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Combined Servo/Regulation Operation of PID Controllers: Performance Considerations and Autotuning Relations

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Abstract: *This paper presents a PID tuning technique with the aim to reduce the degradation of the performance for the optimal settings (servo or regulation), when the operating mode of the system is different from the select one for tuning. The above is achieved from the definition of the Performance Degradation concept, that is a way for measuring this kind of loss on both extreme situations of tuning. From this approach, it is looking for an intermediate tuning that improves the overall performance of the control loop and thus reduces the Performance Degradation. Some extra approaches are developed and presented, specially regarding on autotuning and robustness considerations for this PID trade-off tuning technique.*

Keywords: *PID Control, Optimal Tuning, Performance Degradation*

1. INTRODUCTION

Proportional-Integrative-Derivative (PID) contro-llers are with no doubt the most extensive option that can be found on industrial control applications [1]. Their success is mainly due to its simple structure and to the physical meaning of the corresponding three parameters (therefore making manual tuning possible). This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

Because of the widespread use of PID controllers it is interesting to have simple but efficient methods for tuning the controller. In fact, since Ziegler-Nichols proposed their first tuning rules [2], an intensive research has been done.

O'Dwyer [3] presents a collection of tuning rules for PID controllers, which show their abundance.

Within the wide range of approaches to autotuning, optimal methods have received special interest. These methods provide, given a simple model process description -such as a First-Order-Plus-Dead-Time (FOPDT) model-settings for optimal closed-loop responses.

For One-Degree-of-Freedom (1-DoF) controllers, it is usual to relate the tuning method to the expected operation mode for the control system, known as *servo* or *regulation*. Therefore, controller settings can be found for optimal set-point or load-disturbance responses. This fact allows better performance of the controller when the control system operates on the selected *tuned mode* but, a degradation in the performance is expected when the tuning and operation modes are different. Obviously there is always the need to choose one of the two possible ways to tune the controller, for set-point tracking or to reject load-disturbances. In the case of 1-DoF PID, tuning could be optimal just for one of the two operation modes.

What is provided in this paper is a procedure in order to find an *intermediate* tuning for the controller that improves the overall performance of the system, considered as a *trade-off* between servo and regulation operation modes. The settings are determined from the combination of the optimal ones for set-point and load-disturbance.

The proposed method considers a 1-DoF PID controller as an alternative when an *explicit* 2-DoF PID controller is not available. It should be remembered that for the Two-Degree-of-Freedom (2-DoF) PID controller, tuning is usually optimal for regulation operation and suboptimal for servo-control, where this suboptimal behavior is achieved using a set-point weighting factor as an extra tuning parameter that gives the second Degree-of-Freedom, to improve the tracking action. Also, sometimes is not strictly necessary, or not justified, to increase the number of the tuning parameters in contrast to the benefits that could be obtained. It could be stated that the proposed *intermediate* tuning is a particular case that results in a suboptimal tuning, when both operation modes may happen and it could be seen as an *implicit* 2-DoF structure.

Some previous work have been done and can be found in [4] where it was determined that a suitable *trade-off* tuning can reduce the Performance Degradation when both operating modes are presented. Also, in [5] is shown that the stability margins are bounded between the ones of both tunings. What is presented here is a unified version of the previous study, besides some extensions referring specially to autotuning and also robustness considerations.

The paper is organized as follows. Next section introduces the general problem formulation, with some related concepts and a motivation example. Section 3presents the Performance Degradation (*PD*) analysis which depends on the tuning and on the operation mode for the system. In Section 4, we look for the *intermediate* tuning between the parameters of both operation modes in such a way that overall Performance Degradation is minimized. Some extensions of this main idea, like autotuning and robustness considerations, are presented in Section 5. Examples are in Section 6 and the work conclusions are drawn in Section 7.

2. PROBLEM FORMULATION

2.1. Control System Configuration

We consider the unity-feedback system shown in Fig.1, where $P(s)$ is the process and $K(s)$ is the (1-DoF PID) controller.

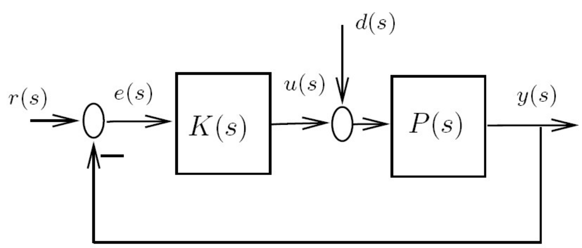

Figure 1: The Considered Feedback Control System

where $y(s)$ is the process output (controlled variable), $u(s)$ is the control signal, $r(s)$ is the set-point for the process output, $d(s)$ is the disturbance and $e(s)$ is the control error $e(s) = r(s)\text{-}y(s)$.

Also, the process $P(s)$ is assumed to be modelled by a FOPDT transfer function of the form:

$$P(s) = \frac{K}{1+Ts}e^{Ls} \tag{1}$$

where K is the process gain, T is the time constant and L is the dead-time. This model is commonly used in process control because is simple and describes the dynamics of many industrial processes approximately [6].

