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INTELLIGENT DIGITAL REDESIGN OF T-S FUZZY CONTROLLER VIA PARTICLE SWARM OPTIMIZATION

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ABSTRACT: In this paper, a novel method is proposed to solve the complex mathematical model of digital redesign of nonlinear systems which is regarded as difficult to approximate. The paper uses the T-S fuzzy model and particle swarm optimization (PSO) to search for the range of digital-controller parameters and to obtain the optimized digital controller using this algorithm. Due to the difficulty in establishing the discrete model of the interval system and designing the digital controller of the interval system, we have formulated the design problem into an optimization problem of a cost function. First, we process the continuous-time nonlinear systems using the T-S fuzzy model, followed by designing a continuous-time controller using individual rules. The next step is to express all possible linear systems as interval systems and search for the range of digital-controller parameters using PSO to narrow down the search range and conveniently search for the optimal solutions. According to the search range of digital controller parameters, the PSO is used to search for the discrete-time controller based on individual rules, so that the states of the discrete-time model based on the fuzzy model approximate to those of the continuous-time nonlinear systems. Finally, one example is given to prove this method is more accurate than the existing one with faster execution speed.

Keywords: intelligent digital resdesign, T-S fuzzy model, particle swarm optimization, nonlinear system.

I. INTRODUCTION

This paper mainly focuses on the digital redesign of continuous-time nonlinear systems. Many studies in recent years have emphasized on linear systems [1]-[14]. Tsai [1] *et al.* applied block-pulse function to approximate digital redesign. Shieh [2] *et al.* applied Pade approximation and inverse-Pade approximation on the digital redesign. Both [1] and [2] used mathematical approximation to obtain the digital controller. Ieko [3] *et al.* presented a digital redesign method for linear time-invariant continuous-time control systems of the one-degree-of-freedom type and the state-feedback type. Shieh [4] *et al.* proposed a state-space self-tuning control using dual-rate sampling and control digital linear time-varying systems. Shieh [5] *et al.* proposed the digital redesign of a cascade analogue controller for continuous-time interval systems. Rafee [6] *et al.* proposed an optimal digital redesign of the state-matching between the continuous-time and discrete-time step responses. Chang [7] *et al.* applied the linear matrix inequalities (LMI) for the digital redesign of time-varying linear systems. Lee [8] *et al.* applied digital redesign on an observer-based feedback controller, treating digital redesign as the minimization of norm distance and applying linear matrix inequalities (LMI) for solution. Lee [9] *et al.* applied Delta-Operator in LMI based digital redesign. Wang [10] *et al.* used linear matrix inequality that treats the sliding mode control of linear time-invariant systems for digital redesign. Kiml [11] *et al.* applied the integral quadratic constraints to treat issues of digital redesign. Cardim [12] applied state-derivative forward gain and feedback gain to control the digital redesign of helicopter vertical

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and landing systems. Lin [13] *et al.* applied evolutionary programming algorithms on the digital redesign of model-reference-based decentralized adaptive tracker. Polyakov [14] *et al.* also proposed a two-level optimization algorithm so that the poles fall within the specified area, in order to reduce the order of the controllers.

Currently, few studies focus on the digital redesign of nonlinear systems [15]-[19]. Because nonlinear systems do not satisfy superposition theorem like linear systems, they are more complex than linear systems and difficult to analyze. Sung [15] *et al.* proposed the digital redesign using fuzzy-model-based global approximation. This method applied linear matrix inequalities (LMI) to minimize the norm distance between the states of continuous time systems and discrete time systems. Joo [16] *et al.* proposed a fuzzy-model-based dual-rate sampling method to process the digital redesign of chaotic systems. Lee [17] *et al.* applied a fuzzy observer-based output-feedback control system to process digital redesign. Lee [18] *et al.* proposed the digital redesign of stabilized linear model for nonlinear systems. This method applied bilinear and linear matrix inequalities to determine the digital controller. Chang [19] *et al.* proposed the use of the T-S fuzzy model for establishing nonlinear models, followed by finding the optimal digital controller using genetic algorithm, to minimize errors between the states of continuous-time and discrete-time systems. Although the approximation accuracy of that paper showed smaller errors in comparison with the method proposed by Joo [16], there is, however, still room for further improvement in terms of approximation accuracy. Furthermore, the computational complexities via the proposed GA are higher. In an attempt to improve the approximation accuracy and computational efficiency, the paper presents a new approximation method to be applied in the digital redesign of a fuzzy-model-based controller. The method applies the continuous-time nonlinear system equivalent to a continuous-time T-S fuzzy model and design continuous-time controller using individual rules. Secondly, the method takes into consideration all possible linear systems and expresses them into interval systems, followed by searching the range of digital controller parameters using PSO, in order to narrow down the search range and to conveniently look for the optimal solution. According to the search range of digital controller parameters, the PSO is used to search for the discrete-time controller based on individual rules, so that the states of discrete-time model based on the fuzzy model closely approximate those of the continuous-time nonlinear systems.

