	-	1.0	-	-	8 node solid
Argoubi *et al*., 1996	L2-L3	-	-	1.5	-	-	20 node poroelastic
Lu *et al*., 1996a, b, 1998	L2-L3	1300	8.0	?	?	495.8	incompressible fluid
Wang *et al*., 1997, 2000	L2-L3	-	-	2.0	-	-	viscoelastic solid
Kong *et al*., 1998, 2003	S1-L1			1.326		40% of total	solid
Natarajan *et al*., 1999	L3-L4	-	-	0.2	-	-	20 node poroelastic
Lee *et al*., 2000	L3-L4	-	-	0.5	-	-	20 node solid
Cao *et al*., 2001	L2-L4	-	-	-	2255	-	8 node solid
Goto *et al*., 2002	L4-L5	-	-	0.54–1.32	-	-	applied pressure
Ezquerro *et al*., 2004	S1- L1	-	-	-	1666.7	-	8 node solid
Chosa *et al*., 2004	L4-L5	-	-	1.00	-	-	8 node solid

Table 3

Description of the Material Properties and Element Type used for Modeling of Annulus of the Lumbar Intervertebral Disc. A '?' Indicates that the Information was not Clear and a '-' Indicates that the Information was not Provided

Study	Level	INTERVERTEBRAL DISC					
		Annulus Fiber			Ground Substance		
		E (MPa)	*n*	*Element Type*	*E (MPa)*	*n*	*Element Type*
Belytschko *et al*., 1974	L2-L3	-	-	combined element	8.3 – 385*	0.45	plane strain triangle
Spilker, 1980	?	-	-	combined element	15.8	0.4	axisymmetric quad
Spilker *et al*., 1984	?	-	-	combined element	14 – 100*	0.4	axisymmetric quad
Shirazi-Adl, 1984, 1986a, b, 1989	L2-L3	initial ~8	-	nonlinear truss	4.2	0.45	8 node solid
Kurowski *et al*., 1986	?	-	-	combined element	10*	0.45	plane strain triangle
Kim *et al*., 1991	L3-L5	175	-	cable	4.2	0.45	8 node solid
Lavaste *et al*., 1992	L1-L5	500	0.3	8 node solid	2.0	0.45	8 node solid
Goel *et al*., 1994	S1-L4	175	-	cable	4.2	0.45	8 node solid
Goel *et al*., 1995a, 1995b.	L3-L5	-	-	combined element	450*	0.3	fiber reinf. concrete
Argoubi *et al*., 1996	L2-L3	initial ~8	-	nonlinear truss	2.5	0.1	poroelastic solid
Lu *et al*., 1996a, 1996b, 1998	L2-L3	500	0.35	viscoelastic truss	4.0	0.4	8 node solid
Wang *et al*., 1997; 2000	L2-L3	-	-	viscoelastic truss	8.0	0.45	viscoelastic solid
Kong *et al*., 1998, 2003	S1-L1	357.7-550	0.3	composite	4.2	0.45	composite
Natarajan *et al*., 1999	L3-L4	480	-	nonlinear truss	4.2	0.45	20 node solid
Lee *et al*., 2000	L3-L4	175	-	truss	0.8	0.35	20 node poroelastic
Cao *et al*., 2001	L2-L4	-	-	combined element	40*	0.45	8 node solid
Goto *et al*., 2002	L4-L5	-	-	nonlinear cable	4.2	0.45	8 node solid
Ezquerro *et al*., 2004	S1- L1	175 (<15%) 450 (>15%)		nonlinear cable	4.2	0.45	8 node solid
Chosa *et al*., 2004	L4-L5	?	-	nonlinear cable	4.2	0.45	8 node solid

Since the disc components are known to exhibit time-dependent behavior, several studies have incorporated temporal effects. One approach to capturing the variations with time is to use viscoelastic material models for the disc components. The model developed by Wang (1997, 1998, 2000) uses this method to examine the creep behavior of discs. Recognizing that the disc properties change, in some part, due to the loss of fluid, several researchers (Martinez, 1997; Lotz *et al.*, 1998; Lee, 2000) have used poroelastic models for the nucleus. A poroelastic model combines an elastic solid element and a porous fluid flow element to look at both effects, and their interaction, simultaneously. Both approaches can simulate the time-dependent behavior of the discs, but the poroelastic models have higher computation requirements.

2.3. Vertebral Endplates

The endplate is a layer of cartilage between the vertebral body and the intervertebral disc. One of the main purposes of the endplate is to path the nutrients by diffusion from the blood supply of the vertebral body to the avascular disc (Roberts *et al.*, 1996). The axial bulging or age related calcification and degeneration of the endplates are an important determinant for the compression characteristics of the spine, especially under cyclic loading (Bernick and Cailliet, 1982; Brinckmann, 1983; Holmes and Hukins, 1993; van Dieen *et al.*, 2001). Another hypothesis exists that the excessive loads applied to the spine may result in damage to vertebral endplates (microfracture) and leave dense scar tissue (Yoganadan, 1994) with the resulting diminution of nutrient supply to the disc. As a mechanical consequence, the damaged endplate, the "weakest link of the lumbar spine", tends to deform more under load, alters the stress distribution to the adjacent disc and leads to progressive structural changes in it (Brinkmann, *et al.*, 1983; Adams *et al.*, 1993; 2000).

