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Performance Study of a Complex System with Three Subsystems in Series Configuration Using Reduction Method and Copula Distribution

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Abstract **- This paper investigates the reliability of a complex system that consists of three subsystems, A, B, and C, linked in series with a power source. Every subsystem is made up of three units that follow the (1–out–of–3: G) policy. For the mathematical model study, power failure was taken into account. There are three types of failure in the system: minor / major partial failure, total failure, and total failure. The failure rates are constant, but the repair rates are distributed in two ways (general and Gumbel–Hougaard family Copula). To study the system, we used the supplementary variable technique and the Laplace transform. The availability, reliability, MTTF, and profit function are all measured as essential reliability measures. A reduction approach is used to increase device efficiency and availability. To illustrate computed results, tables and graphs are used.**

Index Terms - k-out-of-in G: system, Reliability, Availability, *MTTF*, Gumbel–Hougaard Copula, Profit Function, Supplementary Variable Technique.

1. Introduction

The performance of reliability measures in terms of availability, reliability, mean-time-to-system-failure (MTTF), and cost benefits in the operation of repairable systems has been investigated. Previously, numerous researchers and scientists have done the many studies on reliability tests of the complex system. Many efforts have been made to improve component reliability in sequence, parallel, and k-out-of-n configurations. Reference [14] et al. used Copula to investigate the probabilistic evaluation of a complex system with two subsystems in sequence, multiple types of failure, and two types of repair. The performance analysis of a complex repairable system with two subsystems in series configuration and an incomplete switch was addressed in reference [12] et al.. Reference [13] et al. investigated the reliability of a repairable network infrastructure connecting three computer laboratories to a server in a 2-out-of-3-G configuration. Using a copula approach, as [11] et al. investigated stochastic analysis of a two-unit complex repairable system with switch and human failure. Reference [8] et al. used the Gumbel-Hougaard family copula distribution to determine the reliability of a complex system with two subsystems linked in a series configuration. The Gumbel-Hougaard family copula was used in [10] et alstochastic.'s study of a complex system under a preemptive resume repair policy. Reference [6] et al. addressed the reliability analysis of a complex system that consists of two repairable subsystems, A and B, linked in sequence, as well as two forms of failure, deliberate and critical. The complex method was developed by [7] et al. based on information gathered as a result of a marked procedure involving two repairmen with different expertise and availability. The operational conduct of the 2-out-of-3: G device for various situations has been explored in [4] et al. with the definition of preventive maintenance. Many methods, such as reduction and redundancy, are used to improve the performance of a system. The availability / reliability of the system can be enhanced using the reduction approach by reducing the failure rates of some of its units by a factor ρ that is a number between zero and one (i.e. $0 < \rho < 1$). The equivalence of reliability factors for a general series-parallel system with individual units having exponentially distributed lifetimes was discussed in [2]. Reference [1] discussed the reliability equivalence factors of a series-parallel system when the system units are independent and similar and have lifetimes that follow the Weibull distribution. The reliability equivalence factor of a parallel system with time-varying failure rates was studied by [3] et al. Development of the reliability of a dependent system under copula was studied in [9]. The linear-exponential distribution function was used by Reference [5] et al. to improve device reliability. A new form of model of a complex repairable system with three subsystems, subsystem A, subsystem B, and subsystem C in series configuration, has been studied in this paper, with power failure taken into account. The 1-out-of-3; G policy applies to all subsystems. In each of the three subsystems, all of the units are connected in a parallel configuration. After any one unit in one subsystem fails, the device appears to minor partial failure / degraded states, whereas all other subsystem units remain functional. Similarly, failing two units in any subsystem when all other subsystems are functioning normally results in major partial failure / degradation. When more than three units of any subsystem fail and all other subsystems are operational, the system is considered down. The power failure is processed as down state. The failure rates are constant and believed to follow an exponential distribution, but the repair rates follow two different distributions: general and Gumbel-Hougaard family copula. To obtain the system's availability, reliability, MTTF, and profit function, the system is analyzed using the supplementary variable technique and the Laplace transform. The reduction approach improves the original system's availability and reliability. The results are presented in tables and graphs.

2. Assumptions

During model analysis the following suggestions were supposed:

- At first, the system is in full working order in state S_0, with all three subsystems and the power switch functioning properly.
- The system is made up of three subsystems: A, B, and C, which are linked in a series.
- Subsystems A, B and C are running successfully when one or more units are in good working order, i.e., 1-out-of-3: G policy.
- Subsystems A, B, and C are composed of one main unit and two hot standby units that are ready to start after each unit in the subsystem fails for a short period of time.
- General repair fixes minor and major partial failures, but Gumbel-Hougaard family copula distribution fixes the entire failed state.
- Due to a power switch failure, the system is considered to be in a down state, and it is then fixed using copula distribution.
- The system gets repaired instantly; it operates with full efficiency and no weakness during repair.
- The failure rates are constant and follow an exponential distribution.
- The system is as good as new after the repair.