In addition, a common characterization of the process parameters is done in terms of the normalized dead-time $\tau = L/T$ [7]. On the other hand, the ideal PID controller with derivative time filter is considered:

$$K(s) = K_p\left(1+\frac{1}{T_i s}+\frac{T_d s}{1+(T_d/N)s}\right) \tag{2}$$

where K_p is the proportional gain, T_i is the integral time constant and T_d is the derivative time constant. The derivative time noise filter constant N usually takes values within the range 5-20 [6, 8]. Without loss of generality, here we will consider $N = 10$ [9].

2.2. Servo and Regulation Operation Modes

Considering the closed-loop system of Fig. 1 the process output is given by:

$$y(s) = \frac{K(s)P(s)}{1+K(s)P(s)}r(s)+\frac{P(s)}{1+K(s)P(s)}d(s) \tag{3}$$

The system can operate in two different modes, known as *servo control* or *regulatory control*. In the first case, the control objective is to provide a good tracking of the signal reference r, whereas in the second case is to maintain the output variable at the desired value, despite possible disturbances d.

For the servo operation mode, disturbances are not considered ($d(s)$=0), then (3) takes the form:

$$y_{sp}(s) := \frac{K(s)P(s)}{1+K(s)P(s)}r(s) \tag{4}$$

For regulation operation mode, no changes in the set-point reference are supposed (e.g. $r(s)$=0), so process output would be:

$$y_{ld}(s) := \frac{P(s)}{1+K(s)P(s)}d(s) \tag{5}$$

2.3. Set-Point and Load-Disturbance Tuning Modes

Controller tuning is one of the most important aspects in control systems. For the selection of this, it is necessary to take into account some aspects like: the controller structure, the information that is available for the process and the specifications that the output has to fulfill.

The analysis presented in this work is focused on the Integral Square Error (ISE) criteria, which is one of the most well known and most often used [10], however, the general analysis could be developed in terms of any other performance criterion. A general formulation of the performance index including a time weighting factor:

$$J^n = \int_0^\infty (t^n e(t))^2\,dt \tag{6}$$

Criteria (6) with n = 0 corresponds to the usual ISE criterion, with n = 1 is known as the ISTE criterion and with n = 2 is known as the ISTE criterion. In this paper, the optimization of (6) with n = 0 is considered, subject to the control system configuration shown in Fig. 1 where the controller $K(s)$ takes the *explicit* form of a 1-DoF PID controller (2).

The optimal settings presented below correspond to plants with a normalized dead time in the range [0.1-2.0]. Numerical optimization followed by a curve fitting procedure is done for both operating modes. As a result of the curve fitting the controller settings distinguish between $\tau \in [0.1, 1.0]$ and $\tau \in [1.1, 2.0]$, as provided in [9].

2.3.1. Set-Point Tuning Settings

When the settings for optimal set-point (servo control) response are considered, the controller parameters are adjusted according to the following formulae zhuangAthertonIEE1993

$$K_p = \frac{a_1}{K}(\tau)^{b_1},\; T_i = \frac{T}{a_2 + b_2\tau'}\; T_d = a_3 T(\tau)^{b_3} \tag{7}$$

with the values of a_i and b_i given in Table 1.

2.3.2. Load-Disturbance Tuning Settings

When determining the optimal operation in regulation mode, the optimization is performed similarly to the set-point case. In this case the formulae that provide the controller settings are

$$K_p = \frac{a_1}{K}(\tau)^{b_1},\; \frac{1}{T_i} = \frac{a_2}{T}(\tau)^{b_2},\; T_d = a_3 T(\tau)^{b_3} \tag{8}$$

and the corresponding values of a_i and b_i are given in Table 1.

Table 1
Optimal PID Settings for Set-Point (SP) and Load-Disturbance (LD)

τ range	*0.1 - 1.0*		*1.1 - 2.0*	
Tuning	*SP*	*LD*	*SP*	*LD*
a_1	1.048	1.473	1.154	1.524
b_1	-0.897	-0.970	-0.567	-0.735
a_2	1.195	1.115	1.047	1.130
b_2	-0.368	-0.753	-0.220	-0.641
a_3	0.489	0.550	0.490	0.552
b_3	0.888	0.948	0.708	0.851

2.4. Problem Statement

If the control-loop has always to operate on one of the two possible operation modes (servo or regulator) the tuning choice will be clear. However, when both situations occur, it may not be so evident which are the most appropriate controller settings.

The analysis to answer the problem concentrates on the Performance Degradation index which provides a quantitative evaluation of the controller settings with respect to the operation mode and the main objective is to reduce it.

In this paper, the question *"How to improve the performance when the system operates in a different mode that it was tuned for?"* is treated by searching an *intermediate* tuning for the controller, between both optimal parameters settings for set-point and load-disturbance, in order to reduce the global Performance Degradation index.

The problem is faced within the well known quadratic performance criteria and corresponding tuning settings for the PID controller, as there are presented in [9].

2.5. Motivation Example

In order to show the performance of the previously presented settings and how this can degrade when the controller is not operating according to the tuned mode, an example is provided. This motivates the analysis to be presented in the next sections.

We consider the following controlled process, taken from [9], and its corresponding FOPDT model:

$$P(s) = \frac{e^{-0.5s}}{(s+1)^2} \approx \frac{e^{-0.99s}}{1+1.65s} \tag{9}$$

The application of the ISE tuning formulae for optimal set-point response provides: $K_p^{sp} = 1.66$, $T_i^{sp} = 1.69$ and $T_d^{sp} = 0.51$, whereas the tuning for optimal load-disturbance provides: $K_p^{ld} = 2.42$, $T_i^{ld} = 1.01$ and $T_d^{ld} = 0.56$..