Endplate modeling plays an important part in damage prediction. Natarajan (1994) found that failure always started in the endplate while Kurowski (1986) found the highest stresses in the endplates. Shirazi-Adl (1984) showed that for an intact disc, the endplates were most vulnerable, but as the disc degenerated, other components became critical. A summary of the material properties and element types used to model the endplates is given in Table 4. The elastic modulus values used in previous models varied substantially, with some studies assuming the same stiffness as cortical bone (Lee *et al.*, 2000; Cao *et al.*, 2001) while Spilker *et al.* (1984) used a modulus many times larger than those typically used for bone, and both assumptions appear out of line with existing experimental data.

2.4. Spinous Processes and Facet (Zygapophysial) Joints

Spinous processes and facet joints, together with other components, form the posterior element of the vertebrae, which consists of an exterior of cortical bone and a cancellous bony interior (White, 1990). The facet joints are typical synovial joints, covered with hyaline cartilage and are contained in a capsule. They are formed between the superior and inferior articular processes of the vertebrae (Bogduk, 1979; Giles, 1989) and are subject to changes with age (Taylor and Twomey, 1986). The spinous processes and facet joints have a critical role to play in the prevention of excessive deformation (in extension or laterally) of intervertebral discs (Adams *et al.*, 1980; 1983; Shepherd *et al.*, 2000). Twomey and Taylor also showed in their 1983 study the importance

Table 4
Material Properties and Element Type used for Modeling of Vertebral Endplates. A '?' Indicates that the Information was not Clear and a '-' Indicates that the Information was not Provided

Study	*Level*	*ENDPLATES*			
		E (MPa)	*v*	*Element Type*	*Thickness (mm)*
Belytschko *et al.*, 1974	L2-L3	113	0.4	plane strain traingle	-
Spilker *et al.*, 1984	?	56000 – 4x10^5	0.25	axisymmetric quad	-
Lavaste *et al.*, 1992	L1-L5	500	0.4	8 node solid	-
Natarajan, 1994, 1999	L3-L4	24.0	0.4	20 node solid	-
Argoubi *et al.*, 1996	L2-L3	5.0	0.1	poroelastic solid	-
Lu *et al.*, 1996a,b, 1998	L2-L3	23.8	0.4	8 node solid	1.0
Wang *et al.*, 1997, 2000	L2-L3	24	0.4	20 node solid	-
Lee *et al.*, 2000	L3-L4	10000	0.25	20 node solid	-
Cao *et al.*, 2001	L2-L4	12480	0.28	4 node shell	0.512 -0.812
Goto *et al.*, 2002	L4-L5	23.8	0.4	8 node solid	-
Chosa *et al.*, 2004	L4-L5	23.8	0.4	8 node solid	-

and the restraining ability of facet joints during forward flexion coupled with twisting. However, the facet joint load bearing function varies with the position of spine and is found to be greatest during extension (King *et al.*, 1975; Adams and Hutton, 1988). As the spine flexes forward, facet joints tend to disengage, placing tension on the posterior ligaments (Hindle and Pearcy, 1989; Cailliet, 1995).

The spinous process and facets were modeled as linear elastic eight-node elements in most previous studies (Goel *et al.*, 1993). Since they act by separating or contacting based on the spinal joint rotation, contact elements were used to model the interaction between these posterior elements (Goel *et al.*, 1994; Shirazi-Adl, 1991). Contact elements are extremely flexible when open (zero stiffness), but nearly rigid when closed. This has the effect of allowing opening of the joint, but upon closing the posterior elements do not move relative to each other and these elements appear appropriate for modeling these components. Duncan and Ahmed (1991) investigated the effect of the shape of the facets on the behavior of the motion segments. They concluded that, in the absence of damage to the facet, the actual shape had little effect on the spinal behavior.

2.5. Ligaments

There are many ligaments surrounding the spine. Their main purpose is to provide stability and flexibility during motion (Hedtmann *et al.*, 1989; Yahia *et al.*, 1991). The posterior and anterior longitudinal ligaments serve to resist the separation of, respectively, the posterior and anterior ends of the vertebral bodies (Bogduk, 1997). Adams and Hutton (1988) suggested that prolonged full flexion might cause the posterior ligaments to creep and during cyclic shear loading they exhibit nonlinear, viscoelastic and rate-dependent behavior (Weiss *et al.*, 2002). Thus, strained posterior tissues, including ligaments, affected by large shear loads, may increase the risk of injury. The interspinous ligaments connect adjacent spinous processes and they can add resistance to forward bending movements of the lumbar spine (Bogduk, 1997). It is also believed that the interspinous ligament is a major load bearing tissue in the case of high-energy loading in which anterior shear displacement is combined with full flexion (King, 1993). Additionally, biomechanical analysis of the behavior of ligaments can provide important information for an understanding of the injury mechanism.

Most ligaments are active only when under tension. That is, they become “slack” below a threshold length. However, some researchers modeled the ligaments as elastic truss elements, which are active in both tension and compression (Seidel *et al.*, 2001), and are not appropriate. One complication is that the initial (resting) length of the ligaments, and thus the length at which they become active, is unknown. In order to account for the slack-taut behavior of ligaments, Chosa *et al.* (2004), Goto (2002), Kumaresan *et al.* (2001), Goel *et al.* (1995a), and Shirazi-Adl (1989) used a nonlinear cable element to model the ligaments, and these models appear to capture the important aspects of the ligament behavior. The elements had different values of elastic modulus depending upon the strain in the ligament and the threshold strain at which the stiffening occurred varied from ligament to ligament. The major drawback to these models is the large scatter in the threshold strain between models, due to a lack of sufficient experimental data.