3. Notations

t/ s	Time scale / Laplace transform variable
λ_1 / λ_2 / λ_3	Failure rates of each unit in subsystem A / subsystem B / subsystem C
λ_{pw}	Failure rate of the power switch
$\varphi_1(x)$ / $\varphi_2(y)$	Repair rate of each unit in subsystem A / subsystem B
$\varphi_3(z)$	Repair rate of each unit in subsystem C
$P_i(t)$	The probability that the system is in S_i state at an instant t for i = 0 to 10
$P_i^*(s)$	Laplace transform of $P_i(t)$
$P_i(x,t)$	Probability that the system in state S_i , i = 1 to 10; the system is under repair and elapsed repair time is x
K_1 / K_2	Revenue and service cost per unit time, respectively.
$E_p(t)$	Expected profit during the interval [0,t)
$S_\varphi(x)$	$\varphi(x)\exp\left(-\int_0^x \varphi(u)du\right)$
$S_\varphi^*(s)$	Laplace transform of $S_\varphi(x)$
$S_\varphi^*(s)$	$=\int_0^\infty \varphi(x)\exp(-sx-\int_0^x \varphi(u)du)dx$
$\mu_0(x) = C_\theta(u_1(x), u_2(x))$	Coupled repair rate for complete failed state S_i to initial state S_0, then the expression for joint probability according to Gumbel-Hougaard family of copula is given as: $\mu_0(x)=\exp[x^\theta+(\log\varphi(x))^\theta]^{1/\theta}$, where $u_1=\varphi(x)$ and $u_2=e^x$, where θ is a parameter $1\le\theta\le\infty$.

4. Definition of the State

Table I shows the current state of the system following the failure of units in all subsystems, including the transmission power switch. The operative states of the system are {S_0, S_1, S_2, S_4, S_5, S_7 *and* S_8}, and S8, while the failed states of the system are {S_3, S_6, S_9, *and* S_{10}}.

TABLE 1

THE MODEL'S STATE OVERVIEW

State	Description
S_0	The system is perfect, and all units of subsystems A, B, and C are in good working order.
S_1	The system has degraded, with a minor partial failure in subsystem A due to a failure in the main unit's subsystem A. Although the state is undergoing general repairs and the system is in service, both subsystems B and C are in good working order.
S_2	The system is degraded with major partial failure in subsystem A due to the failure of any two units in subsystem A. The system is in operating mode and the state is under general repair, but both subsystems B and C are in good working order.
S_3	After more than two units in subsystem A failed, the system is completely down. Copula distribution is being used to repair the system.
S_4	The system is in a degraded state with minor partial failure in subsystem B due to the failure of the main unit in subsystem B. So the state is in need of general repair and the system is in operation, both subsystems A and C are in good working.
S_5	The system is in degraded state with major partial failure in subsystem B due to the failure of any two units in subsystem B. The state is under general repair, and the system is in operating mode, but both subsystem A and subsystem C are in good operating state.
S_6	After more than two units in subsystem B malfunction, the system is completely broken. The system is being repaired using copula distribution.
S_7	Owing to the failure of the main unit in subsystem C, the system is in a degraded state with minor partial failure in subsystem C. So the state is undergoing general repairs and the system is in service, all subsystems A and B are in good working order.
S_8	The system is degraded with major partial failure in subsystem C due to the failure of any two units in subsystem C. The system is in operating mode and the state is under general repair, but both subsystems A and B are in good working condition.
S_9	The system has completely failed due to the failure of more than two units in subsystem C. Copula distribution is being used to patch the system.
S_{10}	The system is in a down state due to a power switch malfunction that has made it inoperable.