Fig. 2 shows the system responses for both optimal settings when the control system is operating in both, servo and regulation mode. It can be seen that the regulation response of the set-point tuning is close to the optimal regulation one than the load-disturbance tuning to the optimal servo response. Therefore the observed Performance Degradation is larger for the load-disturbance tuning. From

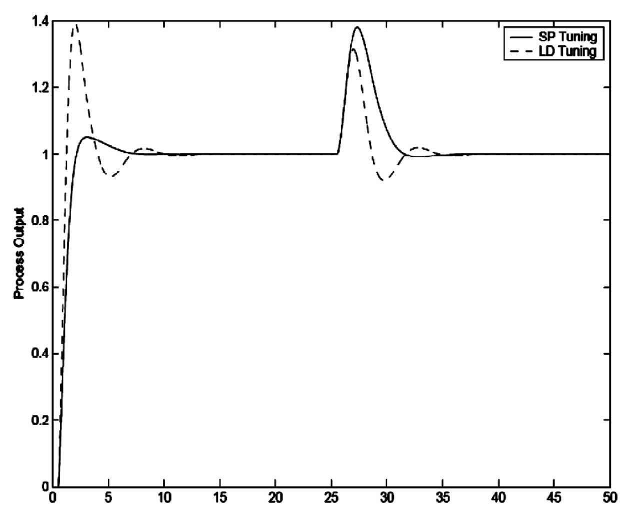

Figure 2: Responses of the Control System (9) Operating in Both Servo and Regulation Modes

a global point of view it will seem better to choose the set-point settings.

3. PERFORMANCE DEGRADATION ANALYSIS

The Performance Degradation concept for set-point and load-disturbance tunings depending on the operation mode was previously presented and developed in [4] and it is succinctly reproduced here for sake of completeness and because it is important to have a clear idea of how is defined and computed. There, the performance of the control system is measured in terms of a performance index that takes into account the possibility of an operation mode different from the selected one. This motivates the redefinition of the performance index (6) as:

$$J_x^n(z) = \int_0^\infty (t^n e(t,x,z))^2 \, dt \qquad (10)$$

where *x* denotes the *operating mode* of the control system and *z* the selected operating mode for tuning, i.e., the *tuning mode*. Thus, we have $x \in \{xp, sl\}$ and $z \in \{xp, sl\}$, where *sp* states for set-point (servo) tuning and *ld* for load-disturbance (regulator) tuning. Obviously, for one specific process it has to be verified that:

$$J_{sp}^n(sp) \leq J_{sp}^n(ld)$$

$$J_{ld}^n(ld) \leq J_{ld}^n(sp)$$

Performance will not be optimal for both situations. The Performance Degradation measure helps in the evaluation of the loss of performance with respect to their optimal value. Performance Degradation, $PD_x z$, will be associated to the *tuning mode* - *z* - and tested on the, opposite, *operating mode* - *x* -. According to this, the Performance Degradation of the load-disturbance tuning, $PD_{sp} ld$, will be defined as:

$$PD_{sp}(ld) = \left| \frac{J_{sp}^n(ld) - J_{sp}^n(sp)}{J_{sp}^n(sp)} \right| \qquad (11)$$

whereas the Performance Degradation associated to the set-point tuning, $PD_{ld}(sp)$, will be:

$$PD_{ld}(sp) = \left| \frac{J_{ld}^n(sp) - J_{ld}^n(ld)}{J_{ld}^n(ld)} \right|. \qquad (12)$$

Note that, because the controller settings expressed through (7) and (8) have explicit dependence on the process normalized dead-time τ, it is worth taking into account that, for the PID application, the Performance Degradation will also depend on τ. Fig. 3 shows the performance analysis for the normalized dead-time ranges where PID controller settings are provided.

Note also that Performance Degradation is a decreasing function of the normalized dead-time, taking very high values for processes with small normalized dead-time.

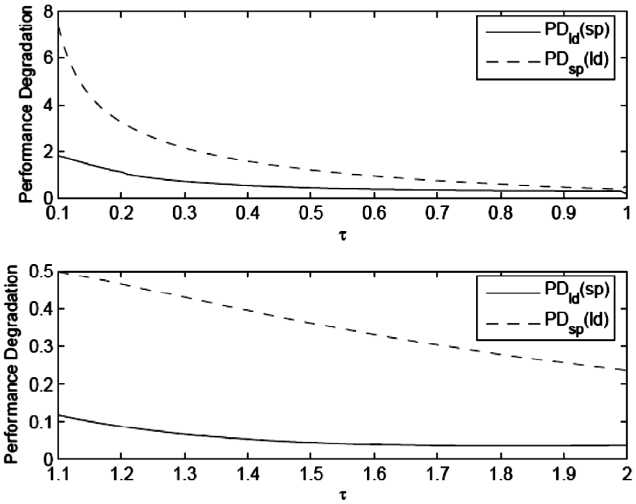

Figure 3: Performance Degradation of Set-point (sp) and load-disturbance (ld) tunings for ISE Criteria with Respect to the Normalized Dead-time τ

The final decision for the choice of the appropriate tuning mode will depend on the preference or importance that is given for the system operation as regulator or as servo. However, if both situations are likely to occur, Fig. 3 suggests a set-point based tuning is to be preferred, because it provides lower Performance Degradation than load-disturbance tuning.

