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Combined Experimental and Analytical Model of the Lumbar Spine Subjected to Large Displacement Cyclic Loads Part I– Model Description

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A model capable of capturing the essential behavior during repeated large displacement cyclic flexure at near real time would greatly enhance the study of the biomechanical functioning of the lumbar spine and improve work design and clinical therapies. This paper presents a model that bridges the gap between detailed finite element models and overly simplified models. All major components of the lumbar spine are modeled using nonlinear elements including viscoelastic effects where appropriate. The element types were selected to provide a balance between computational efficiency and level of detail. To increase the usefulness of the model it is linked to goniometer-measured motion data and the load is applied as a specified displacement at the top of the thoracic vertebra T_{12}. A complete description of the elements, loading derivation, and computational process is presented in this paper. A companion paper provides a validation study of the model results.

Keywords: Lumbar spine; Finite element analysis; Biomechanical modeling; Dynamic motion; Cyclic loading

1. INTRODUCTION

Detailed knowledge of the behavior of the lumbar spine under normal and abnormal postures and loading can lead to improved work design and clinical therapies. However, since it is not possible to perform either acute or cumulative failure tests *in-vivo* with human subjects, two approaches remain feasible, cadaveric studies and modeling. Cadaveric studies investigating cyclic loads have generally subjected the spine to high frequency loads until failure is achieved. These are artificial loading conditions and do not realistically reflect the stresses to which the spine is exposed through bending and lifting during normal daily or occupational activities (eg. manual material handling, etc.).

On the other hand, significant effort has also been invested in developing biomechanical models, in particular finite element (FE) models, for investigating spinal behavior. An advantage of this type of modeling is that the tissue response can be predicted, and the material properties and loads can be varied to a degree that is not possible with human subjects. To date, most finite element analysis studies have applied simplified static loads to a single motion segment. However, since most low back disorders (LBD) found in industry today are due to tasks that are repetitive in nature, static or short duration dynamic analyses may not be adequate.

With finite element models, loads that will damage the spinal structure(s) can be investigated, which cannot be done with human subjects. However, the results obtained are a function of the model assumptions and the applicability of the results depends upon the model specificity. Also, there is a trade-off between the number and complexity of elements (accuracy) and computation time. The implemented models vary widely in the choices made as to material properties, loads, and outcomes investigated.

Over the last two decades, the FE modeling of lumbar motion segments has taken multiple directions. Although mathematical models of the spine have been in use since the 1950's a substantial number of studies began to appear in the 1970's. These early studies used relatively simple representations of the geometry and material properties. Belytschko *et al.* (1976), Spilker (1980), Spilker *et al.* (1984), and Kurowski *et al.* (1986) assumed an axisymmetric geometry, allowing a single two-dimensional slice to represent the entire motion segment. All used linear elastic materials, combined the annulus fibers and ground substance into a single element, and assumed an incompressible nucleus. Kurowski used an applied hydrostatic pressure to represent the nucleus rather than a physical element. A more complex, but still linear, model was developed by Shirazi-Adl *et al.* (1984). A full three-dimensional representation of the motion segment was developed, and the annulus was divided into separate elements for the fibers and ground substance. Further developing the model, Shirazi-Adl *et al.* (1986a, b) added facet joints and the spinous process, and used a nonlinear model for the collagen fibers in the annulus.

Subsequent papers extended the basic models to include various material properties. Kim *et al.* (1991)

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added nonlinear ligaments, and the loss of fluid in the disc was studied by Shirazi-Adl (1992) by changing the disc volume. The time-dependent characteristics of the nucleus were modeled with poroelastic elements by Argoubi and Shirazi-Adl (1996) and with viscoelastic elements by Lu et al. (1996). Kong *et al.* (1998) studied the effect of the thorax and the attached muscles.

