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Robust Adaptive Backstepping Controller for a Class of Nonlinear Cascade Systems via Fuzzy Neural Networks

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Abstract: *In this paper, a robust adaptive backstepping control scheme using fuzzy neural networks, called FNN-ABC, is proposed for a class of nonlinear uncertain systems with cascade structure. Each subsystem is in the form of lower triangular and non-affine systems which contains of external disturbance, uncertainties, or parameters variations. By the backstepping approach, a fuzzy neural network (FNN) based robust adaptive controller is designed in a step by step manner for each subsystem. Two kinds of FNN systems are used to estimate the subsystems' unknown functions. According to the FNNs' estimations, the FNN-ABC control input can be generated by Lyapunov approach such that system output follows the desired trajectory. To enhance the control performance (or FNNs' approximation accuracy), a Taylor expansion method are adopted to derive the update laws of FNNs' antecedent-part parameters. Based on the Lyapunov approach, the adaptive laws of FNNs' parameters and stability analysis of closed-loop system are obtained. Finally, the proposed FNN-ABC is applied to the tracking control of a single-link flexible-joint robot. A simulation study is proposed to illustrate the performances of our approach.*

Keywords: *Nonlinear cascade system, backstepping, fuzzy neural network, adaptive control, Lyapunov*

1. INTRODUCTION

Over the past decade, both fuzzy logic systems and neural networks have found extensive applications for plants that are complex and ill-defined. In most of these cases, they have been used as universal approximators [1-6]. Recently, they have been merged to obtain the fuzzy neural network (FNN) systems (or neuro-fuzzy systems) and have the advantages of both fuzzy logic and neural networks. Based on the approximation ability, many adaptive control techniques are accompanied with them for approximation of system functions or controllers [4, 7-20].

The backstepping design provides a systematic framework and recursive design methodology for nonlinear systems [7, 8, 11, 16, 21]. The design procedure treats the state variables as virtual control inputs, to design step by step to design the virtual controllers and prove the stability by Lyapunov stability theorem. The actual control input can be obtained in the final step. However, the major constraint is that the system functions must be known. If the internal uncertainty and external disturbance exist, then the system maybe become unstable. Therefore, the FNNs are used to approximate the unknown functions to solve this problem.

Recently, newly backstepping control schemes that combine the backstepping technique and neural networks were proposed for nonlinear systems with the lower triangular form (or the nested lower triangular form) [6, 11, 15, 19]. However, most of these approaches still have some constraints, e.g., the gain functions of control input were assumed to be constants (or exactly known). Besides, they are also limited to the feedback linearizable nonlinear systems, which mean that the unknown nonlinearities must satisfy a matching condition. However, many real-world systems do not satisfy such a condition, especial non-affine system. The problem of controlling non-affine system is a difficult one [14, 17-20]. To treat the problem, widely used in practice, is that based or linearization of the nonlinear plant model around an operating point. While the linearization may result in the design of sufficiently accurate controllers in the case of stabilization, in the case of tracking of desired trajectories the problem becomes much more difficult. Hence there is a clear need for the development of systematic control design techniques for nonlinear models that are non-affine in u, and are suitable for the case of tracking of desired trajectories.

In this study, we proposed an FNN-based robust adaptive backstepping control scheme (FNN-ABC) for a class of nonlinear cascade systems with lower triangular non-affine form, i.e., it is not in feedback linearizable form. This controller is designed in the concept of the backstepping control procedure, and the FNNs are used to estimate the unknown system functions online. The adaptive laws of FNNs' parameter are obtained by the Lyapunov approach; therefore the tracking error can be guaranteed uniformly ultimately bounded (UUB).

This paper is organized as follows. Section 2 introduces the problem formulation, design procedure of backstepping controller, and FNN systems. The FNN-ABC robust control scheme is introduced in Section 3. The stability analysis is guaranteed by the Lyapunov theorem. Section 4 shows the simulation results of a single-link flexible-joint robot. Finally, conclusion is given in Section 5.

2. PRELIMINARIES

2.1. Problem Formulation and Backstepping Controller

Consider the following nth-order nonlinear cascade system as

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= x_3 \\ &\cdots \\ \dot{x}_p &= F_1\left(\overline{x}_p,\ x_{p+1},\ d_1\right) \\ \dot{x}_{p+1} &= x_{p+2} \\ \dot{x}_{p+2} &= x_{p+3} \\ &\cdots \\ \dot{x}_n &= F_2\left(\overline{x}_n,\ u,\ d_2\right) \\ y &= x_1 \end{aligned} \tag{1}$$

where $\overline{x}_i = \left[x_1,\ x_2,\ \ldots,\ x_i\right]^T \in \Re^i$, $i = 1, 2, \ldots, n$ denote the system state vectors; $u \in \Re$ and $y \in \Re$ are the input and output, respectively. F_i and d_i, (i=1, 2) are the nonlinear smooth functions and bounded external disturbance, respectively. System (1) cannot be stabilized by feedback linearization approach. Note that system (1) can be viewed as two subsystems in cascade structure, each subsystem is lower triangular and non-affine. The problem of controlling nonlinear non-affine system is a difficult one. In this paper, the control objective is to design the control input u such that the output y follows a desired bounded trajectory y_d. The following assumptions are made for the controllability of system (1).

Assumption 1: The inequality $\left|\partial F_1\left(\overline{x}_p, x_{p+1},\ d_1\right)/\partial x_{p+1}\right| \neq 0$ hold for all $\overline{x}_{p+1} \in \Re^{p+1}$.

Assumption 2: The inequality $\left|\partial F_2\left(\overline{x}_n, u,\ d_n\right)/\partial u\right| \neq 0$ holds for all $\left(\overline{x}_n, u\right) \in \Omega_x \times \Re$ with a controllability region Ω_x.

The above assumptions imply that the $\partial F_1\left(\overline{x}_p, x_{p+1},\ d_1\right)/\partial x_{p+1}$ and $\partial F_2\left(\overline{x}_n, u,\ d_2\right)/\partial u$ are strictly either positive or negative.

We define the state transformation $z_i = x_i$, $q_j = x_{p+j}$, $i = 1, \ldots, p$, $j = 1, \ldots, m$, $p + m = n$. System (1) can be viewed as two connected subsystems

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_p &= F_1\left(\overline{z}_p, q_1,\ d_1\right) \end{aligned} \tag{2}$$

and

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= q_3 \\ &\cdots \\ \dot{q}_m &= F_2\left(\overline{z}_p, \overline{q}_m, u,\ d_2\right). \end{aligned} \tag{3}$$

According to the concept of literature [14], subsystems (2) and (3) can be written as

$$\begin{aligned} \dot{z}_1 &= z_2 \\ \dot{z}_2 &= z_3 \\ &\cdots \\ \dot{z}_p &= h_1\left(\overline{z}_p, q_1,\ d_1\right) + c_1 q_1 \end{aligned} \tag{4}$$

and

$$\begin{aligned} \dot{q}_1 &= q_2 \\ \dot{q}_2 &= q_3 \\ &\cdots \\ \dot{q}_m &= h_2\left(\overline{z}_p, \overline{q}_m, u,\ d_2\right) + c_2 u \end{aligned} \tag{5}$$

where $h_1 = F_1\left(\overline{z}_p, q_1,\ d_1\right) - c_1 q_1$, $h_2 = F_2\left(\overline{z}_p, \overline{q}_m, u,\ d_2\right) - c_2 u$, and c_1, c_2 are design nonzero constants. Then subsystems (4) and (5) are lower triangular and affine-like. Let $\mathbf{z} = \left[z_1\ \dot{z}_1\ \cdots\ z_1^{(p-1)}\right]^T = \left[z_1\ z_2\ \cdots\ z_p\right]^T \in \Re^p$, $\mathbf{q} = \left[q_1\ \dot{q}_1\ \cdots\ q_1^{(m-1)}\right]^T = \left[q_1\ q_2\ \cdots\ q_m\right]^T \in \Re^m$. The desired output z_d of z-subsystem and its derivative are assumed to be bounded. In this paper, we use the backstepping design procedure to design the stabilizing controller for system (4)-(5).