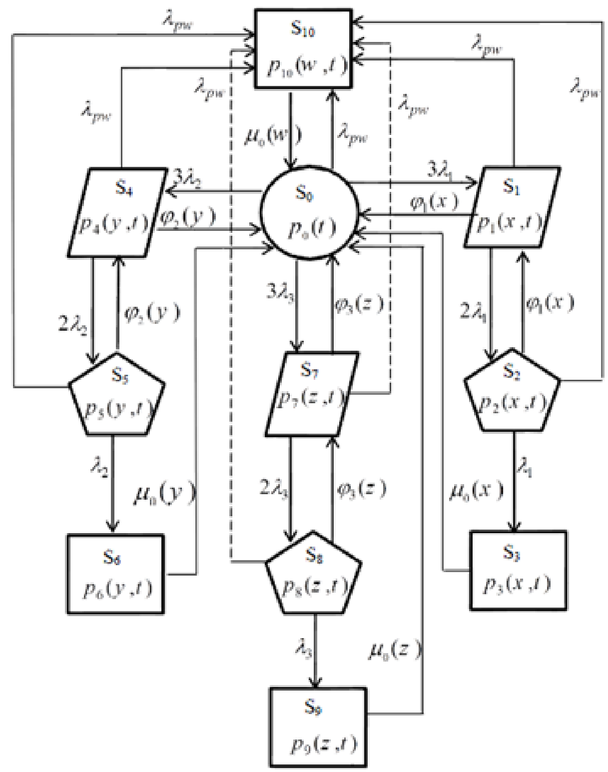

FIGURE 1
DIAGRAM OF THE MODEL'S STATE CHANGE

5. Mathematical Analysis of the Model

The following set of differential equations is correlated with the present mathematical model in relation to the state transformation in Figure 1.

$$\left[\frac{d}{dt}+3\lambda_1+3\lambda_2+3\lambda_3+\lambda_{pw}\right]P_0(t)=\int_0^\infty \varphi_1(x)P_1(x,t)dx$$

$$+\int_0^\infty \varphi_2(y)P_4(y,t)dy+\int_0^\infty \varphi_3(z)P_7(z,t)dz+\int_0^\infty \mu_0(x)P_3(x,t)dx$$

$$+\int_0^\infty \mu_0(y)P_6(y,t)dy+\int_0^\infty \mu_0(z)P_9(z,t)dz \qquad (1)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+2\lambda_1+\lambda_{pw}+\varphi_1(x)\right]P_1(x,t)=0 \qquad (2)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\lambda_1+\lambda_{pw}+\varphi_1(x)\right]P_2(x,t)=0 \qquad (3)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial x}+\mu_0(x)\right]P_3(x,t)=0 \qquad (4)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+2\lambda_2+\lambda_{pw}+\varphi_2(y)\right]P_4(y,t)=0 \qquad (5)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\lambda_2+\lambda_{pw}+\varphi_2(y)\right]P_5(y,t)=0 \qquad (6)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial y}+\mu_0(y)\right]P_6(y,t)=0 \quad (7)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+2\lambda_3+\lambda_{pw}+\varphi_3(z)\right]P_7(z,t)=0 \quad (8)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\lambda_3+\lambda_{pw}+\varphi_3(z)\right]P_8(z,t)=0 \quad (9)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial z}+\mu_0(z)\right]P_9(z,t)=0 \quad (10)$$

$$\left[\frac{\partial}{\partial t}+\frac{\partial}{\partial w}+\mu_0(w)\right]P_{10}(w,t)=0 \quad (11)$$

Boundary conditions:

$$P_1(0,t)=3\lambda_1P_0(t)+\int_0^\infty\varphi_1(x)P_2(x,t)dx \quad (12)$$

$$P_2(0,t)=2\lambda_1P_1(t) \quad (13)$$

$$P_3(0,t)=\lambda_1P_2(t) \quad (14)$$

$$P_4(0,t)=3\lambda_2P_0(t)+\int_0^\infty\varphi_2(y)P_5(y,t)dy \quad (15)$$

$$P_5(0,t)=2\lambda_2P_4(t) \quad (16)$$

$$P_6(0,t)=\lambda_2P_5(t) \quad (17)$$

$$P_7(0,t)=3\lambda_3P_0(t)+\int_0^\infty\varphi_3(z)P_8(z,t)dz \quad (18)$$

$$P_8(0,t)=2\lambda_3P_7(t) \quad (19)$$

$$P_9(0,t)=\lambda_3P_8(t) \quad (20)$$

$$P_{10}(0,t)=\lambda_{pw}\begin{bmatrix}P_0(t)+P_1(t)+P_2(t)+P_4(t)+P_5(t)\\+P_7(t)+P_8(t)\end{bmatrix} \quad (21)$$

Initial condition:

$P_0(0)=1$, and other probabilities are zero at $t = 0$. (22)