The behavior of the collagen forming ligaments (and the fibers in the annulus fibrosus) is known to be strain-rate dependent. Two different models were used to capture this effect. Wang *et al.* (1997, 1998, 2000) used a nonlinear spring and dashpot combination to simulate both the displacement and velocity dependent behaviors. A viscoelastic model was implemented by Lu *et al.* (1996b, 1998). This model used a series approximation to the nonlinear modulus of

$$G_R(t) = G_0\left[1 - \sum_{i=1}^{N} g_i\left(1 - e^{-t/\tau_1}\right)\right]$$

where G_R and G_0 are the current and initial moduli, and g_i and τ_i are series parameters, obtained from matching measured data. The complexity of this model, and the substantial investment in computer time required to calibrate the model at each step, makes it difficult to apply for repetitive loading conditions, especially where the cycles are not of constant magnitude. The cross-sectional areas of the ligaments vary considerably between people, and from one ligament to another. The values used in several studies, as well as the elastic modulus used, where appropriate and given in the literature, are presented in Table 5. As with the areas, the moduli used varied widely between studies, but all values used in these studies appear to be within the scatter of the reported experimental data.

3. DISCUSSION

Finite element models of the lumbar spine have shown a steady, continuous progress in complexity. The earliest studies used relatively simple, linear models that captured the overall behavior of motion segments, but did not provide a great deal of information about component behavior. Approximate validation of those models was accomplished using static testing of tissue or motion segments. Over the years, additional components were added to the models, including ligaments and posterior elements. The element and material models have grown increasingly sophisticated, incorporating advanced nonlinear concepts, viscoelasticity and poroelasticity.

Table 5
Cross-sectional Area, Material Properties and Element Type used for Modeling of the Following Ligaments of the Lumbar Spine: ALL–anterior Longitudinal; PLL – Posterior Longitudinal; LFL-Ligamentum Flavum; ITL – Intertransverse; CL–Capsular; ISL-interspinous; SSL–supraspinous. A '?' Indicates that the Information was Not Clear and a '-' Indicates that the Information was not Provided

Study	*Level*	*LIGAMENTS*						
		ALL	*PLL*	*LFL*	*ITL*	*CL*	*ISL*	*SSL*
		Cross-Sectional Area (mm^2)						
Shirazi-Adl, 1986a, b; 1991 Wang *et al*., 2000	L2-L3	24	14.4	75	12	36	40	30
Kim *et al*., 1991; Goel *et al*., 1994, 1995a, b	L3-L5	63.7	20	40	1.8	30	40	30
Lu *et al*., 1996b, 1998	L2-L3	38	20	60	10	40	35.5	35.5
Kong *et al*., 1998, 2003	S1-L1	63.7	20	40	1.8	30	40	30
Goto *et al*., 2002	L4-L5	75.9	51.8	-	2	-	36.3	75.7
Ezquerro *et al*., 2004	S1- L1	53	16	67	17.3	60	26	23
Chosa *et al*., 2004	L4-L5	75.9	51.8	78.7	2	102.5	36.3	75.7
		TypeYoung's Modulus (MPa)						
Kim *et al*., 1991; Goel et al, 1994, 1995a, b	cable	7.8 (<12%) 20 (>12%)	10(<11%) 20 (>11%)	15 (<6.2%) 19.5(>6.2%)	10 (<18%) 58.7 (>18%)	7.5 (<25%) 32.9 (>25%)	10 (<14%) 11.6 (>14%)	8 (<20%) 15 (>20%)
Lu *et al*., 1996b, 1998	Viscoel. cable	20	70	50	50	20	28	28
Kong *et al*., 1998, 2003	truss	7.8	10	15	10	7.5	10	8

They required additional information, including time-dependent behavior and there have also been attempts to include the effects of muscle forces, to varying degrees of success. The models of today involve fluid dynamics, dynamic loading, and optimization techniques.

However, the accuracy of these models depends upon the material properties, and many uncertainties still exist in this area. For example, little, if any, data is available on the cyclic behavior, and damage accumulation, of spinal components. The data that has been collected is typically based on vibration studies that load the specimen at a constant, high frequency rather than cycling related to human motion. Many of the materials tested are viscoelastic in nature and therefore have rate-dependent properties, and testing at frequencies outside of the normal range experienced *in-vivo* does not provide any useful information. Perhaps the most important information that can come from cyclic testing is the energy dissipation characteristics and possible changes in properties. Unfortunately, complete hysteresis curves are not generally reported; only the maximum values of the output parameters are typically given. Without this information, development of cumulative damage measures for repetitive loading remains impossible.

The data required as model input, and for validation, has all been derived from cadaver studies. The limits of using cadaverous tissue properties are well known, and while bone mechanical properties, for example, may not change significantly, the properties of soft tissue such as ligaments and discs are not the same in living and cadaverous tissue. However, obtaining material properties or behavior measures from living human subjects is impossible, impractical, or unethical.

4. SUMMARY

A review of the literature relating to finite element analysis of non-degenerative lumbar spines reveals that much progress has been made, but also that many questions remain unanswered. The following points summarize areas in which further improvements are recommended.

(1) The range of material properties used in the studies is extremely large. For example, vertebrae modulus varied by a factor of 10 and the nucleus modulus used in some models was up to 200 times that from other studies. While some of the difference in material properties is due to biological variability between individuals, other factors are also working to prevent consensus. The data all comes from cadaveric testing and the limitations of using cadavers to determine *in-vivo* properties are well known. In addition, the age of the tested spinal segments was generally well above the average age of low back pain patients, raising the issue of appropriateness of the results. Also, the lack of

a standard methodology for testing of spinal tissue makes it difficult to compare the material properties from different test series.