Few finite element studies of the cyclic behavior of the lumbar spine have been undertaken to date. Lee et al. (2000) used a poroelastic model and applied an impact load (duration up to 0.2 seconds). Goel *et al.* (1994) investigated the spinal system response to a cyclic load. Five cycles were modeled with a total duration of 1 second. Wang *et al.* (1998) used a viscoelastic model loaded in compression at 15 Hz for 30 seconds to determine the hysteretic energy loss per cycle. A simplified mass-spring model was used by Pankoke *et al.* (1998) to determine the vibration characteristics of a seated subject. Approximately 10 seconds of simulation was performed. The extremely short duration of loading for each of these studies emphasizes the difficulties in applying current FEMs to repetitive motion analysis problems.

Attempts to model the complete lumbar spine using relatively small numbers of elements, so-called "simple" models (Fagan *et al.*, 2002), have been made since the late 50's (Latham, 1957; Orne and Liu, 1970; Roberts and Chen, 1970; Prasad and King, 1974; Sundaram and Feng, 1977; Belytschko *et al.*, 1978; Dietrich *et al.*, 1991). Although often termed simple because of the use of fewer elements, the models of each individual element are actually more complex than typical solid finite elements, with higher orders of displacement and stress approximation, and allow for investigation of the entire spine including motion and the effect of material properties, posture, etc. These models offer a tradeoff between prediction of detailed local behavior and overall global spinous structure behavior. Recent examples of whole-spine models were developed by Pankoke *et al.* (1998) and Ezquerro *et al.* (2004), concentrating on small displacement vibration, or static loads.

In summary, this paper presents a model that addresses both the loading derivation and model complexity issues. Realistic loading during repeated sagittal lifts is obtained by explicitly linking the model with experimental measurements using a lumbar motion monitor (LMM) (Marras, *et al.* 1992). In addition, the analytical model takes a middle way, including enough detail to predict some localized behavior while remaining able to calculate the large displacement dynamic response of the full lumbar spine in near real-time. The model is not intended to replace existing models, but rather to complement them by adding a method of determining the overall behavior with response parameters that identify key areas that should be further investigated.

2. MODEL DESCRIPTION

Overview. Determining the forces to which a spine is subjected during flexion requires consideration of the material properties of the tissue, the subject-specific geometric configuration of the spine, and the particular motion imposed by the flexion task. The method proposed herein involves predicting the response of the lumbar spine using a finite element model and measured motions during flexion. Capturing the essential behavior necessary to accurately predict the spinal motion and forces requires a detailed model. Conversely, the need for near real-time calculations to allow evaluation of the changes in force and to predict damage during repetitive lifting naturally leads to a less complex model.

The finite element model developed in this research bridges this gap. It provides sufficient detail about the internal mechanisms of motion and stress distribution to allow insight into the behavior. On the other hand it executes quickly enough to allow for long-term, multiple cycle calculations. The overall computational methodology is discussed in this section.

Model Description. The model is built-up using elements representing the different components of the lumbar spine. Vertebral bodies, endplates, posterior elements, ligaments, and intervertebral discs are explicitly modeled. Six vertebral bodies are included in the model, L_5 (lumbar) -T_{12} (thoracic) along with their endplates. Posterior elements are integrated for each of the vertebral bodies. Six discs are placed between the endplates and a variable number of ligaments may be included in the model. A total of thirteen nodes are used (one at the top and one at the bottom of each vertebra, plus the top of the sacrum, S). A two-dimensional model was developed and the sacrum was assumed fixed against translation as a reference point, leading to a total of 37 degrees of freedom (Figure 1).