Step 1: Define the tracking error $e_z = y - y_d = z_1 - z_d$ and the corresponding error vector as $\mathbf{e_z} = \left[e_z\ \dot{e}_z\ \cdots\ e_z^{(p-1)}\right]^T \in \Re^p$, then

$$e_z^{(p)} = z_1^{(p)} - z_d^{(p)} = h_1 + c_1 q_1 - z_d^{(p)}. \tag{6}$$

There exists a stabilizing virtual controller for (6)

$$q_d = \frac{1}{c_1}\left(-\mathbf{k}_1^T \mathbf{e_z} + z_d^{(p)} - h_1\right) \tag{7}$$

where $\mathbf{k}_1 = [k_{1p}, \ldots, k_{11}]^T$ is determined such that $s^p + k_{11}s^{p-1} + \ldots + k_{1p}$ is Hurwitz. Let $e_q = q_1 - q_d$, thus

$$e_z^{(p)} = -\mathbf{k}_1^T\mathbf{e}_z + c_1 e_q \tag{8}$$

or

$$\dot{\mathbf{e}}_z = \mathbf{A}_1\mathbf{e}_z + \mathbf{b}_1 c_1 e_q \tag{9}$$

where

$$\mathbf{A}_1 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -k_{1p} & -k_{1p-1} & -k_{1p-2} & \cdots & -k_{12} & -k_{11} \end{bmatrix} \text{ and } \mathbf{b}_1 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

Since $\mathbf{A}_1$ is stable (Hurwitz), there exists a positive definite symmetric matrix $\mathbf{P}_1$ satisfies

$$\mathbf{A}_1^T\mathbf{P}_1 + \mathbf{P}_1\mathbf{A}_1 = -\mathbf{Q}_1 \tag{10}$$

for an arbitrary positive definite matrix $\mathbf{Q}_1$. The Lyapunov candidate function is chosen as

$$V_1 = \frac{1}{2}\mathbf{e}_z^T\mathbf{P}_1\mathbf{e}_z \tag{11}$$

such that

$$\dot{V}_1 = \mathbf{e}_z^T\mathbf{P}_1\dot{\mathbf{e}}_z = -\frac{1}{2}\mathbf{e}_z^T\mathbf{Q}_1\mathbf{e}_z + \mathbf{e}_z^T\mathbf{P}_1\mathbf{b}_1 c_1 e_q. \tag{12}$$

Step 2: As the definition in step 1, $e_q = q_1 - q_d$ and the corresponding error vector as $\mathbf{e}_q = \left[e_q \ \dot{e}_q \ \cdots \ e_q^{(m-1)}\right]^T \in \Re^m$. The derivative of $e_q^{(m-1)}$ is

$$e_q^{(m)} = q_1^{(m)} - q_d^{(m)} = h_2 + c_2 u - q_d^{(m)}. \tag{13}$$

Actual control input u is designed as

$$u = \frac{1}{c_2}\left(-\mathbf{k}_2^T\mathbf{e}_q - \frac{\mathbf{e}_z^T\mathbf{P}_1\mathbf{b}_1 c_1 e_q}{\mathbf{e}_q^T\mathbf{P}_2\mathbf{b}_2} + q_d^{(m)} - h_2\right) \tag{14}$$

where $\mathbf{k}_2 = \left[k_{2m}, \ \ldots, k_{21}\right]^T$ is determined such that $s^m + k_{21}s^{m-1} + \ldots + k_{2m}$ is Hurwitz. Substitute (14) into (13)

$$e_q^{(m)} = -\mathbf{k}_2^T\mathbf{e}_q - \frac{\mathbf{e}_z^T\mathbf{P}_1\mathbf{b}_1 c_1 e_q}{\mathbf{e}_q^T\mathbf{P}_2\mathbf{b}_2} \tag{15}$$

or

$$\dot{\mathbf{e}}_q = \mathbf{A}_2\mathbf{e}_q + \mathbf{b}_2\left(-\frac{\mathbf{e}_z^T\mathbf{P}_1\mathbf{b}_1 c_1 e_q}{\mathbf{e}_q^T\mathbf{P}_2\mathbf{b}_2}\right) \tag{16}$$

where

$$\mathbf{A}_2 = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -k_{2m} & -k_{2m-1} & -k_{2m-2} & \cdots & -k_{22} & -k_{21} \end{bmatrix} \text{ and } \mathbf{b}_2 = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}.$$

$\mathbf{A}_2$ is Hurwitz, there exists a positive definite symmetric matrix $\mathbf{P}_2$ satisfies

$$\mathbf{A}_2^T\mathbf{P}_2 + \mathbf{P}_2\mathbf{A}_2 = -\mathbf{Q}_2 \tag{17}$$

for positive definite matrix $\mathbf{Q}_2$. The following Lyapunov candidate function is chosen

$$V_2 = V_1 + \frac{1}{2}\mathbf{e}_q^T\mathbf{P}_2\mathbf{e}_q, \tag{18}$$

we then have

$$\begin{aligned} \dot{V}_2 &= \dot{V}_1 + \mathbf{e}_q^T\mathbf{P}_2\dot{\mathbf{e}}_q \\ &= \dot{V}_1 - \frac{1}{2}\mathbf{e}_q^T\mathbf{Q}_2\mathbf{e}_q - \mathbf{e}_z^T\mathbf{P}_1\mathbf{b}_1 c_1 e_q \\ &= -\frac{1}{2}\mathbf{e}_z^T\mathbf{Q}_1\mathbf{e}_z - \frac{1}{2}\mathbf{e}_q^T\mathbf{Q}_2\mathbf{e}_q. \end{aligned} \tag{19}$$

Therefore, the asymptotically stability of system (1) is guaranteed. However, since F_i (i=1, 2) are not exactly known that include both internal uncertainties and external disturbance, i.e., h_i (i=1, 2) are also unknown and controller (14) maybe results unstable. Therefore, fuzzy neural network (FNN) systems are used to estimate the unknown functions h_1 and h_2. In addition, h_2 depends on u and u appears in both the left- and right-hand sides of (14). Therefore, a dynamic neural network¡ĐRFNN is introduced to estimate h_2.