The origination of the paper is described as follows: Section II describes the problems to be solved and introduces a continuous-time T-S fuzzy model and a discrete-time T-S fuzzy model. The PSO-based digital redesign of continuous-time nonlinear system is discussed in Section III. Then, in Section IV, an illustrated example is given to verify the proposed method. Finally, some conclusions are drawn in Section V.

II. PRELIMINARIES

Because the majority of actual systems are nonlinear systems which are complex and difficult to analyze, it is necessary to conduct analysis on nonlinear systems. We will discuss on continuous-time and discrete-time T-S fuzzy models in the following.

(A) Continuous-Time T-S Fuzzy Models

Consider a nonlinear system as shown in (1),

$$\dot{x}_c(t) = f(x_c(t)) + g(x_c(t))u_c(t) \tag{1}$$

where $x_c(t) \in R^{n\times 1}$ is the state vector, $f(.)$ and $g(.)$ are nonlinear functions, and $u_c(t) \in R^{m\times 1}$ is the control input vector. The nonlinear system can be approximated by a continuous-time T-S fuzzy model which is composed by multiple linear models and multiple fuzzy inference rules. We use the following T-S fuzzy model to represent a complex nonlinear system:

$$\begin{aligned}&\text{Plant Rule } i:\\&\text{If } s_1(t) \text{ is } M_1^i \text{ and ... and } s_n(t) \text{ is } M_n^i\\&\text{Then } \dot{x}_c(t)=\mathbf{A}_i x_c(t)+\mathbf{B}_i u_c(t),\ i=1,\ 2,\ \ldots,\ q\end{aligned} \tag{2}$$

where M_j^i(j=1,2,…,n) is the jth premise variable of fuzzy sets in the ith fuzzy subspace, $S_1(t)$,..., $S_n(t)$ are the premise variables, $\mathbf{A}_i \in R^{n\times n}$, $\mathbf{B}_i \in R^{n\times m}$, and q is the number of fuzzy rules. A singleton fuzzifier, product inference engine, followed by a center-average defuzzification is adopted to transform (2) into the following equation:

$$\dot{x}_c(t)=\mathbf{A}x_c(t)+\mathbf{B}u_c(t) \tag{3}$$

where

$$\mathbf{A}=\sum_{i=1}^{q}\frac{\mathbf{A}_i'}{\sum_{j=1}^{q}w_j},\mathbf{B}=\sum_{i=1}^{q}\frac{\mathbf{B}_i'}{\sum_{j=1}^{q}w_j}$$

$$\mathbf{A}_i'=w_i\mathbf{A}_i,\ \ \mathbf{B}_i'=w_i\mathbf{B}_i$$

$$w_i=\prod_{k=1}^{q}M_k^i(s_k(t))$$

and $M_k^i(s_k(t))$ is the membership grade of $s_k(t)$. A fuzzy-model-based controller is used as follows:

$$\begin{aligned}&\text{Controller Rule } i:\\&\text{If } s_1(t) \text{ is } M_1^i \text{ and ... and } s_n(t) \text{ is } M_n^i\\&\text{Then } u_c(t)=-\mathbf{K}_{ci}x_c(t),\ i=1,\ 2,\ \ldots,\ q\end{aligned} \tag{4}$$

where $\mathbf{K}_{ci}$ is the feedback gain in the ith fuzzy subspace. After applying some commonly used defuzzification strategies, we obtain

$$u_c(t)=-\mathbf{K}_c x_c(t) \tag{5}$$

where

$$\mathbf{K}_c=\sum_{i=1}^{q}\frac{w_i\mathbf{K}_{ci}}{\sum_{j=1}^{q}w_j}$$

(B) Discrete-Time T-S Fuzzy Models

The discrete-time T-S fuzzy model is shown as follows:

$$\begin{aligned}&\text{Plant Rule } i:\\&\text{If } s_1(t) \text{ is } M_1^i \text{ and ... and } s_n(t) \text{ is } M_n^i\\&\text{Then } x_d(kT+T)=\mathbf{G}_i x_d(kT)+\mathbf{H}_i u_d(kT),\ i=1,\ 2,\ \ldots,\ q.\end{aligned} \tag{6}$$

where T is the sampling time, $\mathbf{G}_i = \exp(\mathbf{A}_i T)$, $\mathbf{H}_i=\int_0^T\exp(\mathbf{A}_i\lambda)\mathbf{B}_i d\lambda$ and $x_1(kT)$, ..., $x_n(kT)$ are the premise variables. Similarly, the dynamics of (6) can be represented as