Using (8-12) and the Laplace transform of (1-7) we get:

$$\left[s+3\lambda_1+3\lambda_2+3\lambda_3+\lambda_{pw}\right]P_0^*(\mathrm{s})=\int_0^\infty\varphi_1(x)P_1^*(x,\mathrm{s})dx$$
$$+\int_0^\infty\varphi_2(y)P_4^*(y,s)dy+\int_0^\infty\varphi_3(z)P_7^*(z,s)dz+\int_0^\infty\mu_0(x)P_3^*(x,s)dx$$
$$+\int_0^\infty\mu_0(y)P_6^*(y,s)dy+\int_0^\infty\mu_0(z)P_9^*(z,s)dz \quad (23)$$

$$\left[s+\frac{d}{dx}+2\lambda_1+\lambda_{pw}+\varphi_1(x)\right]P_1^*(x,s)=0 \quad (24)$$

$$\left[s+\frac{d}{dx}+\lambda_1+\lambda_{pw}+\varphi_1(x)\right]P_2^*(x,s)=0 \quad (25)$$

$$\left[s+\frac{d}{dx}+\mu_0(x)\right]P_3^*(x,s)=0 \quad (26)$$

$$\left[s+\frac{d}{dy}+2\lambda_2+\lambda_{pw}+\varphi_2(y)\right]P_4^*(y,s)=0 \quad (27)$$

$$\left[s+\frac{d}{dy}+\lambda_2+\lambda_{pw}+\varphi_2(y)\right]P_5^*(y,s)=0 \quad (28)$$

$$\left[s+\frac{d}{dy}+\mu_0(y)\right]P_6^*(y,s)=0 \quad (29)$$

$$\left[s+\frac{d}{dz}+2\lambda_3+\lambda_{pw}+\varphi_3(z)\right]P_7^*(z,s)=0 \quad (30)$$

$$\left[s+\frac{d}{dz}+\lambda_3+\lambda_{pw}+\varphi_3(z)\right]P_8^*(z,s)=0 \quad (31)$$

$$\left[s+\frac{d}{dz}+\mu_0(z)\right]P_9^*(z,s)=0 \quad (32)$$

$$\left[s+\frac{d}{dw}+\mu_0(w)\right]P_{10}^*(w,s)=0 \quad (33)$$

Laplace transformation of boundary conditions:

$$P_1^*(0,s)=3\lambda_1P_0^*(s)+\int_0^\infty\varphi_1(x)P_2^*(x,s)dx \quad (34)$$

$$P_2^*(0,s)=2\lambda_1P_1^*(s) \quad (35)$$

$$P_3^*(0,s)=\lambda_1P_2^*(s) \quad (36)$$

$$P_4^*(0,s)=3\lambda_2P_0^*(s)+\int_0^\infty\varphi_2(y)P_5^*(y,s)dy \quad (37)$$

$$P_5^*(0,s)=2\lambda_2P_4^*(s) \quad (38)$$

$$P_6^*(0,s)=\lambda_2P_5^*(s) \quad (39)$$

$$P_7^*(0,s)=3\lambda_3P_0^*(s)+\int_0^\infty\varphi_3(z)P_8^*(z,s)dz \quad (40)$$

$$P_8^*(0,s)=2\lambda_3P_7^*(s) \quad (41)$$

$$P_9^*(0,s)=\lambda_3P_8^*(s) \quad (42)$$

$$P_{10}^*(0,s)=\lambda_{pw}\left[P_0^*(s)+P_1^*(s)+P_2^*(s)+P_4^*(s)+P_5^*(s)+P_7^*(s)+P_8^*(s)\right] \quad (43)$$

The probabilities of the system being in an up or down state can be calculated as follows:

$$P_{up}^*(s)=P_0^*(s)+P_1^*(s)+P_2^*(s)+P_4^*(s)+P_5^*(s)+P_7^*(s)+P_8^*(s)$$

$$P_{up}^{*}(s)=\frac{1}{D(s)}\left[1+(1+2\lambda_1)A(s)\left[\frac{1-S_{\varphi_1}^{*}(s+\lambda_1+\lambda_{pw})}{s+\lambda_1+\lambda_{pw}}\right]+(1+2\lambda_2)B(s)\left[\frac{1-S_{\varphi_2}^{*}(s+\lambda_2+\lambda_{pw})}{s+\lambda_2+\lambda_{pw}}\right]+(1+2\lambda_3)C(s)\left[\frac{1-S_{\varphi_3}^{*}(s+\lambda_3+\lambda_{pw})}{s+\lambda_3+\lambda_{pw}}\right]\right] \quad (44)$$