4. SERVO/REGULATION *TRADE-OFF* TUNING

In this section, we propose a global approach where some degree of optimality is lost, but the Performance Degradation with respect to both operating modes is considered. Therefore, the overall performance is expected to be enhanced.

The tuning approaches presented in subsection 2.3 can be considered extremal situations. The controller settings are obtained by considering exclusively one mode of operation. This may generate, as it has been shown in the previous section, quite poor performance if the non-considered situation occurs. This fact suggests to analyze if, by loosing some degree of optimality with respect to the tuning mode, the Performance Degradation can be reduced when the operation is different to the selected for tuning.

Based on this observation we suggest to look for an *intermediate* controller. In order to define this exploration, we need to define the search-space and the overall Performance Degradation index to be minimized. Obviously the solution will depend on how this factors are defined.

The search of the controller settings that provide a *trade-off* performance for both operating modes could be stated in terms of a completely new optimization procedure. However we would like to take advantage of the autotuning formulae (like (7) and (8)), in order to keep the procedure, as well as the resulting controller expression, in similar simple terms. Therefore the resulting controller settings could be considered as an extension of the optimal ones. On this basis we define a controller settings family parameterized in terms of a single parameter $\gamma \in [0, 1]$. The set-point tuning will correspond to a contour constraint for $\gamma = 0$, whereas the load-disturbance tuning corresponds to $\gamma = 1$. Fig. 4 shows graphically the procedure and the application for the PID controller tuning.

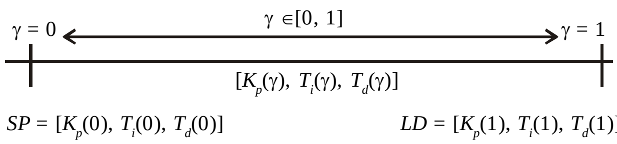

Figure 4: γ-tuning Procedure for the Search of the *Intermediate* Controller

The controller settings family $[K_p(\gamma), T_i(\gamma), T_d(\gamma)]$, can be expressed by observation of formulae (7) and (8). It is seen that the K_p and T_d controller parameters obey to the same expression. Therefore we can think on a generalized parameter $K_p = K_p(\gamma)$ and $T_d = T_d(\gamma)$ and a linear evolution for the integral time constant T_i. Therefore:

$$K_p(\gamma) = \frac{a_1(\gamma)}{K}[\tau]^{b_1(\gamma)}$$

$$T_i(\gamma) = \gamma T_i^{ld} + (1-\gamma)T_i^{sp} \quad (13)$$

$$T_d(\gamma) = a_3(\gamma)T[\tau]^{b_3(\gamma)}$$

with $\gamma \in [0,1]$, T_i^{ld} and T_i^{sp} stand for the load-disturbance and set-point setting for T_i respectively and the $(a_i(\gamma), b_i(\gamma))$ are generated according to:

$$a_i(\gamma) = \gamma a_i^{ld} + (1-\gamma)a_i^{sp} \quad (14)$$

$$b_i(\gamma) = \gamma b_i^{ld} + (1-\gamma)b_i^{sp}$$

Now, in order to define a global Performance Degradation (*PD*) index, the previously defined terms (11) and (12) need to be extended. Note that the Performance Degradation was associated to the *tuning mode*, therefore tested against the opposite *operating mode*. Now, for every value of γ the Performance Degradation needs to be measured with respect to both operating modes (because the corresponding-tuning does not necessarily corresponds to an operating mode). Hence,

- $PD_{sp}(\gamma)$ will represent the Performance Degradation of the γ-tuning on servo operating mode.

$$PD_{sp}(\gamma) = \left| \frac{J_{sp}^n(\gamma) - J_{sp}^n(sp)}{J_{sp}^n(sp)} \right| \quad (15)$$

- *PDld*g will represent the Performance Degradation of the *f*×-tuning on regulation operating mode.

$$PD_{ld}(\gamma) = \left| \frac{J_{ld}^n(\gamma) - J_{ld}^n(ld)}{J_{ld}^n(ld)} \right| \quad (16)$$

From these side Performance Degradation definitions, the overall Performance Degradation is introduced and interpreted as a function of γ. There may be different ways to define the *PD*(γ) function, depending on the importance associated to every operating mode (e.g. applying weighting factors to each component). However, every definition must satisfy the following contour constraints:

$$PD(0) = PD_{ld}(sp),\ PD(1) = PD_{sp}(ld) \quad (17)$$

The most simple definition would be:

$$PD(\gamma) = PD_{ld}(\gamma) + PD_{sp}(\gamma) \quad (18)$$

This expression represents a compromise, or a balance, between both losses of performance.

The *intermediate* tuning will be determined by proper selection of. This choice will correspond to the solution of the following optimization problem:

$$\gamma_{op} = \arg\left[\min_{\gamma} PD(\gamma)\right] \quad (19)$$

The optimal value (19) jointly with (13), give a tuning formula that provides a worse performance than the optimal settings operating in the same way but also a lower degradation in the performance when the *operating mode* is different from the *tuning mode*.