$$x_d(kT+T)=\mathbf{G}x_d(kT)+\mathbf{H}u_d(kT) \tag{7}$$

where

$$\mathbf{G}=\sum_{i=1}^{q}\frac{\mathbf{G}'_i}{\sum_{j=1}^{q}w_j},\quad \mathbf{H}=\sum_{i=1}^{q}\frac{\mathbf{H}'_i}{\sum_{j=1}^{q}w_j}$$

$$\mathbf{G}'_i = w_i\mathbf{G}_i,\quad \mathbf{H}'_i = w_i\mathbf{H}_i$$

and

$$w_i = \prod_{k=1}^{q} M_k^i(s_k(t))$$

where $M_k^i(s_k(kT))$ is the membership grade of $s_k(kT)$ in M_k^i.

The corresponding fuzzy-model-based discrete-time controller is shown in (8):

$$\begin{aligned}&\text{Controller Rule } i:\\ &\text{If } s_1(kT) \text{ is } M_1^i \text{ and ... and } s_n(kT) \text{ is } M_n^i\\ &\text{Then } u_d(kT) = -\mathbf{K}_{di}x_d(kT),\ i = 1,\ 2,\ ...,\ q,\end{aligned} \tag{8}$$

where $\mathbf{K}_{di}$ is the feedback gain in the ith fuzzy subspace. The intelligent digital redesign fuzzy-model-based controller is shown in (9).

$$u_d(kT) = -\mathbf{K}_d x_d(kT), \tag{9}$$

where

$$\mathbf{K}_d = \sum_{i=1}^{q}\frac{w_i\mathbf{K}_{di}}{\sum_{j=1}^{q}w_j}$$

III. PSO-BASED DIGITAL REDESIGN OF CONTINUOUS-TIME NONLINEAR SYSTEMS

(A) Problem Description

Considering the continuous-time system (1), the continuous-time controller (4), and the sampling period, we design a descrete-time controller (8) to minimize the cost function as

$$\begin{aligned}J &= \sum_{i=1}^{n}\int_0^{t_f}\left|x_{ci}(t)-x_{di}(t)\right|dt\\ &\cong \sum_{i=1}^{n}\sum_{j=1}^{N}\left|x_{ci}(jTf)-x_{di}(jTf)\right|Tf,\end{aligned} \tag{10}$$

where $x_{ci}(t)$ and $x_{di}(t)$ are the ith state variables of the state vectors $x_c(t)$ and $x_d(t)$, respectively. $T_f = t_f / N$ is the sampling time with a sufficiently large integer N. $x_{ci}(jTf)$ and $x_{di}(jTf)$ represent the values of state vectors $x_c(t)$ and $x_d(t)$ at $t = jTf$, respectively. We can express the digital redesign issue as the optimization problem in (11).

$$\text{Find } \mathbf{K}_{di} \text{ to minimize J} \tag{11}$$

where $\mathbf{K}_{di}$ is the optimal solution determined by PSO algorithm.

(B) Particle Swarm Optimization

Particle Swarm Optimization (PSO) is a desired optimization technique inspired by the observations of foraging behaviour of birds or fish proposed by Kennedy and Eberhart [20] in

1995. Because of simplicity for implementation, smaller memory usage, distributed search, and ability to quickly converge to a reasonably good solution, PSO is widely adopted in various engineering applications [21]-[23].

The algorithm, which is based on a metaphor of social interaction, searches a space by adjusting the trajectories of individual vectors, called "particles" as they are conceptualized as moving points in multidimensional space. The individual particle in PSO flies in the search space with velocity which is dynamically adjusted according to its own flying experience and its companions' flying experience. The former was termed cognition-only model and the latter was termed social-only model. By integrating these two types of knowledge, the particle behavior in a PSO can be modeled by using the following equations:

$$\begin{aligned} V_i(t+1) = w \times V_i(t) + c_1 \times rand \times (Pbest - x_i) \\ + c_2 \times rand \times (Gbest - x_i) \end{aligned} \tag{12}$$

$$x_i(t+1) = x_i(t) + V_i(t+1) \tag{13}$$

where $V_i(t)$ and $x_i(t)$ refer to the velocity and position of previous particle, $V_i(t+1)$ and $x_i(t+1)$ refer to the velocity and position of next-generation particles, $rand \in [0\ 1]$, constant weight $w \in [0.4\ 0.9]$, c_1 and c_2 refer to the learning factors with typical values of $c_1 = c_2 = 2$. If the position of that particle goes beyond the search range, the particle must be re-initialized.

(C) PSO-Derived Optimal Digital Controller

We must first find out the search range of $\mathbf{K}_d^i$ before using PSO algorithm for intelligent digital redesign. We adopted interval arithmetic to determine the search range of PSO-based intelligent digital redesign.