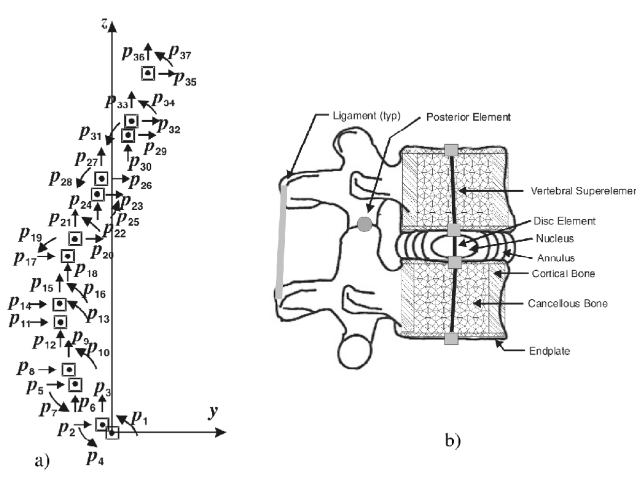

Figure 1: Overall model geometry and components: (a) model nodes and degrees-of-freedom, (b) motion segment detail including all component elements

3. MODEL DETAILS

Vertebrae Model: The vertebral bodies are modeled as linear, elastic, beam-type elements. The behavior of the vertebral bodies appears to follow a basically linear force-displacement relationship for the range of strains considered (Fung, 1993), and the complexity of a nonlinear model does not seem justified. However, the shape of the vertebral bodies, approximately the same height as width, indicates that shear deformations might be important; therefore shear stiffness is included in the element. The cortical and cancellous bone are modeled independently as elements in parallel, one inside the other, having the same displacements at their ends.

The endplates are also modeled as linear, elastic beams including axial, bending, and shear deformations. Although the endplates can exhibit quite complex behavior, only those behavior modes with a significant effect on the overall motion are included in the model. Once the motion and general stress levels have been determined using the current model, more detailed analyses can be performed to predict specific behaviors. For computational efficiency, the endplates are combined with the vertebral bodies into a single superelement using substructuring theory (Sennett, 1994). Geometric stiffness is considered, and is calculated for the superelement rather than for the individual components.

Ligament Model: Since ligaments exhibit mainly a nonlinear (Weiss, 2002; Woo, 1993) uniaxial longitudinal behavior, they are modeled using truss-type elements. In addition, ligaments have little or no ability to carry compression and behave like cables, having an "activation" length (strain) at which they become taut and their stiffness increases dramatically.

The stress-strain behavior of collagen-based ligaments is nonlinear, viscoelastic, and exhibits hysteresis (Fung, 1993, Martin, 1998, Solomonow, 2004). A Voigt model is employed to capture the ligament behavior and consists of a spring and dashpot in parallel. The loading portion of the elastic curve (spring) is characterized by zero stiffness in compression and up to the point where the ligament activates. At higher strains, the stiffness gradually increases, eventually reaching its maximum value.

Ligaments are formed by collagenous tissue consisting mostly of type I collagen. They are tension resistant (Bogduk, 1997) and usually exhibit a behavior similar to isolated collagen fibers, which is characterized by a stress-strain curve (Shah *et al.*, 1977; Shah, 1980; Nordin, 2001) that is divided into four functional regions: a silent zone, transition or toe, linear and yield (Figure 2). Under compression and up to some value of the tensile strain, the ligament exhibits essentially no resistance to deformation – *silent zone*. The center of the transition or *toe zone* between the slack and taut conditions is labeled ε_0 in Figure 2. As the ligament lengthens, the stiffness gradually increases until it reaches a constant value - *linear zone* (starting at Point *A*). Typically, at point *A* the strain values were found to be about 6 to 20% of the initial length (Chazal *et al.*, 1985). As the strain continues to increase, the modulus will eventually start to decrease and failure occurs – *yield zone*. Some ligaments can be strained up to 30% or more without damage (Martin, 1999).