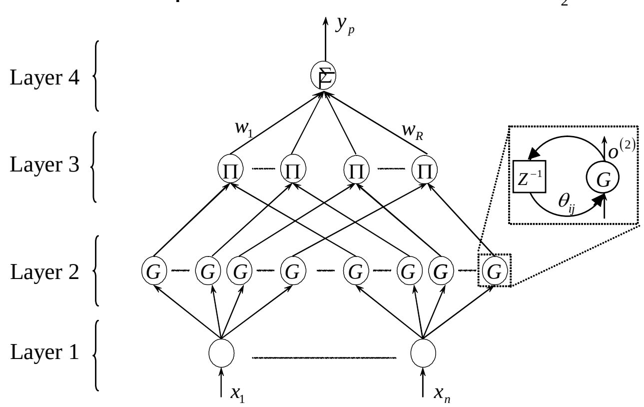

Figure 1: Schematic Diagram of Recurrent Fuzzy Neural Network (R Denotes the Amount of Rule Numbers)

2.2. Fuzzy Neural Network Systems

Herein, we introduce the used recurrent fuzzy neural network (RFNN) systems [9]. The schematic diagram of RFNN is shown in Fig. 1. There are four layers. Layer 1 is input layer and each node represents an input linguistic variable. The nodes in this layer only transmit input variables to the next layer, i.e., $O_i^{(1)}(k) = x_i(k)$. Layer 2- membership layer, is used to calculate Gaussian membership grade, i.e.,

$$O_{ij}^{(2)}(k) = \exp\left[-\frac{\left(\left(O_{ij}^{(2)}(k-1)\cdot\theta_{ij} + O_i^{(1)}(k)\right) - m_{ij}\right)^2}{\left(\sigma_{ij}\right)^2}\right] \tag{20}$$

where m_{ij} and σ_{ij} are the center and the width of the Gaussian function, θ_{ij} is the adjustable feedback parameter. It is worth to mention, if we set $\theta_{ij}=0$, the RFNN can be reduced as the FNN system [4, 9]. Nodes in layer 2 represent the terms of the respective linguistic variables. Nodes in layer 3 represent fuzzy rules, i.e., layer 3 forms the fuzzy rule base. Links before layer 3 represent the antecedent part of fuzzy rules, and the links after layer 3 represent the consequence part. The product operation is used, i.e.,

$$O_j^{(3)}(k) = \prod_i O_{ij}^{(2)}(k). \tag{21}$$

Layer 4 is the output layer. Each node is for actual output to be pumped out this system. The links between layer 3 and layer 4 are connected by the weighting value w_j, i.e.,

$$y_p = O^{(4)}(k) = \mathbf{w}^T \boldsymbol{\psi} = \frac{\sum_{j=1}^{R} w_j O_j^{(3)}(k)}{\sum_{j=1}^{R} O_j^{(3)}(k)} \tag{22}$$

where $\mathbf{w} = [w_1\ w_2,\ \ldots,\ w_R]^T$ is the weighting vector; $\boldsymbol{\psi} = \left[\boldsymbol{\psi}^1\ \boldsymbol{\psi}^2, \ldots, \boldsymbol{\psi}^R\right]$, $\boldsymbol{\psi}^j = O_j^{(3)}(k) \Big/ \sum_j^R O_j^{(3)}(k)$ represents the normalized value. By the results of [9], the RFNN is suitable for on-line estimation. That is, for any given real function hi : $\Re^n \to \Re^p$, there exists optimal parameters, the function h_i can be described by the output of the RFNN with a reconstruction error. From (22), the FNN's and RFNN's outputs are

$$\hat{h}_1(x) = y_{p1} = \hat{\mathbf{w}}_1^T \hat{\boldsymbol{\psi}}_1 = \hat{\mathbf{w}}_1^T \boldsymbol{\psi}\left(\hat{m}^{(1)}, \hat{\sigma}^{(1)}, x\right) \tag{23a}$$

$$\hat{h}_2(x) = y_{p2} = \hat{\mathbf{w}}_2^T \hat{\boldsymbol{\psi}}_2 = \hat{\mathbf{w}}_2^T \boldsymbol{\psi}\left(\hat{m}^{(2)}, \hat{\sigma}^{(2)}, \hat{\theta}^{(2)}, x\right), \tag{23b}$$

therefore, the unknown functions h_i are

$$h_1 = \mathbf{w}_1^{*T} \boldsymbol{\psi}_1^* + \Delta_1 = \mathbf{w}_1^{*T} \boldsymbol{\psi}\left(m^{*(1)}, \sigma^{*(1)}, x\right) + \Delta_1 \tag{24a}$$

$$h_2 = \mathbf{w}_2^{*T} \boldsymbol{\psi}_2^* + \Delta_2 = \mathbf{w}_2^{*T} \boldsymbol{\psi}\left(m^{*(2)}, \sigma^{*(2)}, \theta^{*(2)}, x\right) + \Delta_2 \tag{24b}$$

where $\Delta_i (i=1, 2)$ are the approximation errors; $\mathbf{w}_i^*$, $m^{*(i)}$, $\sigma^{*(i)}$, $\theta^{*(i)}$ are the optimal vectors of $\mathbf{w}_i$, $m^{(i)}$, $\sigma^{(i)}$, $\theta^{(i)}$ of the ith FNN, respectively. From (23), the function estimated error $\tilde{h}_i$ satisfies

$$\begin{aligned}
\tilde{h}_i &= h_i - \hat{h}_i \\
&= \mathbf{w}_i^{*T} \boldsymbol{\psi}_i^* + \Delta_i - \hat{\mathbf{w}}_i^T \hat{\boldsymbol{\psi}}_i \\
&= \mathbf{w}_i^{*T} \boldsymbol{\psi}_i^* - \mathbf{w}_i^{*T} \hat{\boldsymbol{\psi}}_i + \mathbf{w}_i^{*T} \hat{\boldsymbol{\psi}}_i - \hat{\mathbf{w}}_i^T \hat{\boldsymbol{\psi}}_i + \Delta_i \\
&= \mathbf{w}_i^{*T} \tilde{\boldsymbol{\psi}}_i + \tilde{\mathbf{w}}_i^T \hat{\boldsymbol{\psi}}_i + \Delta_i, \qquad i = 1, 2.
\end{aligned} \tag{25}$$

The linearization technique is employed to partially linear form [12], we have

$$\tilde{\boldsymbol{\psi}}_1 = \begin{bmatrix} \tilde{\boldsymbol{\psi}}^{1(1)} \\ \tilde{\boldsymbol{\psi}}^{2(1)} \\ \vdots \\ \tilde{\boldsymbol{\psi}}^{R(1)} \end{bmatrix} = \left.\begin{bmatrix} \frac{\partial \tilde{\boldsymbol{\psi}}^{1(1)}}{\partial m^{(1)}} \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{2(1)}}{\partial m^{(1)}} \\ \vdots \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{R(1)}}{\partial m^{(1)}} \end{bmatrix}\right|_{m^{(1)}=\hat{m}^{(1)}} \left(m^{*(1)} - \hat{m}^{(1)}\right) + \left.\begin{bmatrix} \frac{\partial \tilde{\boldsymbol{\psi}}^{1(1)}}{\partial \sigma^{(1)}} \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{2(1)}}{\partial \sigma^{(1)}} \\ \vdots \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{R(1)}}{\partial \sigma^{(1)}} \end{bmatrix}\right|_{\sigma^{(1)}=\hat{\sigma}^{(1)}} \left(\sigma^{*(1)} - \hat{\sigma}^{(1)}\right) + H_1 \tag{26a}$$