Due to the different values generated under different states for matrices **A**, **B**, and $\mathbf{K}_c$, we take into consideration of all possible state values to find out the upper and lower boundaries of matrices **A**, **B**, and $\mathbf{K}_c$. We can express these matrices into interval matrices $\mathbf{A}^I$, $\mathbf{B}^I$, and $\mathbf{K}_c^I$. The interval expression of (3) is shown in (14).

$$\dot{x}_c(t) = \mathbf{A}^I x_c(t) + \mathbf{B}^I u_c(t) \tag{14}$$

The closed-loop system is shown in (15).

$$\dot{x}_c(t) = (\mathbf{A}^I - \mathbf{B}^I \mathbf{K}_c^I) x_c(t) \tag{15}$$

where $\mathbf{A}^{\mathrm{I}} = [\underline{\mathbf{A}},\ \overline{\mathbf{A}}]$, $\mathbf{B}^{\mathrm{I}} = [\underline{\mathbf{B}},\ \overline{\mathbf{B}}]$, and $\mathbf{K}_c^I = [\underline{\mathbf{K}}_c,\ \overline{\mathbf{K}}_c]$. $\underline{\mathbf{A}}$, $\underline{\mathbf{B}}$, and $\underline{\mathbf{K}}_c$ refer to the lower boundaries of $\mathbf{A}^I$, $\mathbf{B}^I$, and $\mathbf{K}_c^I$, respectively. $\overline{\mathbf{A}}$, $\overline{\mathbf{B}}$, and $\overline{\mathbf{K}}_c$ refer to the upper boundaries of $\mathbf{A}^I$, $\mathbf{B}^I$, and $\mathbf{K}_c^I$, respectively. Each element of the upper and lower boundaries of $\mathbf{A}^I$, $\mathbf{B}^I$, and $\mathbf{K}_c^I$ can be calculated via (16).

$$\begin{aligned} \overline{a}_{uv} &= \left\{ \max_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) a_{uv}^i \right) \right\} \\ \underline{a}_{uv} &= \left\{ \min_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) a_{uv}^i \right) \right\} \\ \overline{b}_{uv} &= \left\{ \max_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) b_{uv}^i \right) \right\} \end{aligned} \tag{16}$$

$$\underline{b}_{uv} = \left\{ \min_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) b_{uv}^i \right) \right\}$$

$$\overline{k}_{cuv} = \left\{ \max_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) k_{cuv}^i \right) \right\}$$

$$\underline{k}_{cuv} = \left\{ \min_{\mu} \left(\sum_{i=1}^{q} \mu_i(s(t)) k_{cuv}^i \right) \right\}$$

where $\overline{a}_{uv}$, a_{uv}^i, $\underline{a}_{uv}$, $\overline{b}_{uv}$, b_{uv}^i, $\underline{b}_{uv}$, $\overline{k}_{cuv}$, k_{cuv}^i, and $\underline{k}_{cuv}$ refer to the vth element at uth column of $\overline{\mathbf{A}}$, $\mathbf{A}_i$, $\underline{\mathbf{A}}$, $\overline{\mathbf{B}}$, $\mathbf{B}_i$, $\underline{\mathbf{B}}$, $\overline{\mathbf{K}}_c$, $\mathbf{K}_{ci}$, and $\underline{\mathbf{K}}_c$, respectively.

Assumption 1: The boundaries (16) can be simplified into the following equations (17) with $\sum_{j=1}^{q} w_j = 1$.

$$\begin{aligned}
\overline{a}_{uv} &= \max_{i=1}^{q} \left(a_{uv}^i \right) \\
\underline{a}_{uv} &= \min_{i=1}^{q} \left(a_{uv}^i \right) \\
\overline{b}_{uv} &= \max_{i=1}^{q} \left(b_{uv}^i \right) \\
\underline{b}_{uv} &= \min_{i=1}^{q} \left(b_{uv}^i \right) \\
\overline{k}_{cuv} &= \max_{i=1}^{q} \left(k_{cuv}^i \right) \\
\underline{k}_{cuv} &= \min_{i=1}^{q} \left(k_{cuv}^i \right)
\end{aligned} \tag{17}$$

The state equation of sampled-data system from (14) is shown in (18).

$$\dot{x}_d(t) = \mathbf{A}^I x_d(t) + \mathbf{B}^I u_d(kT) \tag{18}$$

where $u_d(kT)$ refers to the piecewise function and satisfies $u_d(t) = u_c(kT)$ for $kT \le t \le kT + T$. We redesign the intelligent digital controller as

$$u_d(kT) = -\mathbf{K}_d^I x_d(kT) \tag{19}$$

Substituting (19) into (18), we have (20).

$$x_d(t) = \mathbf{A}^I x_d(t) - \mathbf{B}^I \mathbf{K}_d^I x_d(kT) \tag{20}$$

where $\mathbf{K}_d^I \in R^{m \times n}$ is the digital control gain matrix. The discrete-time model of (15) and (20) are shown in (21) and (22), respectivelly.