This behavior is modeled by varying the tangent modulus based on the length (strain) of the ligament (Figure 2). Required input data include the modulus for both slack and taut conditions, the strain at which the ligament becomes taut, and a parameter describing the sharpness of the transition from slack to taut. The equation used to describe the behavior is

$$E = \left\{1.0 + \tanh\left(\psi\left[\varepsilon - \varepsilon_0\right]\right)\right\}\left\{\left(E_{taut} - E_{slack}\right)/2.0\right\} + E_{slack}$$

where E is the current tangent modulus, E_{taut} is the tangent modulus at large tensile strains, E_{slack} is the tangent modulus at small tensile and all compressive strains, ψ is a parameter defining the shape of the transition curve, ε is the current ligament strain, and ε_0 is the center strain of the ligament modulus transition ("activation" strain). For the ligament model, E_{slack} is taken as zero and E_{taut} is the activated modulus, obtained from experimental data. The value of ε_0 varies based on the ligament and the average strain at which it stiffens. The curve parameter, ψ, is used to adjust the abruptness of the transition to match measured data. For most ligaments the slack modulus is zero (no tensile capacity), but for the ligamentum flavum, which consists of 80% elastin (Bogduk, 1997, Nordin, 2001), the slack modulus is

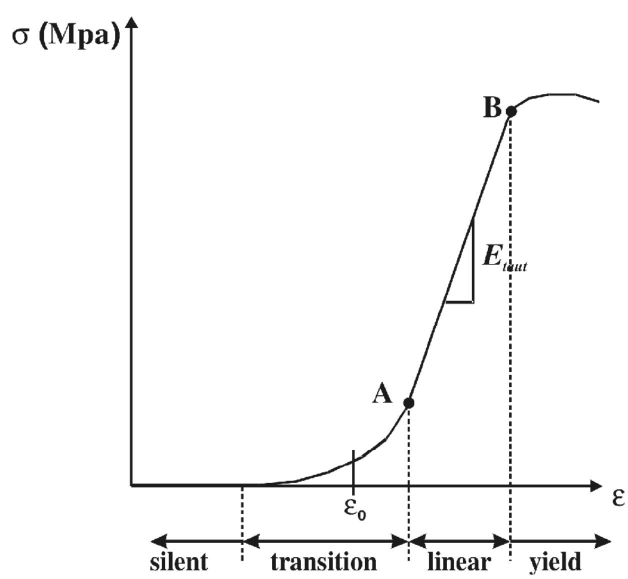

Figure 2: General shape of the ligament stress-strain curve (after Chazal *et al*., 1985)

small, but non-zero. The dashpot is nonlinear with a viscous coefficient, η, relating the ligament stress and strain rate. The viscous coefficient also varies based on the ligament strain using the same transition equation as the modulus but with different parameters. Values for the parameters were determined from an extensive parametric study of the available experimental results (Campbell-Kyureghyan, 2004).

In addition to the ligament material properties and cross sectional area, the attachment location must be part of the input data for each ligament. The ligament axial deformation is determined from the displacement and rotation at the disc centroid assuming the ligament is rigidly attached (Crisfield, 1991) using constraints. Thus, the ligaments carry only axial force, but they contribute to the bending stiffness of the lumbar spine.

Intervertebral Disc Model: The intervertebral discs are the most complicated component in the model. They are made up of three subcomponents: annulus fibers, annulus ground substance, and nucleus. Each of these requires a different model, but they must be combined into a single element for computational efficiency.

The nucleus is modeled as a beam element with shear stiffness. Although it is recognized that the nucleus may behave more like a fluid than a solid, a beam element with properly chosen parameters can satisfactorily model the overall behavior of the nucleus (Campbell-Kyureghyan, 2004). In addition, the beam element is computationally efficient and only a single element is required to adequately capture the overall nucleus behavior.

The annulus consists of collagen fibers embedded in the ground substance. In order to accurately model the spatial variation of the annulus response, the annulus is subdivided into strands running parallel to the element axis (Figure 3a). Each strand has a collagen fiber element and a ground substance element, and is located at a distance from the element centroid. The ground substance is modeled as a linear, viscoelastic truss-type element. Although the truss has only axial stiffness, the distance from the centroid means that it will also contribute to the bending stiffness of the element.