$$\begin{aligned}\tilde{\boldsymbol{\psi}}_2 = \begin{bmatrix} \tilde{\boldsymbol{\psi}}^{1(2)} \\ \tilde{\boldsymbol{\psi}}^{2(2)} \\ \vdots \\ \tilde{\boldsymbol{\psi}}^{R(2)} \end{bmatrix} &= \left.\begin{bmatrix} \frac{\partial \tilde{\boldsymbol{\psi}}^{1(2)}}{\partial m^{(2)}} \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{2(2)}}{\partial m^{(2)}} \\ \vdots \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{R(2)}}{\partial m^{(2)}} \end{bmatrix}\right|_{m^{(2)}=\hat{m}^{(2)}} \left(m^{*(2)} - \hat{m}^{(2)}\right) + \left.\begin{bmatrix} \frac{\partial \tilde{\boldsymbol{\psi}}^{1(2)}}{\partial \sigma^{(2)}} \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{2(2)}}{\partial \sigma^{(2)}} \\ \vdots \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{R(2)}}{\partial \sigma^{(2)}} \end{bmatrix}\right|_{\sigma^{(2)}=\hat{\sigma}^{(2)}} \left(\sigma^{*(2)} - \hat{\sigma}^{(2)}\right) \\
&+ \left.\begin{bmatrix} \frac{\partial \tilde{\boldsymbol{\psi}}^{1(2)}}{\partial \theta^{(2)}} \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{2(2)}}{\partial \theta^{(2)}} \\ \vdots \\ \frac{\partial \tilde{\boldsymbol{\psi}}^{R(2)}}{\partial \theta^{(2)}} \end{bmatrix}\right|_{\theta^{(2)}=\hat{\theta}^{(2)}} \left(\theta^{*(2)} - \hat{\theta}^{(2)}\right) + H_2\end{aligned} \tag{26b}$$

or

$$\tilde{\boldsymbol{\psi}}_1 = O_{m1}^T \tilde{m}^{(1)} + O_{\sigma1}^T \tilde{\sigma}^{(1)} + H_1 \tag{27a}$$

$$\tilde{\boldsymbol{\psi}}_2 = O_{m2}^T \tilde{m}^{(2)} + O_{\sigma2}^T \tilde{\sigma}^{(2)} + O_{\theta2}^T \tilde{\theta}^{(2)} + H_2 \tag{27b}$$

where O_{mi}, $O_{\sigma i}$, and $O_{\theta i}$ are derivatives of $\hat{\boldsymbol{\psi}}_i$ with respective to $m^{*(i)}$, $\sigma^{*(i)}$, and $\theta^{*(i)}$ at $\left[m^{(i)}\ \sigma^{(i)}\ \theta^{(i)}\right] = \left[m^{*(i)}\ \sigma^{*(i)}\ \theta^{*(i)}\right]$; $\tilde{\mathbf{w}}_i = \mathbf{w}_i^* - \hat{\mathbf{w}}_i$, $\tilde{m}^{(i)} = m^{*(i)} - \hat{m}^{(i)}$, $\tilde{\sigma}^{(i)} = \sigma^{*(i)} - \hat{\sigma}^{(i)}$, $\tilde{\theta}^{(n)} = \theta^{*(n)} - \hat{\theta}^{(n)}$; H_i, $i=1, 2$ are higher-order terms. Substitute (27) into (25), we have

$$\begin{aligned}
\tilde{h}_1 &= \mathbf{w}_1^{*T} \tilde{\boldsymbol{\psi}}_1 + \tilde{\mathbf{w}}_1^T \hat{\boldsymbol{\psi}}_1 + \Delta_1 \\
&= \mathbf{w}_1^{*T} \left(O_{m1}^T \tilde{m}^{(1)} + O_{\sigma1}^T \tilde{\sigma}^{(1)} + H_1\right) + \tilde{\mathbf{w}}_1^T \hat{\boldsymbol{\psi}}_1 + \Delta_1 \\
&= \left(\hat{\mathbf{w}}_1 + \tilde{\mathbf{w}}_1\right)^T \left[O_{m1}^T \left(m^{*(1)} - \hat{m}^{(1)}\right) + O_{\sigma1}^T \left(\sigma^{*(1)} - \hat{\sigma}^{(1)}\right)\right] \\
&\qquad + \mathbf{w}_1^{*T} H_1 + \tilde{\mathbf{w}}_1^T \hat{\boldsymbol{\psi}}_1 + \Delta_1 \\
&= \tilde{\mathbf{w}}_1^T \left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T \hat{m}^{(1)} - O_{\sigma1}^T \hat{\sigma}^{(1)}\right) + \hat{\mathbf{w}}_1^T \left(O_{m1}^T \tilde{m}^{(1)} + O_{\sigma1}^T \tilde{\sigma}^{(1)}\right) \\
&\qquad + \mathbf{w}_1^{*T} H_1 + \tilde{\mathbf{w}}_1^T \left(O_{m1}^T m^{*(1)} + O_{\sigma1}^T \sigma^{*(1)}\right) + \Delta_1 \\
&= \tilde{\mathbf{w}}_1^T \left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T \hat{m}^{(1)} - O_{\sigma1}^T \hat{\sigma}^{(1)}\right) + \hat{\mathbf{w}}_1^T \left(O_{m1}^T \tilde{m}^{(1)} + O_{\sigma1}^T \tilde{\sigma}^{(1)}\right) + \omega_1
\end{aligned} \tag{28a}$$

$$\begin{aligned}
\tilde{h}_2 &= \mathbf{w}_2^{*T}\tilde{\boldsymbol{\psi}}_2 + \tilde{\mathbf{w}}_2^T\hat{\boldsymbol{\psi}}_2 + \Delta_2 \\
&= \mathbf{w}_2^{*T}\left(O_{m2}^T\tilde{m}^{(2)} + O_{\sigma 2}^T\tilde{\sigma}^{(2)} + O_{\theta 2}^T\tilde{\theta}^{(2)} + H_2\right) + \tilde{\mathbf{w}}_2^T\hat{\boldsymbol{\psi}}_2 + \Delta_2 \\
&= \left(\hat{\mathbf{w}}_2 + \tilde{\mathbf{w}}_2\right)^T\left[O_{m2}^T\left(m^{*(2)} - \hat{m}^{(2)}\right) + O_{\sigma 2}^T\left(\sigma^{*(2)} - \hat{\sigma}^{(2)}\right) + O_{\theta 2}^T\left(\theta^{*(2)} - \hat{\theta}^{(2)}\right)\right] \\
&\qquad + \mathbf{w}_2^{*T}H_2 + \tilde{\mathbf{w}}_2^T\hat{\boldsymbol{\psi}}_2 + \Delta_2 \\
&= \tilde{\mathbf{w}}_2^T\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) \\
&\qquad + \hat{\mathbf{w}}_2^T\left(O_{m2}^T\tilde{m}^{(2)} + O_{\sigma 2}^T\tilde{\sigma}^{(2)} + O_{\theta 2}^T\tilde{\theta}^{(2)}\right) + \mathbf{w}_2^{*T}H_2 \\
&\quad + \tilde{\mathbf{w}}_2^T\left(O_{m2}^T m^{*(2)} + O_{\sigma 2}^T\sigma^{*(2)} + O_{\theta 2}^T\theta^{*(2)}\right) + \Delta_2 \\
&= \tilde{\mathbf{w}}_2^T\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) \\
&\qquad + \hat{\mathbf{w}}_2^T\left(O_{m2}^T\tilde{m}^{(2)} + O_{\sigma 2}^T\tilde{\sigma}^{(2)} + O_{\theta 2}^T\tilde{\theta}^{(2)}\right) + \omega_2
\end{aligned} \tag{28b}$$

where $\omega_1 = \mathbf{w}_1^{*T}H_1 + \tilde{\mathbf{w}}_1^T\left(O_{m1}^T m^{*(1)} + O_{\sigma 1}^T\sigma^{*(1)}\right) + \Delta_1$ and $\omega_2 = \mathbf{w}_2^{*T}H_2 + \tilde{\mathbf{w}}_2^T(O_{m2}^T m^{*(2)} + O_{\sigma 2}^T\sigma^{*(2)}\ O_{\theta 2}^T\theta^{*(2)}) + \Delta_2$ are assumed to be bounded by $|\omega_i| \le D_i$, $i=1, 2$.