5. EXTRA CONSIDERATIONS

This section provides some extra approaches about the main proposal presented in the previous section. These ideas move towards: autotuning settings, weighting factor in the cost function, a stability margins characterization and from this, a reformulation of the trade-off tuning, taking into account robustness considerations.

5.1. Automatic Tuning Settings

Tuning relations (13) allow to select γ on the basis of *trade-off* performance for both operating modes. However, it would be desirable an automatic methodology to choose this parameter without the need to run the whole Performance Degradation analysis.

In order to pursue the previous idea, by repeating the problem optimization posed in (19) for different values of the normalized dead-time τ, we can find an optimal set for γ parameter. For this set, it is possible to approximate a function to determine a general procedure, that allows to find the suitable value for the that provides the best intermediate tuning. Results are adjusted to the following expressions as:

$$\gamma(\tau) = a + b\tau + c\tau^2 \quad \tau \in [0.1, 1.0] \quad (20)$$

$$\gamma(\tau) = a + b\tau^c \quad \tau \in [1.1, 2.0] \quad (21)$$

where a, b and c are given in Table 2 depending of the τ range. Fig. 5 shows the followed procedure.

Table 2
Settings for *Trade-off* Autotuning

	τ range	
constant	*0.1 - 1.0*	*1.1 - 2.0*
a	0.5778	0.2382
b	-0.5753	0.2313
c	0.5528	-7.1208

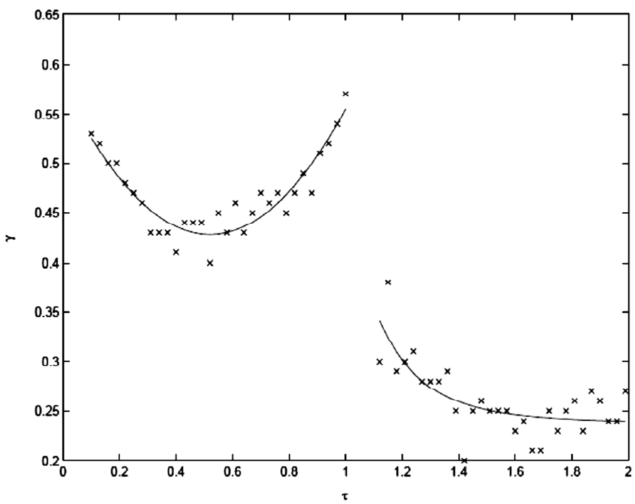

Figure 5: Optimal set for Parameter (x points) and the Corresponding Approximated Function (Solid-line)

Equations (20) and (21) for γ along with the settings (13) and (15) provide what we call here γ-autotuning for servo/regulation operation, that is one of the main contributions of this paper.

5.2. Weighting Factors for Balanced Performance Degradation

As it has been mentioned before, the greatest loss of performance occurs when the load-disturbance tuning operates as a servo mode. Therefore, $PD_{sp}(\gamma)$ will be the largest component of the global expression of $PD(\gamma)$ and in the oppositive side $PD_{ld}(\gamma)$ the smallest one. This causes that the percentage reduction of PD that can be obtained from the PD_{ld} side is lower than the one for the PD_{sp} part. A balanced reduction of $PD(\gamma)$ from both Performance Degradations is possible by introducing weighting factors associated to each operating mode. This idea can be applied rewriting (18) as:

$$PD(\gamma) = \alpha PD_{ld}(\gamma) + (1-\alpha)PD_{sp}(\gamma) \quad (22)$$

where $\alpha \in [0, 1]$ is the weight factor and indicates which of the two possible operation modes is preferred or more important. One way to express the importance between both operation modes, could be the total time that the system operates in each one of them. For example, a system that operates the 75% of the time as a regulator (or viceversa 25% as a servo), $\alpha = 0.75$. However, the α parameter allows to make a more general choice for the preference of the system operation (not only taking into account the time for each operation mode). Note also that (22) with $\alpha = 0.50$, represents an equivalent expression obtained previously in (18) that gives the same significance for both operation modes.

One possible way to select α, therefore the weighting factors, can be stated as follows:

$$\alpha = W_{ld} = \frac{PD_{sp}(ld)}{PD_{sp}(ld) + PD_{ld}(sp)} \quad (23)$$

$$(1-\alpha) = W_{sp} = \frac{PD_{ld}(sp)}{PD_{sp}(ld) + PD_{ld}(sp)}$$

From (22) with the selection (23) and subjected to the optimization (19), we can get a balance of the performance improvement introduced with the γ-tuning.

5.3. Stability Margins Characterization

Stability and robustness are two of the most important properties in process control. Every tuning rule has to provide a proper stability for the closed-loop system and a suitable robustness, because process model often has an amount parametric uncertainty.

Stability for both, set-point and load-disturbance, optimal tuning settings can be studied determining the robustness that they provide [5]. Then, Gain and Phase Margins are traditionally used to measure robustness. The expressions for those stability margins can be obtained from the basic definitions as:

$$GM = \frac{1}{|K(j\omega_p)P(j\omega_p)|}$$

$$PM = arg[K(j\omega_g)\, P(j\omega_g] + \pi \quad (24)$$

where ω_p and ω_g are given by:

$$arg[K(j\omega_p)P(j\omega_p)] = -\pi \quad (25)$$

$$|K(j\omega_g)\, P(j\omega_g)| = 1 \quad (26)$$

For our stability margins characterization, applying equations from (25) to (26), it is possible to determinate the stability margins for the whole transition of Fig. 6 shows the obtained results.