The annulus collagen fibers are modeled as nonlinear truss-type elements. They are oriented at an angle a_v to the element axis, both longitudinally and transversely (Figure 3b). The orientation of the fibers with the respect to the vertical axis is of constant magnitude and alternating sign. This allows the fibers to contribute to the axial, bending, and shear stiffness. The varying horizontal orientation of the fibers also allows for an estimation of the disc bulge. Collagen fiber nonlinear behavior is modeled with the method already used in the ligament property variation.

The nucleus and the annulus strands are parallel to each other. The equivalent element stiffness can be calculated from

$$EA = (EA)_n + \sum E_{gi} A_i (1 - r_i) + E_{ci} A_i r_i \cos \alpha_{vi}$$

$$EI = (EI)_n + \sum E_{gi} A_i (1 - r_i) d_i^2 + E_{ci} A_i r_i d_i^2 \cos \alpha_{vi}$$

$$GA' = (GA')_n + \sum E_{ci} A_i r_i \sin \alpha_{vi} \cos \alpha_{hi}$$

where E is the elastic modulus, A is the cross-sectional area, I is the moment of inertia, G is the shear modulus, A' is the effective shear area, r is the fraction of the annulus made up of collagen, d is the distance between the strand and the centroid, α_{vi} is the angle formed by collagen fiber i with the vertical (element axis) direction, α_{hi} is the angle formed by collagen fiber i with the horizontal direction, and the subscripts n, g, and c refer

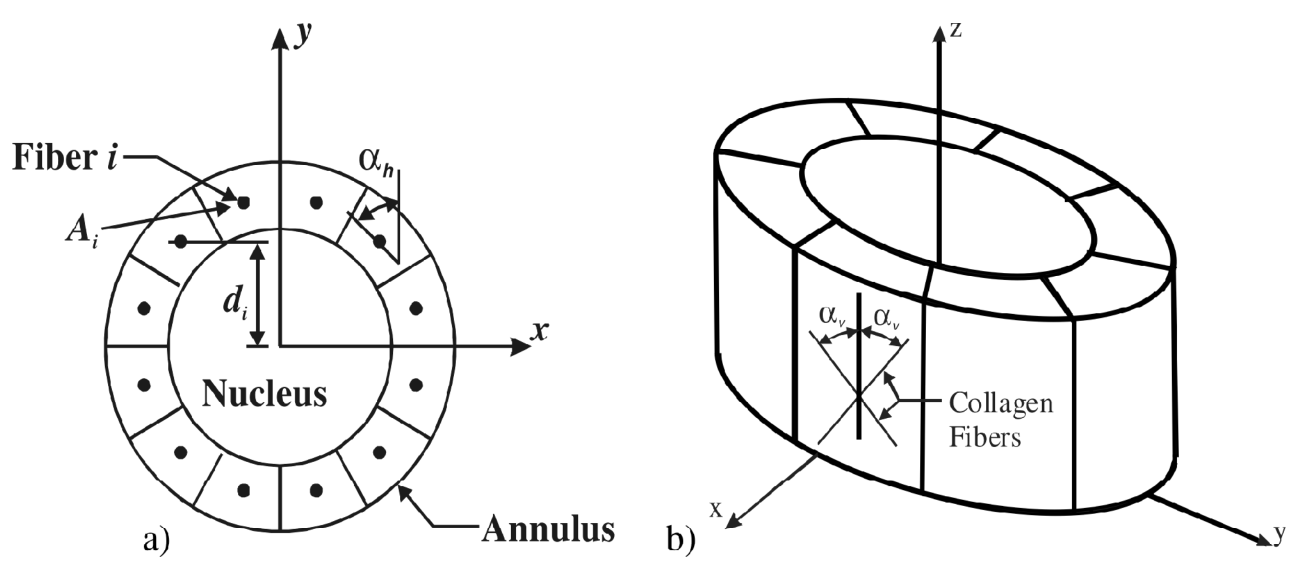

Figure 3: Intervertebral disc model: (a) basic cross-sectional geometry; (b) annulus collagen fiber orientation

to the nucleus, ground substance, and collagen respectively. The design of the disc element allows the behavior and contribution of each of the components to be determined. In addition, the strand design allows the spatial variation in the response of the annulus to be calculated. With a reasonable number of strands, approximately 6-8, quite complex behavior can be obtained without great computational expense.