$$x_d(kT + T) = \exp(\mathbf{A}^I - \mathbf{B}^I \mathbf{K}_c^I) \tag{21}$$

$$x_d(kT + T) = (\mathbf{G}^I - \mathbf{H}^I \mathbf{K}_d^I) x_d(kT) \tag{22}$$

where $\mathbf{G}^I = \exp(\mathbf{A}^I T)$ and $\mathbf{H}^I = (\mathbf{G}^I - \mathbf{I}_n)\mathbf{A}^{I^{-1}}\,\mathbf{B}^I$. We can find $\mathbf{K}_d^I$ through $\mathbf{K}_c^I$ so that the continuous-time state vector $x_c(t)$ approximate to the discrete-time state vector $x_d(t)$. This method is known as state compatibility digital redesign. From (21) and (22), we can obtain (23).

$$\exp(\mathbf{A}^I - \mathbf{B}^I\mathbf{K}_c^I) = \mathbf{G}^I - \mathbf{H}^I\mathbf{K}_d^I \tag{23}$$

Then we apply the Pade method [2] to mathematically approximate $\exp(\mathbf{A}^I - \mathbf{B}^I\mathbf{K}_c^I)$ and obtain an equation as shown in (24).

$$\begin{aligned}\mathbf{K}_d^I &= \frac{1}{2}\left(I_m + \frac{1}{2}\mathbf{K}_c^I(\mathbf{I}_n - \frac{T}{6}(\mathbf{P}+\mathbf{Q}))\mathbf{H}^I\right)^{-1} \\ &\times \mathbf{K}_c^I\left(\left(\mathbf{G}^I - \mathbf{I}_n\right) - \frac{T}{6}(\mathbf{P}+\mathbf{Q})\left(\mathbf{G}^I - \mathbf{I}_n\right)\right)\end{aligned} \tag{24}$$

where $\mathbf{P} = \left(\mathbf{B}^I\mathbf{K}_c^I\right)^{+}\mathbf{A}^I\left(\mathbf{B}^I\mathbf{K}_c^I\right)$ and $\mathbf{Q} = \mathbf{A}^I - \mathbf{B}^I\mathbf{K}_c^I$.

We find the search range of $\mathbf{K}_d^I$ for PSO, as shown in (24), and rewrite (11) as (25).

$$\begin{gathered}\text{Find } \mathbf{K}_{di} \text{ to minimize } J \\ \text{Subject to } \mathbf{K}_{di} \in \mathbf{K}_d^I.\end{gathered} \tag{25}$$

We can use PSO to research a set of $\mathbf{K}_{di}$ in order to minimize (25), and consequently approximate the discrete-time states to the continuous-time states.

The key processes to design an optimal digital controller are described as follows:

(1) Obtain the continuous-time state vector x_c.

(2) Choose a sampling time T to obtain $\mathbf{G}_i$ and $\mathbf{H}_i$ as shown in (6).

(3) Encode $\mathbf{K}_{di}$ to become a particle, as shown in (26)

$$\mathbf{P} = \left[K_{d11}\ K_{d12}\ \cdots\ K_{d21}\ K_{d22}\ \cdots\ K_{dn1}\ K_{dn2}\ \cdots\ K_{dnm}\right] \tag{26}$$

(4) Obtain the discrete-time state vector x_d.

(5) Calculate the cost function J from (10).

(6) Apply PSO to obtain a set of $\mathbf{K}_{di}$ that minimizes J.

We can find a set of optimal solution for $\mathbf{K}_{di}$ through the above design process so that the discrete-time system can closely approximate the continuous-time system.

IV. SIMULATION RESULTS

In this section, we use the example of Chen's chaotic attractor [19] to express the effectiveness of the proposed method. The dynamic system of Chen's chaotic attractor is shown in (27).

$$\begin{aligned}\dot{x}_1 &= -35x_1 + 35x_2 \\ \dot{x}_2 &= -7x_1 - x_1x_3 + 28x_2 \\ \dot{x}_3 &= x_1x_2 - 3x_3\end{aligned} \tag{27}$$

The state trackers of Chen's chaotic attractor at initial state are $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$ as shown in Figure 1. To establish the T-S fuzzy model, we define the range of x_1 between –30 and 30. We can transform (27) into the following T-S fuzzy model, as shown in (28).