where,

$$A(s)=3\lambda_1\left[\frac{1-S_{\varphi_1}^{*}(s+2\lambda_1+\lambda_{pw})}{s+2\lambda_1+\lambda_{pw}}\right]\Bigg/\left\{1-2\lambda_1 S_{\varphi_1}^{*}(s+\lambda_1+\lambda_{pw})\left[\frac{1-S_{\varphi_1}^{*}(s+2\lambda_1+\lambda_{pw})}{s+2\lambda_1+\lambda_{pw}}\right]\right\} \quad (45)$$

$$B(s)=3\lambda_2\left[\frac{1-S_{\varphi_2}^{*}(s+2\lambda_2+\lambda_{pw})}{s+2\lambda_2+\lambda_{pw}}\right]\Bigg/\left\{1-2\lambda_2 S_{\varphi_2}^{*}(s+\lambda_2+\lambda_{pw})\left[\frac{1-S_{\varphi_2}^{*}(s+2\lambda_2+\lambda_{pw})}{s+2\lambda_2+\lambda_{pw}}\right]\right\} \quad (46)$$

$$C(s)=3\lambda_3\left[\frac{1-S_{\varphi_3}^{*}(s+2\lambda_3+\lambda_{pw})}{s+2\lambda_3+\lambda_{pw}}\right]\Bigg/\left\{1-2\lambda_3 S_{\varphi_3}^{*}(s+\lambda_3+\lambda_{pw})\left[\frac{1-S_{\varphi_3}^{*}(s+2\lambda_3+\lambda_{pw})}{s+2\lambda_3+\lambda_{pw}}\right]\right\} \quad (47)$$

$$\begin{aligned}D(s)&=s+3\lambda_1+3\lambda_2+3\lambda_3+\lambda_{pw}\\&-\left\{S_{\varphi_1}^{*}(s+2\lambda_1+\lambda_{pw})\left\{3\lambda_1+2\lambda_1 A(s)S_{\varphi_1}^{*}(s+\lambda_1+\lambda_{pw})\right\}\right\}\\&-\left\{S_{\varphi_2}^{*}(s+2\lambda_2+\lambda_{pw})\left\{3\lambda_2+2\lambda_2 B(s)S_{\varphi_2}^{*}(s+\lambda_2+\lambda_{pw})\right\}\right\}\\&-\left\{S_{\varphi_3}^{*}(s+2\lambda_3+\lambda_{pw})\left\{3\lambda_3+2\lambda_3 C(s)S_{\varphi_3}^{*}(s+\lambda_3+\lambda_{pw})\right\}\right\}\\&-2\lambda_1^2 A(s)S_{\mu_0}^{*}(s)\left[\frac{1-S_{\varphi_1}^{*}(s+\lambda_1+\lambda_{pw})}{s+\lambda_1+\lambda_{pw}}\right]\\&-2\lambda_2^2 B(s)S_{\mu_0}^{*}(s)\left[\frac{1-S_{\varphi_2}^{*}(s+\lambda_2+\lambda_{pw})}{s+\lambda_2+\lambda_{pw}}\right]\\&-2\lambda_3^2 C(s)S_{\mu_0}^{*}(s)\left[\frac{1-S_{\varphi_3}^{*}(s+\lambda_3+\lambda_{pw})}{s+\lambda_3+\lambda_{pw}}\right]\\&-\lambda_{pw}S_{\mu_0}^{*}(s)\left\{1+A(s)+2\lambda_1 A(s)\left[\frac{1-S_{\varphi_1}^{*}(s+\lambda_1+\lambda_{pw})}{s+\lambda_1+\lambda_{pw}}\right]+B(s)+2\lambda_2 B(s)\left[\frac{1-S_{\varphi_2}^{*}(s+\lambda_2+\lambda_{pw})}{s+\lambda_2+\lambda_{pw}}\right]+C(s)+2\lambda_3 C(s)\left[\frac{1-S_{\varphi_3}^{*}(s+\lambda_3+\lambda_{pw})}{s+\lambda_3+\lambda_{pw}}\right]\right\}\end{aligned} \quad (48)$$

$$P_{down}^{*}(s)=1-P_{up}^{*}(s) \quad (49)$$

5.1 Availability analysis

Availability is a performance norm for a repairable system that is linked to the principles of reliability and maintainability. The term "availability" refers to the system's ability to operate without fail at any given time. Setting,

$$S_{\mu_0}^{*}(s)=\frac{exp[x^{\theta}+(log\,\varphi(x))^{\theta}]^{\frac{1}{\theta}}}{s+exp[x^{\theta}+(log\,\varphi(x))^{\theta}]^{\frac{1}{\theta}}},$$