It can be seen that Gain and Phase Margins, for each $\gamma \in [0,1]$, provide stability guarantees for the closed-loop system achieved by the trade-off tuning in the whole range of τ. In addition, these margins are bounded by the margins corresponding to the optimal tuning settings: set-point ($\gamma = 0$) and load-disturbance ($\gamma = 1$) meaning that if increases,

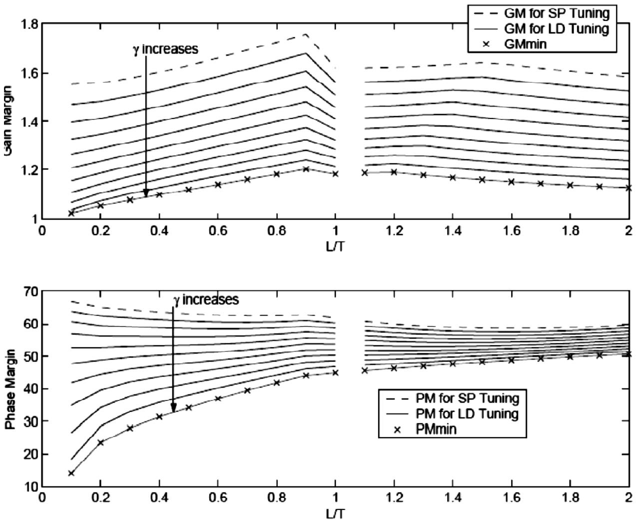

Figure 6: Gain and Phase Margins Variation for Set-point and load-disturbance turnings and the Variation

the stability margins decrease and consequently the robustness. Another aspect that can be observed, in Fig. 6, is that Phase Margin has almost always values higher than 30o, that is recommended as the acceptable minimum [10].

5.4. Trade-off Tuning with Robustness Considerations

From the stability margins characterization, exposed in the previous section and showed graphically in Fig. 6, it is possible to see that set-point based settings provide better stability margins than load-disturbance settings, which means a greater robustness for the control system. Also, as it was mentioned before, Phase Margin has almost always values higher than 30o, but in the other site, Gain Margin has small values, so the proposed extension is about how to obtain the most GM with the less PD.

According to that, a Gain Margin reduction expression will be defined, in terms of γ, as:

$$GM_{red}(\gamma) = \frac{GM(sp) - GM(\gamma)}{GM(sp} \qquad (27)$$

note that:

$$GM_{red}(0) = GM_{red}(sp) = 0, \quad GM_{red}(1) = GM_{red}(ld) \qquad (28)$$

Obviously it is desirable that the reduction of *GM* be as minimum as possible, that means close to the *GM* for set-point tuning settings, which is the maximum value that can be obtained.

Now, from (18) and (27), a new global expression that evaluates both Performance Degradation and Gain Margin reduction can be define as:

$$PDGM_{red}(\gamma) = PD(\gamma) + GM_{red}(\gamma) \qquad (29)$$

This redefinition of the previous index defined in (18) represents a balance between both kinds of performance degradations (servo and regulation), as well as the minimum reduction as possible of the Gain Margin term. Therefore achieving a better trade-off with the best robustness that is possible.

6. EXAMPLES

This section presents several examples of the above ideas, exposed in the previous sections, that illustrate how the implementation of the intermediate tuning improves the performance of the closed-loop system, when the operation mode is different to the tuning mode. Therefore achieving a better *trade-off*.

Let us consider the system (9), shown before as a Motivation Example. In all the examples it is shown the process output for the three tunings: set-point, load-disturbance and the proposed ?-tuning (with its variations), when a step change in the set-point occurs at t=0 and in the disturbance input at $t = 25$.

6.1. γ-Autotuning

From the automatic tuning settings exposed in Section 5.1 as the proposed γ-autotuning, the PID controller parameters will be: $K_p^\gamma = 1.98$, $T_i^\gamma = 1.40$ and $T_d^\gamma = 0.53$. Fig. 7 shows the corresponding process outputs.

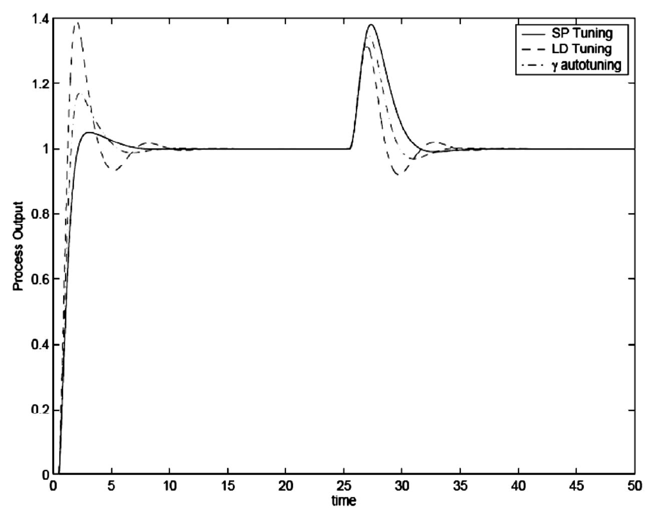

Figure 7: Performance of the Set-point (Solid), Load-disturbance (Dashed) and -autotuning (Dot-dashed), Operating in Both Servo and Regulation Modes