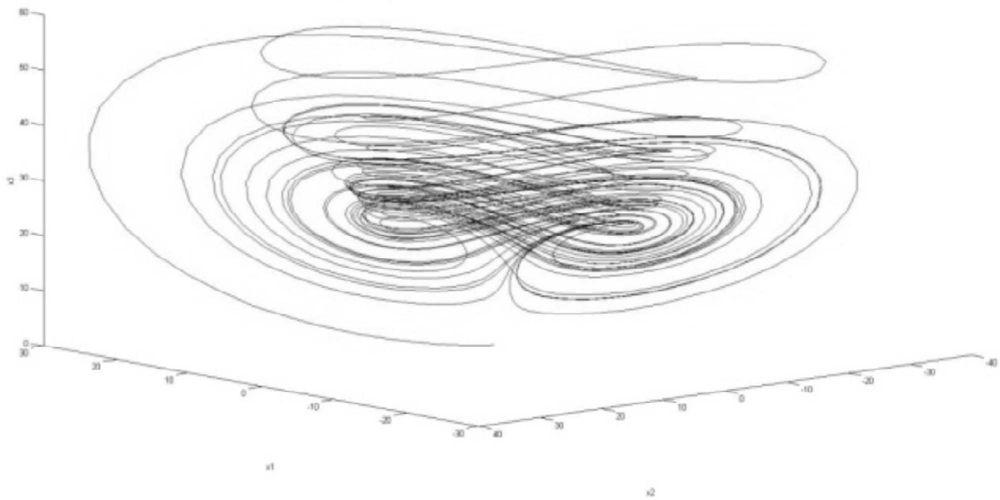

Figure 1: The Phase Diagram of the Chen's Chaotic Attractor with Initial States $x_1(0) = 1$, $x_2(0) = 1$, $x_3(0) = 1$

$$\begin{aligned}&\text{Plant Rule } i:\\ &\text{If } x_1(t) \text{ is } M_1^i\\ &\text{Then } \dot{x}_c(t) = \mathbf{A}_i x_c(t) + \mathbf{B}_i u_c(t)\ (i = 1, 2)\end{aligned} \tag{28}$$

where

$$M_1^1 = \max\left(\min\left(\frac{-x_1 + 30}{60},\ 1\right),\ 0\right)$$

$$M_1^2 = \max\left(\min\left(\frac{x_1 - 30}{60},\ 1\right),\ 0\right)$$

$$\mathbf{A}_1 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & 30 \\ 0 & -30 & -3 \end{bmatrix}$$

$$\mathbf{A}_2 = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & -30 \\ 0 & 30 & -3 \end{bmatrix}$$

$$\mathbf{B}_i = \mathbf{I}_3,\ (i = 1, 2)$$

By using the proposed algorithm in Section III, the continuous-time T-S fuzzy model (28) can be represented as follows:

$$\dot{x}_c(t) = \mathbf{A}^I x_c(t) + \mathbf{B}^I u_c(t) \tag{29}$$

where

$$\mathbf{A}^I = \begin{bmatrix} -35 & 35 & 0 \\ -7 & 28 & [-30\ \ 30] \\ 0 & [-30\ \ 30] & -3 \end{bmatrix}$$

$$\mathbf{B}^I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

The corresponding discrete-time system is shown in (30).

$$x_d(kT+T) = \mathbf{G}^I x_d(kT) + \mathbf{H}^I u_d(kT) \tag{30}$$

where

$$\mathbf{G}^I = \exp(\mathbf{A}^I T),\ \ \mathbf{H}^I = (\mathbf{G}^I - \mathbf{I}_n)\mathbf{A}^{I^{-1}}\mathbf{B}^I$$

$$\mathbf{G}^I = \begin{bmatrix} [-0.0798 \quad 0.0125] & [1.1723 \quad 2.736] & [-1.6162 \quad 1.1568] \\ [-0.5982 \quad -0.3754] & [1.2705 \quad 5.7941] & [-4.0297 \quad 3.1016] \\ [-0.3236 \quad 0.2094] & [-4.0401 \quad 2.8518] & [-0.4842 \quad 0.9819] \end{bmatrix}$$

$$\mathbf{H}^I = \begin{bmatrix} [-0.0195 \quad 0.0206] & [0.0356 \quad 0.0566] & [-0.0243 \quad 0.0242] \\ [-0.0113 \quad -0.0071] & [0.0694 \quad 0.1422] & [-0.07 \quad 0.0707] \\ [-0.0048 \quad 0.0048] & [-0.0697 \quad 0.0707] & [0.0245 \quad 0.0745] \end{bmatrix}$$

The rule structure of the continuous-time T-S fuzzy model is shown in (31).

$$\begin{aligned} &\text{Controller Rule } i: \\ &\text{If } x_1(t) \text{ is } M_1^i, \\ &\text{Then } u_c(t) = -\mathbf{K}_c^i x_c(t), q = 1,\ 2, \end{aligned} \tag{31}$$

where

$$\mathbf{K}_c^1 = \begin{bmatrix} -20 & 35 & 0 \\ -7 & 43 & 30 \\ 0 & -30 & 12 \end{bmatrix},\ \mathbf{K}_c^2 = \begin{bmatrix} -20 & 35 & 0 \\ -7 & 43 & -30 \\ 0 & 30 & 12 \end{bmatrix}$$