$$S_{\varphi_i}^{*}(s)=\frac{\varphi_i}{s+\varphi_i},\ i=1,2,3 \quad (50)$$

Case I: Using the following values for various parameters $\lambda_1=0.04, \lambda_2=0.03, \lambda_3=0.02,\ \lambda_{pw}=0.022,$ $\varphi_1=\varphi_2=\varphi_3=1,\ \theta=1,\ x=1$, then putting all values in (44). By taking inverse Laplace transformation, we have the availability of the origin system as follows:

$$\begin{aligned}P_{up}(t)&=0.991817+0.00812037\,e^{-2.74054\,t}\\&-0.000607903\,e^{-1.47688\,t}+0.0000194516\,e^{-1.33273\,t}\\&+0.000024043\,e^{-1.2657\,t}+0.000575935\,e^{-0.945719\,t}\\&+0.0000306053\,e^{-0.841054\,t}+0.0000207375\,e^{-0.809664\,t}\\&+6.87309\times10^{-13}e^{-0.102\,t}-3.38109\times10^{-12}e^{-0.082\,t}\\&+7.50783\times10^{-7}e^{-0.062\,t}-1.02214\times10^{-6}e^{-0.062\,t}\\&+5.74992\times10^{-12}e^{-0.052\,t}-4.86847\times10^{-13}e^{-0.042\,t}\end{aligned} \quad (51)$$

Case II: Reduction method

This method is used to increase system availability by reducing the failure rates of the system's units by multiplying them by a factor ρ such that $0<\rho<1$. Considering the values of different parameters as $\lambda_1=0.04, \lambda_2=0.03,\ \lambda_3=0.02,\ \lambda_{pw}=0.022,$ $\varphi_1=\varphi_2=\varphi_3=1,\ \theta=1,\ x=1$, and put $\rho=0.2$, in (44), then using the inverse Laplace transformation, we can obtain the system's improved availability as follows:

$$P_{up}(t) = 0.998382 + 0.00161667\, e^{-2.72268\, t} - 0.0000108351 e^{-1.16158\, t} - 2.94609\times10^{-7} e^{-1.12872\, t} + 1.37527\times10^{-7} e^{-1.10347\, t} + 7.68663\times10^{-6} e^{-0.933669\, t} + 2.70067\times10^{-6} e^{-0.911807\, t} + 1.6161\times10^{-6} e^{-0.895147\, t} + 4.68802\times10^{-13} e^{-0.0204\, t} - 1.55349\times10^{-12} e^{-0.0164\, t} + 9.77085\times10^{-7} e^{-0.0124\, t} - 1.06216\times10^{-6} e^{-0.0124\, t} + 5.7635\times10^{-12} e^{-0.0104\, t} - 5.95789\times10^{-13} e^{-0.0084\, t} \quad (52)$$

Now, if we change t = 0 to 10 in (51) and (52) above, we get Table 1 and correspondingly Figure 2, which show how availability changes over time in two cases.

TABLE 2
COMPARISON OF THE ORIGINAL AND IMPROVED SYSTEMS' AVAILABILITY

Time	availability	
	Case I	***Case II***
0	1	1
1	0.99246	0.99997
2	0.991919	0.999911
3	0.99185	0.999872
4	0.99183	0.999852
5	0.99182	0.999843
6	0.991819	0.999839
7	0.991817	0.999837
8	0.991817	0.999837
9	0.991817	0.999837
10	0.991817	0.999836

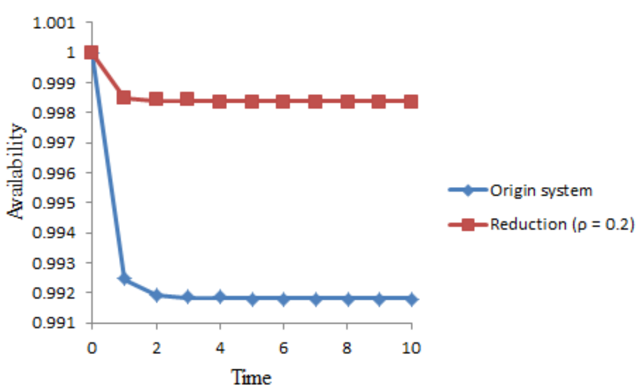

FIGURE 2
COMPARISON OF THE ORIGINAL AND IMPROVED SYSTEMS' AVAILABILITY

Case III: a comparison of the improved system's availability for various values of ρ vs time, where $0<\rho<1$. Take the values of the various parameters as in (44), and then apply the inverse Laplace transformation; it can be shown that decreasing the value of the factor ρ raises the value of the system's availability.