Our objective is to design a control scheme for system (1) under unknown system functions such that the output y follows a desired trajectory y_d. FNNs are used to estimate the unknown functions as above discussion, and then the FNN based adaptive backstepping control scheme can be designed.

3. ROBUST ADAPTIVE BACKSTEPPING CONTROL USING FNNS (FNN-ABC)

The block diagram of our proposed robust FNN-based adaptive backstepping control scheme (FNN-ABC) for nonlinear cascade systems (1) is shown in Fig. 2. Each virtual controller and actual controller contains three parts- linear state feedback controller, FNN's nonlinear function estimation, and robust controllers. The inputs of FNN and RFNN are $\bar{x}_p = [x_1, x_2, ..., x_p]^T$ and $\bar{x}_n = [x_1, x_2, ..., x_n]^T$. They are used to estimate the unknown functions h_1 and h_2, denote as $\hat{h}_1$ and $\hat{h}_2$, respectively. Thus, we have the following theorem.

Theorem 1: Consider nonlinear cascade system (1) satisfying *Assumptions* 1 and 2, the corresponding virtual controller and control input are designed as

$$q_d = \frac{1}{c_1}\left(-\mathbf{k}_1^T\mathbf{e_z} + z_d^{(p)} - \hat{h}_1 + u_{r1}\right) \tag{29}$$

$$u = \frac{1}{c_2}\left(-\mathbf{k}_2^T\mathbf{e_q} - \frac{\mathbf{e_z^T P_1 b_1}c_1 e_q}{\mathbf{e_q^T P_2 b_2}} + q_d^{(m)} - \hat{h}_2 + u_{r2}\right) \tag{30}$$

$$u_{r1} = -\hat{\delta}_1 \mathrm{sat}\left(\mathbf{e_z^T P_1 b_1},\ \mathrm{J}_1\right) \tag{31}$$

$$u_{r2} = -\hat{\delta}_2 \mathrm{sat}\left(\mathbf{e_q^T P_2 b_2},\ \mathrm{J}_2\right) \tag{32}$$

where u_{ri} (i = 1, 2) represent the robust control inputs, $\hat{h}_i$ are the FNNs' approximation of function h_i (i = 1, 2), $\hat{\delta}_i$ are the estimations of δ_i (i=1, 2), the estimated errors is defined as $\tilde{\delta}_i = \delta_i - \hat{\delta}_i$. The following corresponding parameter update laws of FNNs are chosen

$$\dot{\hat{\mathbf{w}}}_1 = r_{\mathbf{w}1}\mathbf{e_z^T P_1 b_1}\left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T\hat{m}^{(1)} - O_{\sigma 1}^T\hat{\sigma}^{(1)}\right) \tag{33}$$

$$\dot{\hat{m}}^{(1)} = r_{m1}\mathbf{e_z^T P_1 b_1}O_{m1}\hat{\mathbf{w}}_1 \tag{34}$$

$$\dot{\hat{\sigma}}^{(1)} = r_{\sigma 1}\mathbf{e_z^T P_1 b_1}O_{\sigma 1}\hat{\mathbf{w}}_1 \tag{35}$$

$$\dot{\hat{\mathbf{w}}}_2 = r_{\mathbf{w}2}\mathbf{e_q^T P}_2\mathbf{b}_2\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) \tag{36}$$

$$\dot{\hat{m}}^{(2)} = r_{m2}\mathbf{e_q^T P}_2\mathbf{b}_2 O_{m2}\hat{\mathbf{w}}_2 \tag{37}$$

$$\dot{\hat{\sigma}}^{(2)} = r_{\sigma 2}\mathbf{e_q^T P}_2\mathbf{b}_2 O_{\sigma 2}\hat{\mathbf{w}}_2 \tag{38}$$

$$\dot{\hat{\theta}}^{(2)} = r_{\theta 2}\mathbf{e_q^T P}_2\mathbf{b}_2 O_{\sigma 2}\hat{\mathbf{w}}_2 \tag{39}$$

$$\dot{\hat{\delta}}_1 = a_1\left|\mathbf{e_z^T P_1 b_1}\right| \tag{40}$$

$$\dot{\hat{\delta}}_2 = a_2\left|\mathbf{e_q^T P_2 b_2}\right| \tag{41}$$

where r_{wi}, r_{mi}, $r_{\sigma i}$, i = 1, 2 and $r_{\theta 2}$ are the parameter adaptive rates. Then, the tracking error is guaranteed uniformly ultimately bounded (UUB) if $J_1 > 0$ and $J_2 > 0$ keep sufficiently small.

Proof: Herein, we prove it by the concept of backstepping approach.

Step 1: According to equations (4) and (29), the error dynamics is

$$\begin{aligned}
e_z^{(p)} &= z_1^{(p)} - z_d^{(p)} \\
&= -\mathbf{k_1}^T\mathbf{e_z} + c_1 e_q + \tilde{h}_1 + u_{r1}
\end{aligned} \tag{42}$$

or in matrix representation

$$\begin{aligned}
\dot{\mathbf{e}}_\mathbf{z} &= \mathbf{A_1 e_z} + \mathbf{b_1}\left(c_1 e_q + \tilde{h}_1 + u_{r1}\right) \\
&= \mathbf{A_1 e_z} + \mathbf{b_1}\Big[c_1 e_q + \tilde{\mathbf{w}}_1^T\left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T\hat{m}^{(1)} - O_{\sigma 1}^T\hat{\sigma}^{(1)}\right) \\
&\qquad + \hat{\mathbf{w}}_1^T\left(O_{m1}^T\tilde{m}^{(1)} + O_{\sigma 1}^T\tilde{\sigma}^{(1)}\right) + \omega_1 + u_{r1}\Big]
\end{aligned} \tag{43}$$