(2) Both material and geometric nonlinearity play an important role in determining the response of the lumbar spine. Many studies have considered material nonlinearity and a few have also included geometric nonlinearity. These factors become increasingly important at large loads and for considering motion of the spine, and the inclusion of both material and geometric nonlinearity are essential for accurately predicting lumbar spine response.

(3) The reviewed FE models tend to be more complex, using a large number of elements and including material and geometric nonlinearities and, in some cases, fluid dynamics. Although these models can provide detailed results, their complexity makes them very computationally expensive and time-consuming. Only static or very short time dynamic loading have been applied. Finding the right balance between detail and usability is crucial for taking analysis to the next level for clinical or practical applications.

(4) Very few of the current FE models are validated against experimental data, and none are validated for realistic loads. In addition, systematic sensitivity analysis into the effect of the multiple parameters required as input has yet to be performed. If any variation in input data is allowed, only a single parameter has been varied over a limited range.

The models reviewed in this paper made important contributions to the understanding of lumbar spine response under load. As with any model, only when based on realistic estimates of the material properties, geometry, and boundary conditions can the results be considered useful. In addition, the models must be thoroughly validated against experimental data to provide confidence in their ability to predict response in cases where no experiments are available. Although it is well known that finite element models cannot exactly reproduce the results from experimental studies, with due diligence to selection of parameters and validation, finite element studies can continue to increase in applicability.

REFERENCES

[1] Adams, M. A., Hutton, W. C., Stott, JR. R. (1980), The Resistance to Flexion of the Lumbar Intervertebral Joint. *Spine,* 5: 245-252.

[2] Adams, M. A., Hutton, W. C. (1983), The Mechanical Function of the Lumbar Apophyseal Joints. *Spine,* 8: 326-330.

[3] Adams, M. A., Hutton, W. C. (1988), Mechanics of the Intervertebral Disc. In Ghosh P (editor): *The Biology of the Intervertebral Disc,* Boca Raton, CRC Press.

[4] Adams, M. A., McNally, D. S., Wagstaff, J., Goodship, A. E. (1993), Abnormal Stress Concentration in Lumbar Intervertebral Discs Following Damage to the Vertebral Body: A Cause of Disc Failure. *Eur. Spine Journal*, 1: 214-221.

[5] Adams, M. A., Dolan, P. (1996), Time-dependent Changes in the Lumbar Spine's Resistance to Bending. *Clinical Biomechanics* 11: 194-200.

[6] Adams, M. A., Freeman, B. J. C., Morrison HP, Nelson IW, Dolan P. (2000), Mechanical Initiation of Intervertebral Disc Degeneration. *Spine* 25: 1625-1636.

[7] Argoubi, M., Shirazi-Adl, A. (1996), Poroelastic Creep Response Analysis of a Lumbar Motion Segment in Compression. *Journal of Biomechanics* 29: 1331-1339.

[8] Belytschko, T., Kulak, R. F., Schultz, A. B. (1974), Finite Element Stress Analysis of an Intervertebral Disc. *Journal of Biomechanics*, 7: 277–285.

[9] Belytschko, Tl, Schwer, L., Privitzer, E. (1978), Theory and Application of a Three-dimensional Model of the Human Spine. *Aviation, Space Environmental Med* 49: 158-165.

[10] Bernick, S., Cailliet, R. (1982), Vertebral End-plate Changes with Aging of Human Vertebrae. *Spine* 7: 97-102.

[11] Bogduk, N. (1976), The Anatomy of the Lumbar Intervertebral Disc Syndrome. *The Medical Journal of Australia* 1: 878-881.

[12] Bogduk, N., Long D. M. (1979), The Anatomy of the So-called "Articular Nerves" and their Relationship to Facet Denervation in the Treatment of Low-back Pain. *Journal of Neurosurgery* 51: 172-177.

[13] Bogduk, N. (1997), *Clinical Anatomy of the Lumbar Spine and Sacrum*. Churchill Livingstone, New York.

[14] Brinckmann, P., Frobin, W., Hierholzer, E., Horst, M. (1983), Deformation of the Vertebral End-plate under Axial Loading of the Spine. *Spine,* 8: 851-856.

[15] Brinckmann, P., Biggemann, M., Hilweg, D. (1988), Fatigue Fracture of Human Lumbar Vertebrae. *Clinical Biomechanics* Suppl. 1: S1-S23.

[16] Brinckmann, P., Biggemann, M., Hilweg, D. (1989), Prediction of the Compressive Strength of Human Lumbar Vertebrae. *Spine,* 14: 606-610.

[17] Brown, N., Saputa, C., Black, J. (1981), Young's Modulus of Living Human Bone. *Trans 27th Ann Meeting*, Orthop Res Soc, 6: 44.

[18] Buckwalter, J. A. (1995), Aging and Degeneration of the Human Intervertebral Disc. *Spine* 20: 1307-1314.

[19] Cailliet, R. (1995), *Low Back Syndrome*. F. A. Davis Company, Philadelphia.

[20] Carter, D. R., Caler, W. E., Spengler, D. M., Frankel, V. H. (1981), Cortical Bone Fatigue: the Effect of Strain Range, Stress Range, and Elastic Modulus. *Trans 27th Ann Meeting*, Orthop Res. Soc., 6: 44.

[21] Cavanaugh, J. M., Ozaktay, A. C., Yamashita, T., Avramov, A., Getchell, T. V., King, A. I. (1997), Mechanism of Low Back Pain. A Neurophysiologic and Neuroanatomic Study. *Clinical Orthopaedics and Related Research* 335: 166-180.