TABLE 3
CALCULATED AVAILABILITY CORRESPONDING TO TIME WITH VARIOUS ρ VALUES

Time	Availability			
	$\rho=0.2$	$\rho=0.4$	$\rho=0.6$	$\rho=0.8$
0	1	1	1	1
1	0.99849	0.996982	0.995475	0.993968
2	0.99839	0.996781	0.995168	0.993548
3	0.998383	0.996765	0.995139	0.993502
4	0.998382	0.996762	0.995133	0.99349
5	0.998382	0.996762	0.995131	0.993486
6	0.998382	0.996761	0.99513	0.993484
7	0.998382	0.996761	0.99513	0.993483
8	0.998382	0.996761	0.995129	0.993483
9	0.998382	0.996761	0.995129	0.993482
10	0.998382	0.996761	0.995129	0.993482

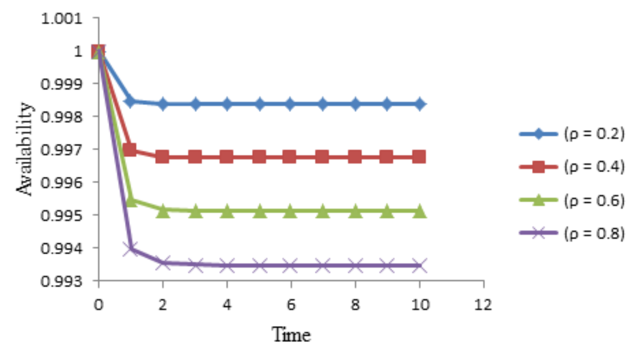

FIGURE 3
COMPARISON OF THE ENHANCED SYSTEM'S AVAILABILITY FOR VARIOUS ρ VALUES

5.2 Reliability Analysis

The probability that a system will operate satisfactorily for the intended period of time under the specified operating conditions is referred to as reliability. Take all repair rates to zero to achieve system reliability. We use the same cases as in the previous section.

Case I: Using the values of various parameters as $\lambda_1=0.04, \lambda_2=0.03, \lambda_3=0.02,\ \lambda_{pw}=0.022,\ \varphi_1=\varphi_2=\varphi_3=1,$. $\theta=1,\ x=1$. The reliability of the origin system is obtained by plugging all of these values into (44), then applying the inverse Laplace transformation:

$$R(t) = 0.0475417\, e^{-0.292\, t} - 0.631579\, e^{-0.102\, t} - 0.428571\, e^{-0.082\, t} + 0.782609\, e^{-0.062\, t} + 0.75\, e^{-0.052\, t} + 0.48\, e^{-0.042\, t} \quad (53)$$

Case II: It is assumed that the failure rates of the system's units are reduced using the reduction method by multiplying by a factor ρ such that $0<\rho<1$. Using the values of various parameters as $\lambda_1=0.04, \lambda_2=0.03, \lambda_3=0.02$, $\lambda_{pw}=0.022$, $\varphi_1=\varphi_2=\varphi_3=1$, . $\theta=1$, $x=1$, and take $\rho=0.2$. The reliability of the system is obtained by putting all of these values into (44), then applying the inverse Laplace transformation:

$$R(t)=0.0475417\, e^{-0.0584\, t}-0.631579\, e^{-0.0204\, t} - 0.428571\, e^{-0.0164\, t}+0.782609\, e^{-0.0124\, t} + 0.75\, e^{-0.0104\, t}+0.48\, e^{-0.0084\, t} \quad (54)$$

Table 4 and corresponding Figure 4 reflect the variance of reliability with respect to time for two cases when t = 0 to 10 in (54).

TABLE 4
COMPARISON OF THE ORIGINAL SYSTEM'S AND THE IMPROVED SYSTEM'S RELIABILITY

Time	Reliability R(t)	
	Case I	*Case II*
0	1	1
1	0.978152	0.995609
2	0.956324	0.991233
3	0.934227	0.986867
4	0.911713	0.982508
5	0.88873	0.978152
6	0.86529	0.973796
7	0.841447	0.969438
8	0.817278	0.965075
9	0.792876	0.960704
10	0.768336	0.956324