An increase in stiffness with cyclic loading has been noted in studies on both single components and complete motion segments (Koeller *et al.*, 1984a, 1984b; Farfan, 1973; Hansson *et al.*, 1987; Goel *et al.*, 1988; Yoganandan *et al.*, 1994; Callaghan and McGill, 2001). A common characteristic of the results from each study is a rapid increase in stiffness at the early stages of cyclic loading, followed by a more gradual increase with an asymptotic approach to an ultimate stiffness value. In order to model this effect, the elastic modulus of the components found to exhibit this behavior (discs and ligaments) was varied with cycling. Variation based on the number of cycles would introduce a number of problems including determining what constituted a cycle, partial cycles, and the increase in stiffness within a single cycle. The use of energy as the measure of cycling addresses all of these problems associated with standard cycle determination methodologies.

Energy will increase with loading and decrease with unloading, allowing for variations within a cycle. In addition, the energy will increase or decrease during any loading sequence, eliminating the need to measure "cycles". And finally, the energy will gradually increase with time as some of the energy is dissipated through creep and damping. Addressing all of these issues, an equation relating the elastic modulus with the energy was developed as

$$E = E_{ini} + \left(1 - e^{-\frac{ENG}{ENG_{ref}}}\right)\left(E_{ult} - E_{ini}\right)$$

where E is the current modulus, E_{ini} and E_{ult} are the initial and final values of the modulus, ENG is the current energy in the component, and ENG_{ref} is a reference energy. The reference energy is chosen to set the rate of modulus increase. Figure 4 shows the general curve of the stiffness (modulus) modification versus energy dissipation for a final modulus equal to twice the initial modulus.

4. LOADING AND MODEL CALCULATIONS

Loading: The spinal column without the supporting muscles, rib cage, etc. is an unstable structure. The application of load directly to the unsupported spine would result in immediate instability. To overcome this issue, the current model loads the spine through imposing displacements on the top of T_{12}. The loading

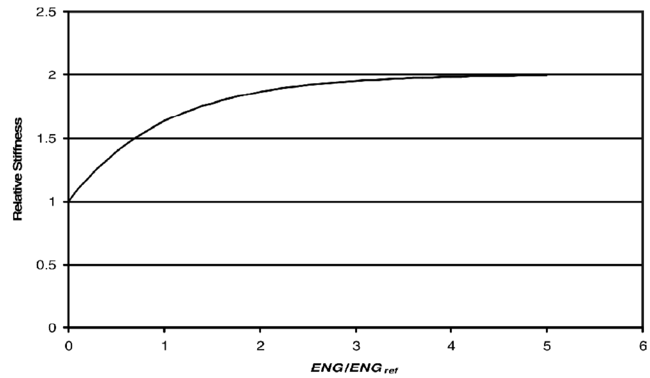

Figure 4: Variation of elastic modulus with component energy

(displacement) for the model is derived from LMM data recorded during actual sagittal lifting tasks as described in the following section. The motion of the top of vertebra T_{12} (top node of the model) and the rotation of the sacrum have been calibrated against the LMM measurements (Campbell-Kyureghyan, *et al.*, 2005). This allows for a determination of the position of the top node in the model and the rotation of the bottom node at each step. The displacements are applied to the model using a Penalty Method (Bathe, 1996).