Design the Lyapunov candidate function

$$\begin{aligned}
V_1 &= \frac{1}{2}\mathbf{e_z^T P_1 e_z} + \frac{1}{2r_{\mathbf{w}1}}\tilde{\mathbf{w}}_1^T\tilde{\mathbf{w}}_1 + \frac{1}{2r_{m1}}\tilde{m}^{(1)^T}\tilde{m}^{(1)} \\
&\quad + \frac{1}{2r_{\sigma 1}}\tilde{\sigma}^{(1)^T}\tilde{\sigma}^{(1)} + \frac{1}{2a_1}\tilde{\delta}_1^2
\end{aligned} \tag{44}$$

such that

$$\dot{V}_1 = \mathbf{e_z^T P_1}\dot{\mathbf{e}}_\mathbf{z} - \frac{1}{r_{\mathbf{w}1}}\tilde{\mathbf{w}}_1^T\dot{\hat{\mathbf{w}}}_1 - \frac{1}{r_{m1}}\tilde{m}^{(1)^T}\dot{\hat{m}}^{(1)} - \frac{1}{r_{\sigma 1}}\tilde{\sigma}^{(1)^T}\dot{\hat{\sigma}}^{(1)} - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1$$

$$
\begin{aligned}
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\Big[c_1 e_q + \tilde{\mathbf{w}}_1^T\left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T\hat{m}^{(1)} - O_{\sigma 1}^T\hat{\sigma}^{(1)}\right) \\
&\qquad + \hat{\mathbf{w}}_1^T\left(O_{m1}^T\tilde{m}^{(1)} + O_{\sigma 1}^T\tilde{\sigma}^{(1)}\right) + \omega_1 \\
&\qquad + u_{r1}\Big] - \frac{1}{r_{\mathbf{w}1}}\tilde{\mathbf{w}}_1^T\dot{\hat{\mathbf{w}}}_1 - \frac{1}{r_{m1}}\tilde{m}^{(1)^T}\dot{\hat{m}}^{(1)} - \frac{1}{r_{\sigma 1}}\tilde{\sigma}^{(1)^T}\dot{\hat{\sigma}}^{(1)} - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\Big[c_1 e_q + \tilde{\mathbf{w}}_1^T\left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T\hat{m}^{(1)} - O_{\sigma 1}^T\hat{\sigma}^{(1)}\right) \\
&\qquad + \tilde{m}^{(1)^T}O_{m1}\hat{\mathbf{w}}_1 + \tilde{\sigma}^{(1)^T}O_{\sigma 1}\hat{\mathbf{w}}_1 \\
&\qquad + \omega_1 + u_{r1}\Big] - \frac{1}{r_{\mathbf{w}1}}\tilde{\mathbf{w}}_1^T\dot{\hat{\mathbf{w}}}_1 - \frac{1}{r_{m1}}\tilde{m}^{(1)^T}\dot{\hat{m}}^{(1)} - \frac{1}{r_{\sigma 1}}\tilde{\sigma}^{(1)^T}\dot{\hat{\sigma}}^{(1)} - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \tilde{\mathbf{w}}_1^T\left[\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\left(\hat{\boldsymbol{\psi}}_1 - O_{m1}^T\hat{m}^{(1)} - O_{\sigma 1}^T\hat{\sigma}^{(1)}\right) - \frac{1}{r_{\mathbf{w}1}}\dot{\hat{\mathbf{w}}}_1\right] \\
&\quad + \tilde{m}^{(1)^T}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}O_{m1}\hat{\mathbf{w}}_1 - \frac{1}{r_{m1}}\dot{\hat{m}}^{(1)}\right) + \tilde{\sigma}^{(1)^T}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}O_{\sigma 1}\hat{\mathbf{w}}_1 - \frac{1}{r_{\sigma 1}}\dot{\hat{\sigma}}^{(1)}\right) \\
&\quad + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\left(c_1 e_q + \omega_1 + u_{r1}\right) - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 .
\end{aligned}
\tag{45}
$$

Substituting (33)-(35) into (45), we obtain

$$
\dot{V}_1 = -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\left(c_1 e_q + \omega_1 + u_{r1}\right) - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \tag{46}
$$

Step 2: Finally, we have

$$
\begin{aligned}
\dot{\mathbf{e}}_{\mathbf{q}} &= \mathbf{A}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \mathbf{b}_{\mathbf{2}}\Bigg[-\frac{\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}c_1 e_q}{\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}} + \tilde{\mathbf{w}}_2^T\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) \\
&\quad + \hat{\mathbf{w}}_2^T\left(O_{m2}^T\tilde{m}^{(2)} + O_{\sigma 2}^T\tilde{\sigma}^{(2)} + O_{\theta 2}^T\tilde{\theta}^{(2)}\right) + \omega_2 + u_{r2}\Bigg].
\end{aligned}
\tag{47}
$$

Choose the Lyapunov candidate function

$$
\begin{aligned}
V_2 &= V_1 + \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \frac{1}{2r_{\mathbf{w}2}}\tilde{\mathbf{w}}_2^T\tilde{\mathbf{w}}_2 + \frac{1}{2r_{m2}}\tilde{m}^{(2)^T}\tilde{m}^{(2)} \\
&\quad + \frac{1}{2r_{\sigma 2}}\tilde{\sigma}^{(2)^T}\tilde{\sigma}^{(2)} + \frac{1}{2r_{\theta 2}}\tilde{\theta}^{(2)^T}\tilde{\theta}^{(2)} + \frac{1}{2a_2}\tilde{\delta}_2^2
\end{aligned}
\tag{48}
$$

such that

$$
\begin{aligned}
\dot{V}_2 &= \dot{V}_1 + \mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\dot{\mathbf{e}}_{\mathbf{q}} - \frac{1}{r_{\mathbf{w}2}}\tilde{\mathbf{w}}_2^T\dot{\hat{\mathbf{w}}}_2 - \frac{1}{r_{m2}}\tilde{m}^{(2)^T}\dot{\hat{m}}^{(2)} - \frac{1}{r_{\sigma 2}}\tilde{\sigma}^{(2)^T}\dot{\hat{\sigma}}^{(2)} - \frac{1}{r_{\theta 2}}\tilde{\theta}^{(2)^T}\dot{\hat{\theta}}^{(2)} - \frac{1}{a_2}\tilde{\delta}_2\dot{\hat{\delta}}_2 \\
&= \dot{V}_1 - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\Bigg[-\frac{\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}c_1 e_q}{\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}} + \hat{\mathbf{w}}_2^T\left(O_{m2}^T\tilde{m}^{(2)} + O_{\sigma 2}^T\tilde{\sigma}^{(2)} + O_{\theta 2}^T\tilde{\theta}^{(2)}\right) \\
&\quad + \tilde{\mathbf{w}}_2^T\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) + \omega_2 + u_{r2}\Bigg] - \frac{1}{r_{\mathbf{w}2}}\tilde{\mathbf{w}}_2^T\dot{\hat{\mathbf{w}}}_2 - \frac{1}{r_{m2}}\tilde{m}^{(2)^T}\dot{\hat{m}}^{(2)} \\
&\quad - \frac{1}{r_{\sigma 2}}\tilde{\sigma}^{(2)^T}\dot{\hat{\sigma}}^{(2)} - \frac{1}{r_{\theta 2}}\tilde{\theta}^{(2)^T}\dot{\hat{\theta}}^{(2)} - \frac{1}{a_2}\tilde{\delta}_2\dot{\hat{\delta}}_2 \\
&= \dot{V}_1 - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\Bigg[-\frac{\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}c_1 e_q}{\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}} + \tilde{m}^{(2)^T}O_{m2}\hat{\mathbf{w}}_2 + \tilde{\sigma}^{(2)^T}O_{\sigma 2}^T\hat{\mathbf{w}}_2 + \tilde{\theta}^{(2)^T}O_{\theta 2}\hat{\mathbf{w}}_2 \\
&\quad + \tilde{\mathbf{w}}_2^T\left(\hat{\boldsymbol{\psi}}_2 - O_{m2}^T\hat{m}^{(2)} - O_{\sigma 2}^T\hat{\sigma}^{(2)} - O_{\theta 2}^T\hat{\theta}^{(2)}\right) + \omega_2 + u_{r2}\Bigg] - \frac{1}{r_{\mathbf{w}2}}\tilde{\mathbf{w}}_2^T\dot{\hat{\mathbf{w}}}_2 - \frac{1}{r_{m2}}\tilde{m}^{(2)^T}\dot{\hat{m}}^{(2)} \\
&\quad - \frac{1}{r_{\sigma 2}}\tilde{\sigma}^{(2)^T}\dot{\hat{\sigma}}^{(2)} - \frac{1}{r_{\theta 2}}\tilde{\theta}^{(2)^T}\dot{\hat{\theta}}^{(2)} - \frac{1}{a_2}\tilde{\delta}_2\dot{\hat{\delta}}_2
\end{aligned}
\tag{49}
$$