[22] Chosa, E., Totoribe, K., Tajima, N. (2004), A Biomechanical Study of Lumbar Spondylolysis Based on a Three-dimensional Finite Element Model. *Journal of Orthopaedic Research* 22: 158-163.

[23] Crawford, R. P., Rosenberg, W. S., Keaveny, T. M. (2003), Quantitative Computed Tomography-based Finite Element Models of the Human Lumbar Vertebral Body: Effect of

Element Size on Stiffness, Damage, and Fracture Strength Predictions. *J. Biomech Eng*. 125: 434-438.

[24] Dar, F. H., Meakin, J. R., Aspden, R. M. (2002), Statistical Methods in Finite Element Analysis. *Journal of Biomechanics* 35: 1155-1161.

[25] DePalma, A. F., Rothman, R. H. (1970), *The Intervertebral Disc*, W. B. Sounders Company, Philadelphia.

[26] Dietrich, M., Kedzior, K., Zagrajek, T. (1991), A Biomechanical Model of the Human Spinal System. *Proc Instn Mech Engrs, Part II: J Engineering in Medicine* 205: 19-26.

[27] Duncan, N. A., Ahmed, A. M. (1991), The Role of Axial Rotation in the Etiology of Unilateral Disc Prolapse, An Experimental and Finite Element Analysis. *Spine* 16: 1089-1098.

[28] Evans, F. G. (1973), *Mechanical Properties of Bone*, CC Thomas, Springfield, IL.

[29] Fagan, M. J., Julian, S., Mohsen, A. M. (2002), Finite Element Analysis in Spine Research. Proc Instn Mech Engrs, *Part II: J. Eng. in Medicine*, 216: 281-298.

[30] Farfan, H. F. (1973), *Mecanical Disorders of the Low Back Pain*, Lea & Febiger, Philadelphia.

[31] Fyhrie, D. P., Schaffler, M. B. (1994), Failure Mechanism in Human Vertebral Cancellous Bone, *Bone*, 15: 105-109.

[32] Giles, L.G.F. (1989), *Anatomical Basis of Low Back Pain*, Williams & Wilkins, Baltimore.

[33] Goel, V. K., Kong, W., Han, J. S., Weinstein, J. N., Gilbertson, L. G. (1993), A Combined Finite Element and Optimization Investigation of Lumbar Spine Mechanics with and without Muscles. *Spine* 18: 1531-1541.

[34] Goel, V. K., Park, H., Kong, W. (1994), Investigation of Vibration Characteristics of Ligamentous Lumbar Spine using the Finite Element Approach. *Journal of Biomechanical Engineering* 116: 377-383.

[35] Goel, V. K., Monroe., M. S., Gilbertson, L. G., Brinckmann, P., (1995a), Interlaminar Shear Stress and Laminae Separation in a Disc. *Spine*, 20: 689-698.

[36] Goel, V. K., Ramirez, S. A., Kong, W., Gilbertson, L. G. (1995b), Cancellous Bone Young's Modulus Variation within the Vertebral Body of a Ligamentous Lumbar Spine - Application of Bone Adaptive Remodeling Concepts. *Journal of Biomechanical Engineering* 117: 266-271.

[37] Gordon, S. J., Yang, K. H., Mayer, P. J., Mace, A. H., Kish, V. L., Radin, E. L. (1991), Mechanism of Disc Prolapse. *Spine* 16: 450-456.

[38] Goto, K., Tajima, N., Chosa, E., Totoribe, K., Kuroki, H., Arizumi, Y. (2002), Mechanical Analysis of the Lumbar Vertebrae in a Three-dimensional Finite Element Method Model in which Intradiscal Pressure in the Nucleus Pulposus was used to Establish the Model. *Journal of Orthopaedic Science* 7: 243-246.

[39] Gruber, H. E., Hanley, E. N. (1998), Analysis of Aging and Degeneration of the Human Intervertebral Disc. Comparison of Surgical Specimens with Normal Controls. *Spine* 23: 751-757.

[40] Gruber, H. E., Hanley, E. N. (2002), Ultrastructure of the Human Intervertebral Disc during Aging and Degeneration. *Spine* 27: 798-805.

[41] Hampton, D., Laros, G., McCarron, R., Franks, D. (1989), Healing Potential of the Anulus Fibrosus. *Spine* 14: 398-401.

[42] Hansson, T., Roos, B., Nachemson, A. (1980), The Bone Mineral Content and Ultimate Compressive Strength of Lumbar Vertebrae. *Spine* 5: 46-55.

[43] Hedtmann, A., Steffen, R., Methfesses, J., Kolditz, D., Kramer, J., Thols, M. (1989), Measurement of Human Lumbar Spine Ligaments during Loaded and Unloaded Motion. *Spine* 14: 175-185.

[44] Hindle, R. J., Pearcy, M. J. (1989), Rotational Mobility of the Human Back in Forward Flexion. *Journal of Biomech Eng*, 11: 219.

[45] Holmes, A. D., Hukins, D. W. L. (1993), Response of the End-plate to Compression of the Spine. *European Spine Journal* 2: 16-21.

[46] Hutton, W. C., Adams, M. A. (1982), Can the Lumbar Spine be Crushed in Heavy Lifting? *Spine*, 7: 586-590.

[47] Imai, K., Ohnishi, I., Yamamoto, S., Nakamura, K. (2008), *In-vivo* Assessment of Lumbar Vertebral Strength in Elderly Women using Computer Tomography-based Nonlinear Finite Element Model. *Spine* 33: 27-32.

[48] Jones, A. C., Wilcox, R. K. (2008), Finite Element Analysis of the Spine: Towards a Framework of Verification, Validation and Sensitivity Analysis. *Med. Eng. & Phys*., 30: 1287-1304.