Table 3
PD Values for the System (9) and the Improvement Obtained with γ-autotuning

tuning	PD_{sp}	PD_{ld}	PD
set-point(sp)	-	0.4128	0.4128
load-disturbance (ld)	0.9341	-	0.9341
-autotuning	0.1131	0.0654	0.1785
improvement in % of -autotuning	87.9%(ld)	84.3%(sp)	57.1%(sp)
(respect to)			80.9%(ld)

It is seen that the proposed γ-autotuning gives lower performance than the optimum settings when the system operates in the same way as it was tuned. However, higher performance can be obtained for the whole system operation (servo and regulatory control), when an intermediate tuning is used.

Table 3 shows the Performance Degradation values calculated from (15) to (18) for each tuning. The last row presents the improvement in percentage that can be achieved with the γ-autotuning respect to the extreme tunings (set-point and load-disturbance).

All the values confirm the fact that, in global terms, when both operation modes could appear, the proposed γ-autotuning is the best choice to tune the PID controller.

6.2. Weighting Factors

Fig. 8 shows various plots of (22) for different values of α. Note that the line for α = 0.50, represents the equivalent to the Performance Degradation obtained previously in (18).

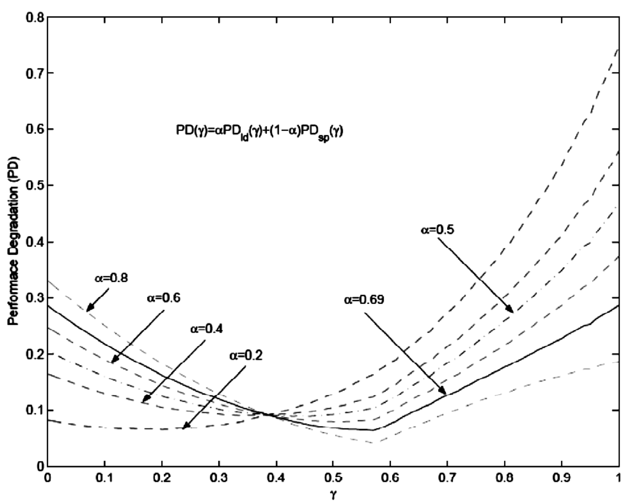

Figure 8: Performance Degradation (22) using Different Weighting Factors

For the system example, α = 0.69 is the corresponding weight factor obtained according to (23), in order to get a balanced performance improvement. For this value it is found that γ = 0.57 is the value that minimizes (22) and then using tuning relations (13), the PID parameters are: $K_p^\gamma = 2.08$, $T_i^\gamma = 1.30$ and $T_d^\gamma = 0.54$. .

The process outputs are shown in Fig. 9 and the values of the PD and the corresponding improvement can be seen in Table 4.

It is possible to see in Table 4 that the improvement that can be achieved from the both sides (servo and regulation) are the same, so a balanced improvement is feasible by introducing the appropriate weighting factors in (18).

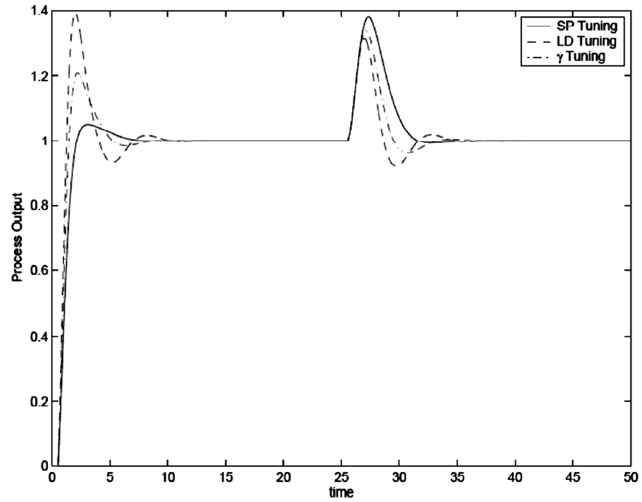

Figure 9: Performance of the Set-point (Solid), Load-disturbance (Dashed) and γ-tuning (Dot-dashed), Operating in Both Servo and Regulation Modes

Table 4
PD(α = 0.69) Values for the System (9) and the Improvement Obtained with γ-tuning

tuning	PD_{sp}	PD_{ld}	$PD_{\alpha=0.69}$
set-point(sp)	-	0.4128	0.2862
load-disturbance (ld)	0.9341	-	0.2863
= 0.57	0.2039	0.0017	0.0637
improvement in % of	78.2%(ld)	99.5%(sp)	77.8%(sp)
-tuning (respect to)			77.8%(ld)

6.3. Trade-off Tuning with Minimum Gain Margin Reduction

The associated side Performance Degradation (18) and Gain Margin reduction (27) as well as the overall index computed by using (29) for the system (9) are shown in Fig. 10.