By substituting (36)-(39) and (46) into (49), we obtain

$$
\begin{aligned}
\dot{V}_2 &= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\left(\omega_1 + u_{r1}\right) - \frac{1}{a_1}\tilde{\delta}_1\dot{\hat{\delta}}_1 \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\left(\omega_2 + u_{r2}\right) - \frac{1}{a_2}\tilde{\delta}_2\dot{\hat{\delta}}_2 .
\end{aligned}
\tag{50}
$$

The robust controllers and $\hat{\delta}$'s adaptive laws are designed as (31), (32), (40) and (41). Therefore, we obtain

$$
\begin{aligned}
\dot{V}_2 &= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\left[\omega_1 - \hat{\delta}_1\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)\right] - \left(\delta_1 - \hat{\delta}_1\right)\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\left[\omega_2 - \hat{\delta}_2\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)\right] - \left(\delta_2 - \hat{\delta}_2\right)\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| \\
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \omega_1\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right) - \hat{\delta}_1\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right) - \delta_1\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| + \hat{\delta}_1\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \omega_2\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right) - \hat{\delta}_2\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right) - \delta_2\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| + \hat{\delta}_2\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| \\
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \omega_1\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right) - \delta_1\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| + \hat{\delta}_1\left[\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| - \left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)\right] \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \omega_2\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right) - \delta_2\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| + \hat{\delta}_2\left[\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| - \left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)\right] \\
&\le -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \left|\omega_1\right|\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| - \delta_1\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| + \hat{\delta}_1\left[\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| - \left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)\right] \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \left|\omega_2\right|\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| - \delta_2\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| + \hat{\delta}_2\left[\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| - \left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)\right] \\
&= -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \left(\left|\omega_1\right| - \delta_1\right)\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| + \hat{\delta}_1\left[\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| - \left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)\right] \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \left(\left|\omega_2\right| - \delta_2\right)\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| + \hat{\delta}_2\left[\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| - \left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)\right] \\
&\le -\frac{1}{2}\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{1}}\mathbf{e}_{\mathbf{z}} + \hat{\delta}_1\left[\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| - \left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)\right] \\
&\quad - \frac{1}{2}\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{Q}_{\mathbf{2}}\mathbf{e}_{\mathbf{q}} + \hat{\delta}_2\left[\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| - \left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right)\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)\right]
\end{aligned}
\tag{51}
$$

where $\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right)$ and $\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right)$ are the saturate functions that define as

$$
\mathrm{sat}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}},\ \mathrm{J}_1\right) = \begin{cases} \mathrm{sign}\left(\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right), & \left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| \ge \mathrm{J}_1 \\ \dfrac{\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}}{\mathrm{J}_1}, & \left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| < \mathrm{J}_1 \end{cases}, \tag{52}
$$

$$
\mathrm{sat}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}},\ \mathrm{J}_2\right) = \begin{cases} \mathrm{sign}\left(\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right), & \left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| \ge \mathrm{J}_2 \\ \dfrac{\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}}{\mathrm{J}_2}, & \left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| < \mathrm{J}_2 \end{cases}. \tag{53}
$$

Thus, the condition $\left|\mathbf{e}_{\mathbf{z}}^{\mathbf{T}}\mathbf{P}_{\mathbf{1}}\mathbf{b}_{\mathbf{1}}\right| \ge \mathrm{J}_1$ and $\left|\mathbf{e}_{\mathbf{q}}^{\mathbf{T}}\mathbf{P}_{\mathbf{2}}\mathbf{b}_{\mathbf{2}}\right| \ge \mathrm{J}_2$ imply $\dot{V}_2 < 0$, the errors $\mathbf{e}_{\mathbf{z}}$ and $\mathbf{e}_{\mathbf{q}}$ are guaranteed UUB within a small region if J_1 and J_2 are positive and keep arbitrarily small. This completes the proof. ¡%

4. SIMULATION RESULT

In this section, a single-link flexible-joint robot is presented to show the performance of our proposed adaptive backstepping control scheme. The single-link flexible-joint robot arm is shown in Fig. 3 [22] and the corresponding system model is

$$\begin{aligned} I_1\ddot{\theta}_1 + mgl\sin\theta_1 + k(\theta_1 - \theta_2) &= 0 \\ I_2\ddot{\theta}_2 - k(\theta_1 - \theta_2) &= u \end{aligned} \tag{54}$$

where θ_1 and θ_2 are the angles of the link and of the motor shaft, respectively; m is the total mass of the link; I_1 and I_2 are rotor inertias of the link and the motor, respectively. l denote the distance from the motor shaft to the center of mass of the link, g is the acceleration constant due to gravity, k is the torsional spring constant, and u is the torque applied to the motor shaft. System (54) can be expressed in the form of (1) by $x_1 = \theta_1, x_2 = \dot{\theta}_1, x_3 = \theta_2, x_4 = \dot{\theta}_2$

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\frac{mgl}{I_1}\sin x_1 - \frac{k}{I_1}(x_1 - x_3) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= \frac{k}{I_2}(x_1 - x_3) + \frac{1}{I_2}u. \end{aligned} \tag{55}$$

The desired trajectory is $x_{1d} = \sin(t)$. The parameters values are given by mgl=10, k=100, I_1=100, I_2=10. The design parameters are $\mathbf{k}_1 = \mathbf{k}_2 = [64, 16]^T$; $c_1 = 1$, $c_2 = 0.1$; $\mathbf{P}_1 = \mathbf{P}_2 = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$; $\alpha_1 = \alpha_2 = 0.01$; $\hat{\delta}_1(0) = \hat{\delta}_2(0) = 2$; $J_1 = J_2 = 0.1$. And the parameters of RFNN and FNN are chosen as shown in Table 1.