[49] Kim, Y. E., Goel, V. K., Weinstein, J. N., Lim, T. H. (1991), Effect of Disc Degeneration at One level on the Adjacent Level in Axial Mode. *Spine* 16: 331-350.

[50] Kim, Y. (2000), Prediction of Peripheral Tears in the Annulus of the Intervertebral Disc. *Spine* 25: 1771-1774.

[51] King, A. I., Prasad, P., Ewing, C. L. (1975), Mechanism of Spinal Injury Due to Caudocephalad Acceleration. *Orthop Clin North Am*, 6: 19.

[52] King, A. I. (1993), Injury to the Thoracolumbar Spine and Pelvis. In Nahum A. M., Melvin J. W. (editors). *Accidental Injury, Biomechanics and Presentation*, Springer-Verlag, New York.

[53] Kong, W. Z., Goel, V. K., Gilbertson, L. G., Weinstein, J. N. (1996), Effects of Muscle Dysfunction on Lumbar Spine Mechanics. *Spine* 21: 2197-2207.

[54] Kong, W. Z., Goel, V. K., Gilbertson, L. G. (1998), Prediction of Biomechanical Parameters in the Lumbar Spine during Static Sagittal Plane Lifting. *Journal of Biomechanical Engineering* 120: 273-280.

[55] Kong, W. Z., Goel, V. K. (2003), Ability of the Finite Element Models to Predict Response of the Human Spine to Sinusoidal Vertical Vibration. *Spine* 28: 1961-1967.

[56] Kopperdahl, D. L., Roberts, A. D., Keaveny, T. M. (1999), Localized Damage in Vertebral Bone is Most Detrimental in Regions of High Strain Energy Density. *Journal of Biomechanical Engineering* 121: 622-628.

[57] Kumaresan, S., Yoganandan, N., Pintar, F. (1999a), Finite Element Analysis of the Cervical Spine: A Material Property Sensitivity Study. *Clinical Biomechanics* 14: 41-53.

[58] Kumaresan, S., Yoganandan, N., Pintar, F., Maiman, D. J. (1999b), Finite Element Modeling of the Cervical Spine: Role of Intervertebral Disc under Axial and Eccentric Loads. *Medical Engineering and Physics* 689-700: 689-700.

[59] Kumaresan, S., Yoganandan, N., Pintar, F. A., Maiman, D. J., Goel, V. K. (2001), Contribution of Disc Degeneration to Osteophyte Formation in the Cervical Spine: A Biomechanical Investigation. *Journal of Orthopaedic Research* 19: 977-984.

[60] Kurowski, P., Kubo, A. (1986), The Relationship of Degeneration of the Intervertebral Disc to Mechanical Loading Conditions on Lumbar Vertebrae. *Spine* 11: 726-731.

[61] Latham, F. (1957), A Study in Body Ballistics. *Proc R. Soc. Lond*, 174: 121.

[62] Lavaste, F., Skalli, W., Robin, S., Roy-Camille, R., Mazel, C. (1992), Three-dimensional Geometrical and Mechanical Modeling of the Lumbar Spine. *Journal of Biomechanics*, 10: 1153–1164.

[63] Lee, C., Kim, Y. E., Lee, C. S., Hong, Y. M,, Jung, J., Goel, V. K. (2000), Impact Response of the Intevertebral Disc in a Finite Element Model. *Spine* 25: 2431-2439.

[64] Liebschner, M. A. K., Kopperdahl, D. L., Rosenberg, W. S., Keaveny, T. M. (2003), Finite Element Modeling of the Human Thoracolumbar Spine. *Spine* 28: 559-565.

[65] Lim, T. H., Goel, V. K., Weinstein, J. N., Kong, W. (1994), Stress Analysis of a Canine Spinal Motion Segment using the Finite Element Technique. *Journal of Biomechanics* 27: 1259-1269.

[66] Lin, H. S., Liu, Y. K., Adams, K. H. (1978), Mechanical Response of the Lumbar Intervertebral Joint under Physiological (Complex) Loading. *Journal of Bone and Joint Surgery* 60-A: 41-55.

[67] Lotz, J., Colliou, O. K., Chin, J., Duncan, N. A., Liebenberg, E. (1998), Compression-induced Degeneration of the Intervertebral Disc: An in Vivo Model and Finite-element Study. *Spine* 23: 2493-2506.

[68] Lu, Y. M., Hutton, W. C., Gharpuray, V. M. (1996a), Can Variations in Vertebral Disc Height Affect the Mechanical Function of the Disc? *Spine* 21: 2208-2217.

[69] Lu, Y. M., Hutton, W. C., Gharpuray, V. M. (1996b), Do Bending, Twisting, and Diurnal Fluid changes in the Disc Affect the Propensity to Prolapse? A Viscoelastic Finite Element Model. *Spine* 21: 2570-2579.

[70] Lu, Y. M., Hutton, W. C., Gharpuray, V. M. (1998), The Effect of Fluid Loss on the Viscoelastic Behavior of the Lumbar Intervertebral Disc in Compression. *Journal of Biomechanical Engineering* 120: 48-54.

[71] Martinez, J. B., Oloyede, V. O. A., Broom, N. D. (1997), Biomechanics of Load-bearing of the Intervertebral Disc: An Experimental and Finite Element Model. *Medical Engineering and Physics* 19: 145-156.

[72] Natarajan, R., N., Ke, J. H., Andersson, G. B. J. (1994), A Model to Study the Disc Degeneration Process. *Spine* 19: 259-265.