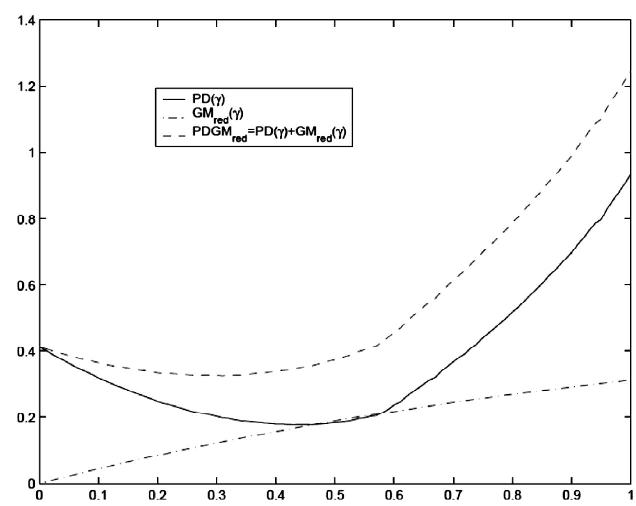

Figure 10: Global index (29), Performance Degradation (18) and Gain Margin reduction (27)

Fig. 10 shows the Performance Degradation and *GM* reduction analysis for this system from where it is found that γ = 0.31 is the value that minimizes expression (29). With this optimal value for γ and the FOPDT model, the three parameters of the PID controller are: $K_p^\gamma = 1.89$, $K_i^\gamma = 1.48$ and $T_d^\gamma = 0.53$.

Process outputs of the system are shown in Fig. 11. Moreover, the Gain Margin (*GM*) and Phase Margin (*PM*) values for each tuning are: *GM*(*sp*) = 1.66, *PM*(*sp*) = 62.77°, *GM*(*ld*) = 1.14, *PM*(*ld*) = 36.92° and *GM*(γ) = 1.45 and *PM*(γ) = 55.92°.

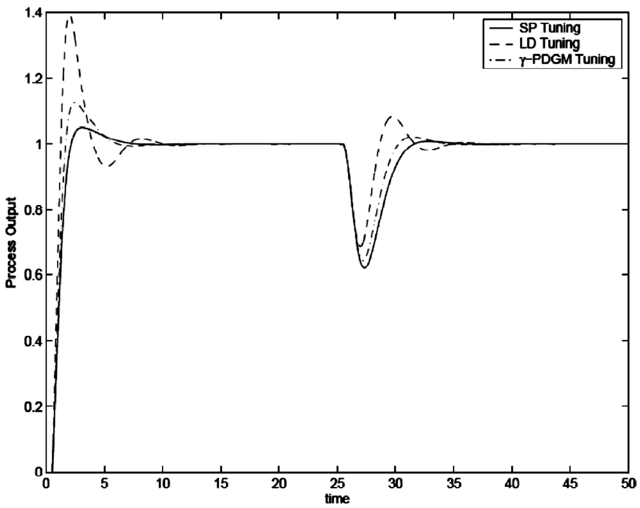

Figure 11: Performance of the Set-point (Solid), Load-disturbance (Dashed) and γ-*PDGM* (Dot-dashed) Tunings, Operating in Both Servo and Regulation Modes

Table 5 shows the Performance Degradation values calculated from equations (15) to (18) and the Gain Margin reduction from (27). Also, the global index (29) is showed and the last row presents the improvement in percentage when the trade-off tuning is used.

Table 5
***PD* and GM_{red} Values for the System (9) and the Improvement Obtained with PDGM Tuning**

tuning	PD_{sp}	PD_{ld}	PD	GM_{red}	$PDGM_{red}$
set-point(sp)	-	0.4128	0.4128	-	0.4128
load-disturbance (ld)	0.9341	-	0.9341	0.3128	1.2469
= 0.31	0.0587	0.1404	0.1991	0.1271	0.3262
improvement in % of PDGM tuning (respect to)	93.7% (ld)	66.0% (sp)	78.7% (ld) 51.8%(sp)	59.38% (ld)	73.84% (ld) 20.98%(sp)

All those values confirm the fact that, in global terms (when both operating modes could appear), the γ-PDGM tuning has a better global performance (taking into account Gain Margin reduction consideration) than the optimal settings and obviously, it leads to have a smaller value for *PD* and GM_{red} and the improvement can be measured as a diminution of that degradation and/or reduction.

7. CONCLUSIONS

In process control it is very usual to have changes in the set-point, as well as in the disturbance. This causes the need to face with both servo and regulatory control problems. For 1-DoF PID controllers, when the tuning objective is different to the real system operation, a degradation in the performance is expected and it can be evaluated. A reduction in the overall Performance Degradation can be obtained by searching an intermediate controller between the optimal ones proposed for set-point and load-disturbance tunings.

Some extensions around the general main idea have been presented, like: autotuning, the use of weighting factors and considerations about stability and robustness into the trade-off, showing a general approach depending on the system operation. The examples showed the improvement obtained with each one of the extensions.

Even if the results were presented and exemplified using the ISE performance criteria and the Zhuang and Atherton tuning [9], it can be possible to apply a similar methodology to other PID controller tunings with different performance objectives. Also, different transitions for the intermediate tuning parameter, can be studied.

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