In the next step, we will use (24) to find out the search range $\mathbf{K}_{di}^I$ of $\mathbf{K}_{di}$, as shown in (32).

$$\mathbf{K}_{di}^I = \begin{bmatrix} [-31.644 \ -7.9951] & [-53.2901\ 118.18171] & [\text{-}99.4139\ 76.8313] \\ [-36.1757\ \text{-}1.9501] & [-9.4868\ 151.7798] & [-109.4525\ 75.3013] \\ [\text{-}7.3147\ 6.7761] & [-79.1132\ 91.199] & [\text{-}30.4794\ 76.695] \end{bmatrix} \tag{32}$$

We can use (32) as the search range for PSO to find out the optimal set of $\mathbf{K}_{di}$ that minimizes the J in (10). Parameters of the PSO include: sampling time $T = 0.05$ sec., $c_1 = c_2 = 2$, $w = 0.4$, the maximal iterations of 1000, and a population of 20 particles. We can find a set of $\mathbf{K}_{di}$, as shown in (33).

$$\mathbf{K}_{d1} = \begin{bmatrix} -19.4922 & 42.9394 & -36.1631 \\ -8.3599 & 36.6113 & 18.1556 \\ -5.3463 & -54.1652 & 12.2281 \end{bmatrix} \tag{33}$$

$$\mathbf{K}_{d2} = \begin{bmatrix} -14.9083 & 114.3146 & -53.1769 \\ -11.6169 & 21.779 & -38.0012 \\ -5.3111 & -27.2025 & 4.5611 \end{bmatrix}$$

Then we can obtain a fuzzy-model-based discrete-time controller as shown in (34).

$$\begin{aligned}&\text{Controller Rule } i:\\&\text{If } x_1(kT) \text{ is } M_1^i\\&\text{Then } u_c(kT) = -\mathbf{K}_{di} x_d(kT),\ i = 1,\ 2\end{aligned} \tag{34}$$

Figs. 2-4 show the state responses with the initial conditions $(x_1(0), x_2(0), x_3(0)) = (1, 1, 1)$.

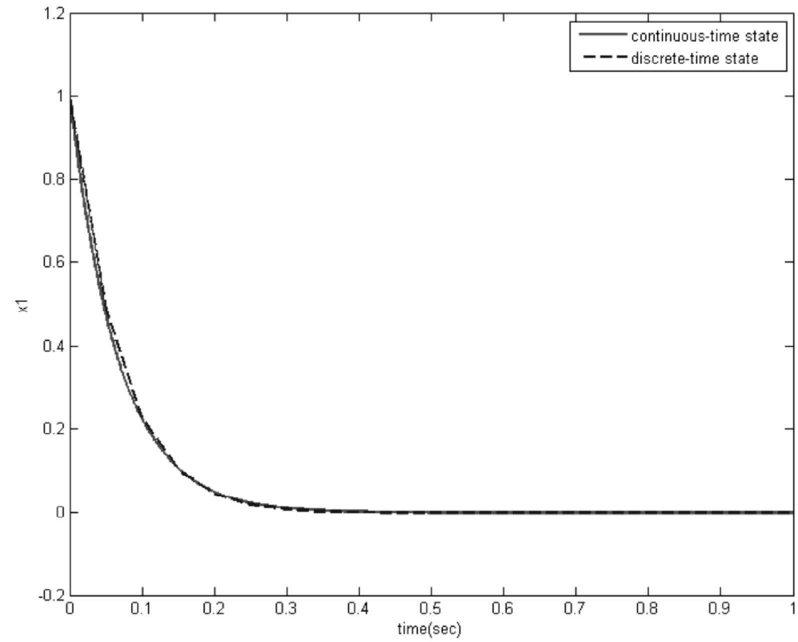

Figure 2: Response of the State x_1 of the Chen's Chaotic Attractor by the Proposed Method (Dashed Line) and the Analogue Controller (Solid Line)