[73] Natarajan, R. N., Andersson, G. B. J. (1999), The Influence of Lumbar Disc Height and Cross-sectional Area on the Mechanical Response of the Disc to Physiologic Loading. *Spine* 24: 1873-1881.

[74] Natarajan, R. N., Andersson, G. B. J., Patwardhan, A. G., Verma, S. (2002), Effect of Annular Incision Type on the Change in Biomechanical Properties in a Herniated Lumbar Intervertebral Disc. *Journal of Biomechanical Engineering* 124: 229-236.

[75] Natarajan, R. N., Williams, J. R., Andersson, G. B. J. (2004), Recent Advances in Analytical Modeling of Lumbar Disc Degeneration. *Spine*, 29: 2733-2741.

[76] Natarajan, R. N., Lavender, S. A., An, H. A., Andersson, G. B. J. (2008), Biomechanical Response of a Lumbar Intervertebral Disc to Manual Lifting Activities. *Spine* 33: 1958-1965.

[77] Nordin, M., Frankel, V. H. (2001), *Basic Biomechanics of the Musculoskeletal System*. Lippincott Williams and Wilkins, Baltimore.

[78] Ogata, K., Whiteside, L. A. (1981), Nutritional Pathways of the Intervertebral Disc. An Experimental Study using Hydrogen washout Technique. *Spine* 6: 211-216.

[79] Orne, D., Liu, Y. (1970), A Mathematical Model of Spinal Response to Impact. *J. Biomechanics*, 4: 49-71.

[80] Osti, O. L., Vernon-Roberts, B., Fraser, R. D. (1990), Anulus Tears and Intervertebral Disc Degeneration. An Experimental Study using an Animal Model. *Spine* 15: 762-767.

[81] Pankoke, S., Buck, B., Woelfel, H. P. (1998), Dynamic FE Model of Sitting Man Adjustable to Body Height, Body Mass and Posture used for Calculating Internal Forces in the Lumbar Vertebral Disc. *Journal of Sound and Vibration* 215: 827-839.

[82] Prasad, P., King, A. I. (1974), An Experimentally Validated Dynamic Model of the Human Spine. *J. Appl. Mechanics* 546-550.

[83] Prendergast, P. J. (1997), Review Paper: Finite Element Models in Tissue Mechanics and Orthopaedic Implant Design. *Clinical Biomechanics* 12: 343-366.

[84] Repanti, M., Korovessis, P. G., Stamatakis, M. V., Spastris, P., Kosti, P. (1998), Evolution of Disc Degeneration in Lumbar Spine: A Comparative Histological Study between Herniated and Postmortem Retrieved Disc Specimens. *Journal of Spinal Disorders* 11: 41-45.

[85] Roberts, S. B., Chen, P. H. (1970), Elastostatic Analysis of the Human Thoracic Skeleton. *J. Biomechanics* 3: 527-545.

[86] Roberts, S., Urban, J. P. G., Evans, E. H., Eisenstein, S. M. (1996), Transport Properties of the Human Cartilage Endplate in Relation to its Composition and Calcification. *Spine*, 21: 415-420.

[87] Salminen, J. J., Erkintalo, M. O., Pentti, J., Oksanen, A., Kormano, M. J. (1999), Recurrent Low Back Pain and Early Disc Degeberation in the Young. *Spine* 24: 1316-1321.

[88] Schwarzer, A. C., Aprill, C. N., Derby, R., Fortin, J., Kine, G., Bogduk, N. (1994), The relative contributions of the Disc and Zygapophyseal joint in Chronic Low Back Pain. *Spine* 19: 801-806.

[89] Seidel, H., Bluthner, R., Hinz, B. (2001), Application of Finite-element Models to Predict Forces Acting on the Lumbar Spine during Whole-body Vibration. *Clin Biomech.(Bristol., Avon.)* 16 Suppl 1: S57-S63.

[90] Shepherd, D. E. T, Leahy, J. C,, Mathias, K. J., Wilkinson, S. J., Hukins, D. W. (2000), Spinous Process Strength. *Spine* 25: 319-323.

[91] Shirazi-Adl, S. A., Shrivastava, S. C., Ahmed, A. M. (1984), Stress Analysis of the Lumbar Disc-body unit in Compression. A Three-dimensional Nonlinear Finite Element Study. *Spine* 9: 120-134.

[92] Shirazi-Adl, S. A., Ahmed, A. M., Shrivasta, S. C. (1986a), Mechanical Response of a Lumbar Motion Segment in Axial Torque Alone and Combined with Compression. *Spine* 11: 914-927.

[93] Shirazi-Adl, S. A., Ahmed, A. M., Shrivastava, S. C. (1986b), A Finite Element Study of a Lumbar Motion Segment Subjected to Pure Sagittal Plane Moments. *Journal of Biomechanics* 19: 331-350.

[94] Shirazi-Adl, S. A. (1989), Strain in Fibers of a Lumbar Disc. Analysis of the Role of Lifting in Producing Disc Prolapse. *Spine* 14: 96-103.

[95] Shirazi-Adl, S. A. (1991), Finite-element Evaluation of Contact Loads on Facets of an L2-L3 Lumbar Segment in Complex Loads. *Spine* 16: 533-541.

[96] Shirazi-Adl, S. A. (1992), Finite Element Simulation of Changes in the Fluid Content of Human Lumbar Discs. Mechanical and Clinical Investigation. *Spine* 17: 206-212.

[97] Shirazi-Adl, S. A., Parnianpour, M. (1993), Nonlinear Response Analysis of the Human Ligamentous Lumbar Spine

in Compression. On Mechanisms Affecting the Postural Stability. *Spine* 18: 147-158.