Calculations. Calculating the lumbar spine response consists of three phases (Figure 5). First, a model for the specific subject and task must be defined. Based on the anthropometric data of the subject and the initial LMM readings, the initial position of the nodes (Campbell-

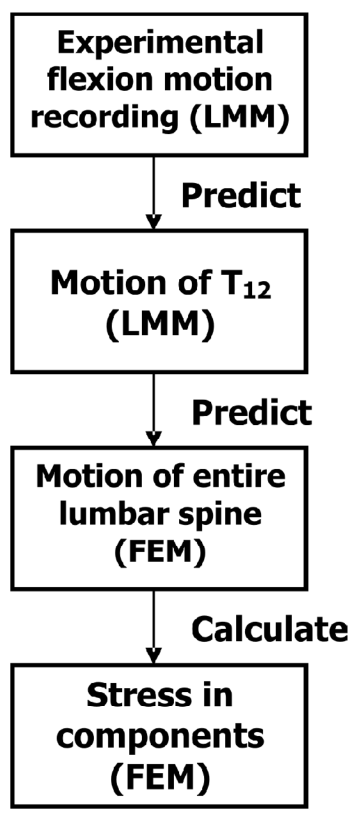

Figure 5: Flow chart for the overall solution procedure

Kyureghyan *et al.*, 2005) is determined. The user defines the material and cross sectional properties, including the number of fibers used to model the disc annulus. Second, at each step of the calculation the position of T_{12} is determined from the measured LMM data during a sagittal lift. Finally, after the boundary displacements are imposed, the displacements of the remaining nodes are calculated and used to determine the component deformations and stresses.

5. DISCUSSION

This paper described a model that can be used to analyze the response of the lumber spine to realistic, dynamic, cyclic loads and bridges the gap between complex finite element models and simplified models. The model incorporates all the major spinal components and their behaviors that contribute to the overall deformation. Sufficient detail is included to allow for the prediction of detailed response quantities such as the maximum and minimum stress in the disc, ligament forces, and energy dissipation. The model is intended to interface with the Lumbar Motion Monitor for determination of the input motion. Within the limitation imposed by the model complexity, it can be a valuable tool for investigating the response of the lumbar spine to typical flexion motions.

There are several advantages of such a model. First, many finite element models have been developed that provide detailed information on the response of single motion segments. However, these models have only been applied to simple loading conditions for static or extremely short load durations. While providing a valuable insight into the motion segment response, the models do not provide any information on the long-term cyclic behavior. This model provides a method for investigating these effects.

Second, this model provides information for the entire range of motion during a flexion cycle. Other models exist that provide snapshots of the forces for a given posture or peak values, but do not give a full time history. The temporal information provided by this model allows for the calculation of secondary response parameters such as energy dissipation and to investigate the relationship between posture, load, velocity, and the resulting response. This information may provide valuable insight into the injury mechanism.

This model could be used in conjunction with existing tools to provide a more complete picture of lumbar spine behavior. One could use a simple, goniometer-driven model to determine the motions most likely to cause injury. The model presented in this paper would then be used to provide a more detailed look at the motion and the resulting forces and deformations. The results would pinpoint the segments and behavior modes most likely to cause injury, and provide input deformations, postures, and forces to more complex finite element models of individual motion segments for detailed analysis.

Possible improvements to this model include adding a muscle model and extending the model to three-dimensions. Without muscle force in the model only passive forces are predicted. The active muscle forces would also act to stabilize the model during deformation. At present only two-dimensional, sagitally symmetric motion can be analyzed. The individual components of the model were developed to be easily adapted to three dimensions and such an extension would enhance the utility of the model.

Finally, this model has been tested and validated against experimental results in the literature. Each component was examined for its ability to reproduce the range of observed behavior and parametric analyses were conducted to determine the effect of each input variable on the results. The complete model was then tested using experimentally measured cyclic sagittal flexion motions and the predictions were compared to reported values. A companion article (Campbell-Kyureghyan and Marras, 2007 [this issue]) describes the experimental conditions and validation study. The ability to rapidly generate and execute models, including linking the model to real-world loading conditions, allows the use of this model in conditions where more traditional analytical models are either too time consuming or of inadequate detail.

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