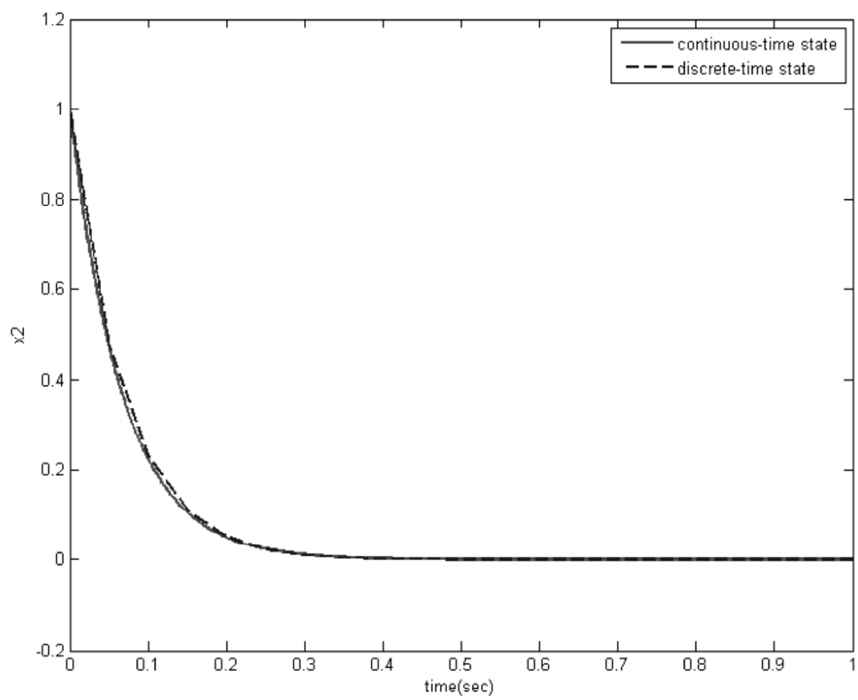

Figure 3: Response of the State x_2 of the Chen's Chaotic Attractor by the Proposed Method (Dashed Line) and the Analogue Controller (Solid Line)

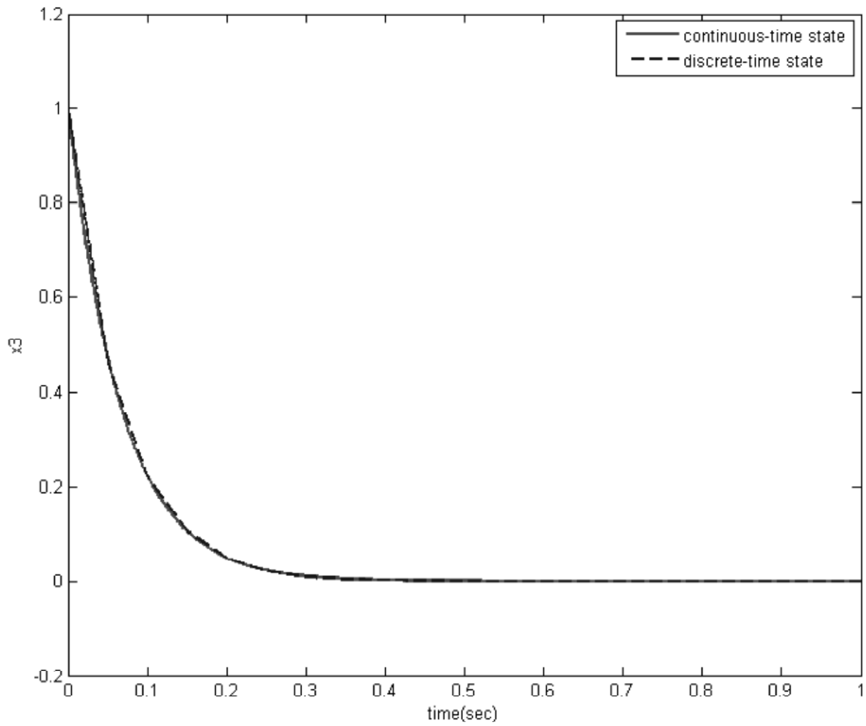

Figure 4: Response of the State x_3 of the Chen's Chaotic Attractor by the Proposed Method (Dashed Line) and the Analogue Controller (Solid Line)

In Table I, we compare the performance of the proposed method with the previous method [19]. For sampling T = 0.05s, the error between the states of the original continuous-time and the discrete-time systems by using the proposed method is 0.0039. Note that the cost function value revealed in [19] is incorrect, which should be 0.004 after our re-evaluation. This result shows that the error using the proposed PSO-based intelligent digital redesign method is still relatively smaller than that presented in the previous paper [19].

Table I
Comparison of Cost Function Value

Digital redesign method	*J*
Wook's	1.4445 §
Proposed	0.0039

V. CONCLUSION

In this paper, we have presented a novel intelligent digital redesign method by using the T-S fuzzy model for complex continuous-time systems. Instead of approximating the original continuous-time system directly, the proposed T-S fuzzy model approximates a digital redesign system. Moreover, the PSO algorithm with a simple structure is adopted to achieve a better performance with a smaller error in comparison with the previous article. Therefore, the PSO algorithm is applied to establish a more accurate discrete-time model to narrow down the search range of the parameter matrix and to conveniently search for the optimal solution. Finally, we have used an example to illustrate the feasibility and effectiveness of the proposed scheme for designing an optimal digital controller.

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