[98] Shirazi-Adl, S. A., Parnianpour, M. (1996a), Role of Posture in Mechanics of the Lumbar Spine in Compression. *Journal of Spinal Disorders* 9: 277-286.

[99] Shirazi-Adl, S. A., Parnianpour, M. (1996b), Stabilizing Role of Moments and Pelvic Rotation on the Human Spine in Compression. *Journal of Biomechanical Engineering* 118: 26-31.

[100] Shirazi-Adl, S. A., Parnianpour, M. (1999), Effect of Changes in Lordosis on Mechanics of the Lumbar Spine-lumbar Curvature in Lifting. *Journal of Spinal Disorders* 12: 436-447.

[101] Shirazi-Adl, S. A., Parnianpour, M. (2000), Load Bearing and Stress Analysis of the Human Spine under a Novel Wrapping Compression Loading. *Clinical Biomechanics* 15: 718-725.

[102] Spilker, R. L. (1980), Mechanical Behavior of a Simple Model of an Intervertebral Disc under Compressive Loading. *Journal of Biomechanics*, 13: 495-901.

[103] Spilker, R. L., Daugirda, D. M., Schultz, A. B. (1984), Mechanical Response of a Simple Finite Element Model of the Intervertebral Disc under Complex Loading. *Journal of Biomechanics*, 17(2): 103–112.

[104] Stairmand, J. W., Holm, S., Urban, J. P. G. (1991), Factors Influencing Oxygen Concentration Gradients in the Intervertebral Disc. A Theoretical Analysis. *Spine* 16: 444-449.

[105] Sundaram, S. H., Feng, C. C. (1977), Finite Element Analysis of the Human Thorax. *J. Biomechanics* 10: 505-516.

[106] Taylor, J. R., Twomey, L. T. (1986), Age Changes in Lumbar Zygapophyseal Joints. Obsevations on Structure and Function. *Spine* 11: 739-745.

[107] Tchirhart, C. E., Nagpurkar, A., Whyne, C. M. (2004), Effects of Tumor Location, Shape and Surface Serration on Burst Fracture Risk in the Metastatic Spine. *J. Biomech* 37: 653-60.

[108] Teo, E. C., Ng, H. W. (2001), Evaluation of the Role of Ligaments, Facets and Discs Nucleus in Lower Spine under Compression and Sagittal Moments using Finite Element Method. *Medical Engineering and Physics* 23: 155-164.

[109] Twomey, L. T., Taylor, J. R. (1983), Saggital Movements of the Human Lumbar Vertebral Column: A Quantitative Study of the Role of the Posterior Vertebral Elements. *Arch Phys Med Rehabil*, 64: 322-325.

[110] Urban, J. P. G., McMullin, J. F. (1988), Swelling Pressure of the Lumbar Intervertebral Discs: Influence of Age, Spinal Level, Composition, and Degeneration. *Spine*, 13(2): 179-187.

[111] Wang, J. L., Parnianpour, M., Shirazi-Adl, A., Engin, A. E., Li, S., Patwardhan, A. G. (1997), Development and Validation of a Viscoelastic Finite Element Model of an L2/L3 Motion Segment. *Theoretical and Applied Fracture Mechanism* 28: 81-93.

[112] Wang, J., Parnianpour, M., Shirazi-Adl, A., Engin, A. E. (1998), The Dynamic Response of L2/L3 Motion Segment in Cyclic Axial Compressive Loading. *Clinical Biomechanics*, 13: S16-S25.

[113] Wang, J., Parnianpour, M., Shirazi-Adl, A., Engin, A. E. (2000), Viscoelastic Finite Element Analysis of a Lumbar Motion Segment in Combined Compression and Sagittal Flexion. Effect of Loading Rate, *Spine*, 25: 310-318.

[114] Weiss, J. A., Gardiner, J. C., Bonifasi-Lista, C. (2002), Ligament Material Behavior is Nonlinear, Viscoelastic and Rate-independent under Shear Loading. *Journal of Biomechanics* 35: 943-950.

[115] White, A. A., Panjabi, M. M. (1990), *Clinical Biomechanics of the Spine*, 2nd edition, Lippincott Company, Philadelphia.

[116] Yahia, L. H., Audet, J., Droulin, G. (1991), Rheological Properties of the Human Lumbar Spine Ligaments. *Journal of Biomedical Engineering* 13: 399-406.

[117] Yasuma, T., Makino, E., Saito, S., Inui, M. (1986), Histological Development of Intervertebral Disc Herniation. *Journal of Bone and Joint Surgery* 68-A: 1066-1072.

[118] Yasuma, T., Arai, K., Yamauchi, Y. (1993), The Histology of Lumbar Intervertebral Disc Herniation. The Significance of Small Blood Vessels in the Extruded Tissue. *Spine* 18: 1761-1765.

[119] Yingling, V. R., McGill, S. M. (1996), Mechanical Properties and Injuries Resulting from Anterior and Posterior Shear Loading of the Spine at Different Load Rates, *In Proceeding of the American Society for Biomechanics*, Atlanta.

[120] Yoganandan, N., Kumaresan, S. C., Voo, L., Pintar, F. A. (1996a), Finite Element Application in Human Cervical Spine Modeling. *Spine* 21: 1824-1834.

[121] Yoganandan, N., Kumaresan, S. C., Voo, L., Pintar, F. A., Larson, S. J. (1996b), Finite Element Modeling of the C4-C6 Cervical Spine Unit. *Medical Engineering and Physics*, 18: 569-574.