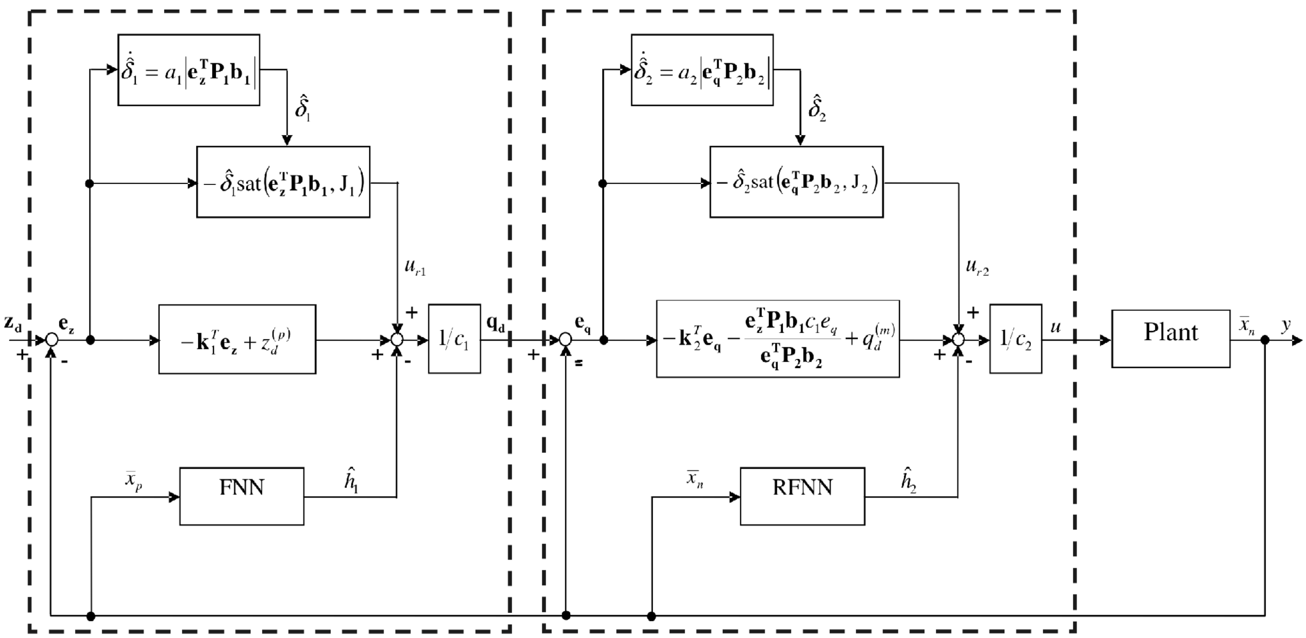

Figure 2: The Robust FNN-based Adaptive Backstepping Control Scheme for Nonlinear Cascade System (1)

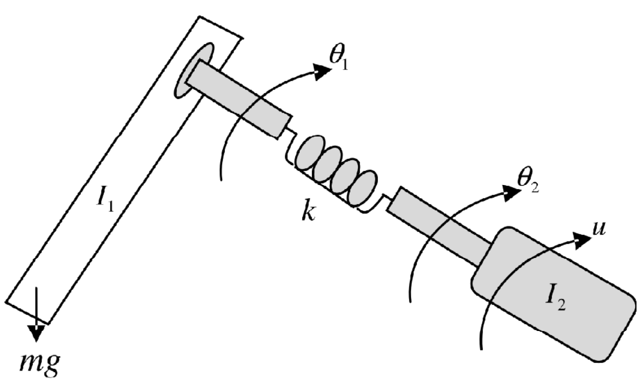

Figure 3: Single-link Flexible-joint Robot Arm [22]

Table 1
Network Structures and Initializations of the FNN and RFNN

	FNN	*RFNN*
Network structure	[3-15-5-1]	[4-20-5-1]
m_{ij}	[-2, -1, 0, 1, 2]	[-2, -1, 0, 1, 2]
σ_{ij}	1	1
θ_{ij}	None	0

Simulation 1: Stability illustration.

The comparison of these two approaches (FNN-ABC$_1$ and FNN-ABC$_2$) is introduced in Table 2. The initial condition is set to be $\bar{x}_4(0) = [\pi/12,\ 0, \pi/6\ ,\ 0]^T$. Figure 4 shows the simulation results with tracking error- RMSE=

4.1525×10^{-4}. Figure 4(a) shows reference and output trajectories x_d and x_1, respectively (dashed-line: desired trajectory; solid-line: FNN-ABC result). Figures 4(b) and 4(c) show the tracking error and control effort, respectively. These indicate that the closed-loop system is stable and tracking error approaching to zero. The tracking control can be achieved and performs well by our approaches.

Simulation 2: Robustness of FNN-ABC

Herein, the robustness of FNN-ABC is discussed. Parameters I_1 and I_2 of nominal plant have 20% modeling error, i.e., nominal plant- I_1=100, I_2=10; actual plant- I_1=80, I_2=8. In addition, parameter k has 50% error, i.e., nominal plant- k=100; actual plant- k=150. Figure 8 shows the simulation results of FNN-ABC (state trajectory, tracking error, and control effort). The stabilizing time is larger than the simulation-1. The FNN-ABC performs well even if the system modeling error occurs.

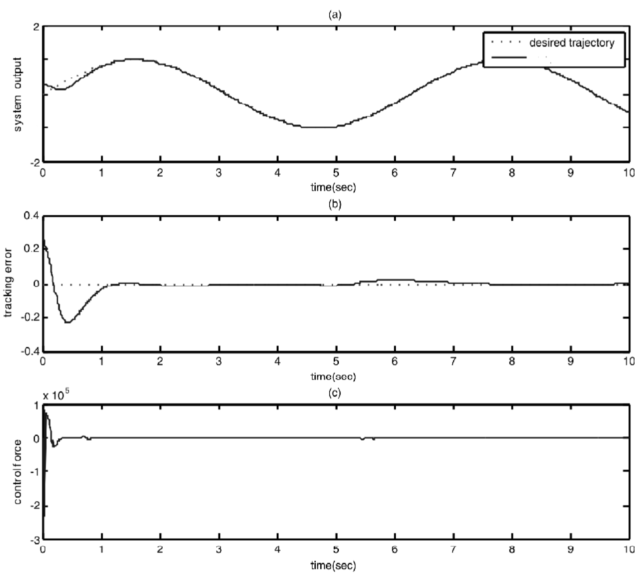

Figure 4: Simulation Results of Single-link Robot Arm, (a) State Trajectories x_1(Dashed-line: Reference Trajectory x_d; and silid-line: FNN-ABC Result); (b) Tracking Error; (c) Control Effort

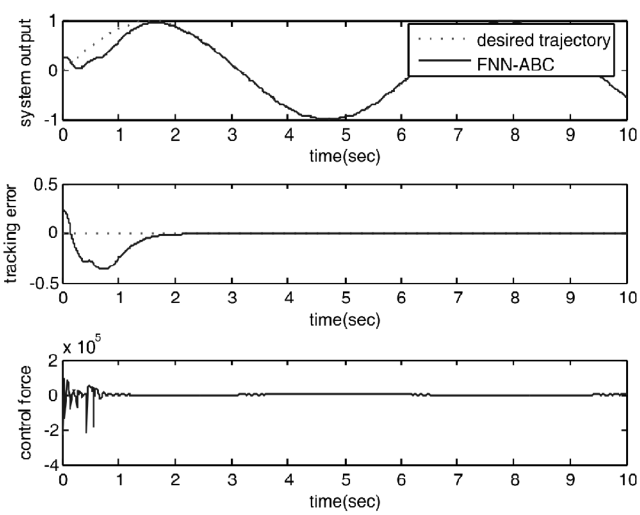

Figure 5: Simulation Result-2 of FNN-ABC (State Trajectory, Tracking Error, and Control Effort)

5. CONCLUSION

This paper has presented an FNN-based adaptive backstepping control scheme (FNN-ABC) for a class of nonlinear uncertain systems with cascade and lower triangular non-affine form. Two kinds of FNN systems (FNN and RFNN) are used to estimate the unknown functions. By the Lyapunov stability approach, the stability of the control system is guaranteed and the adaptive laws of FNNs' parameters have been obtained. Besides, Taylor expansion also has used to derive another kind of parameter adaptive laws for reducing the effect of initialization and improving the control performance. A single-link flexible-joint robot has been presented to illustrate the effectiveness and performance of FNN-ABC. By the backstepping design procedure, FNN-ABC approach can be extended to the nonlinear cascade system more than two subsystems.

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