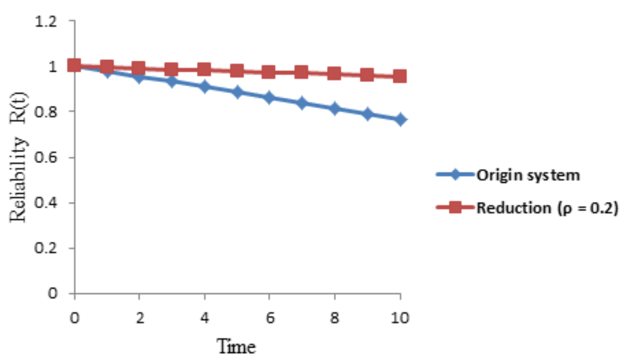

FIGURE 4
COMPARISON OF RELIABILITY OF ORIGINAL SYSTEM AND IMPROVED SYSTEM

Case III: Comparison of the improved system's reliability for various values of ρ vs time, where, $0<\rho<1$. Using the values of various parameters as $\lambda_1=0.04, \lambda_2=0.03, \lambda_3=0.02$, $\lambda_{pw}=0.022$, $\varphi_1=\varphi_2=\varphi_3=1$, . $\theta=1$, $x=1$. It can be shown that lowering the value of factor ρ raises the value of the system's reliability.

TABLE 5
COMPUTED OF RELIABILITY CORRESPONDING TO TIME WITH DIFFERENT VALUES OF ρ

Time	Reliability R(t)			
	$\rho=0.2$	$\rho=0.4$	$\rho=0.6$	$\rho=0.8$
0	1	1	1	1
1	0.995609	0.991233	0.986867	0.982508
2	0.991233	0.982508	0.973796	0.965075
3	0.986867	0.973796	0.960704	0.947528
4	0.982508	0.965075	0.947528	0.92976
5	0.978152	0.956324	0.934227	0.911713
6	0.973796	0.947528	0.920773	0.893364
7	0.969438	0.938676	0.907154	0.874718
8	0.965075	0.92976	0.893364	0.855797
9	0.960704	0.920773	0.879407	0.836636
10	0.956324	0.911713	0.86529	0.817278

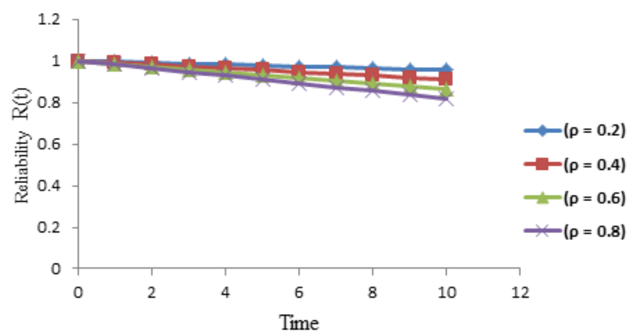

FIGURE 5
CALCULATED RELIABILITY RELATING TO TIME WITH VARIOUS ρ VALUES

5.3 Mean time to failure (MTTF)

The mean time to failure (MTTF) is defined as the estimated time for a system to be operational. We get the MTTF of the system by taking all repairs to zero and the limit as s tends to zero in (44) for the exponential distribution:

$$MTTF=\lim_{s\to 0} P_{up}^{*}(s)=\frac{1}{3(\lambda_1+\lambda_2+\lambda_3)+\lambda_{wp}}\left[1+\frac{6\lambda_3}{\lambda_3+\lambda_{wp}} -\frac{3\lambda_3}{2\lambda_3+\lambda_{wp}}+\lambda_1\left(\frac{6}{\lambda_1+\lambda_{wp}}-\frac{3}{2\lambda_1+\lambda_{wp}}\right) +\lambda_2\left(\frac{6}{\lambda_2+\lambda_{wp}}-\frac{3}{2\lambda_2+\lambda_{wp}}\right)\right] \quad (55)$$

Putting the values of the failure rates $\lambda_1=0.04$, $\lambda_2=0.03, \lambda_3=0.02$, and $\lambda_{pw}=0.022$, varying λ_1, $\lambda_2, \lambda_3, \lambda_{pw}$ one by one, respectively from 0.01 to 0.1 in

(55). Table 6 and Figure 6 display the difference in mean time to failure (MTTF) corresponding to failure rates.

TABLE 6
VARIATION OF *MTTF* WITH RESPECT TO FAILURE RATES

Failure rate	$MTTF_1(\lambda_1)$	$MTTF_2(\lambda_2)$	$MTTF_3(\lambda_3)$	$MTTF_4(\lambda_{pw})$
0.01	31.7529	29.0716	27.5542	37.432
0.02	30.7878	28.524	27.2188	28.5345
0.03	29.0739	27.2188	26.1567	22.9351
0.04	27.2188	25.7093	24.8664	19.1014
0.05	25.4364	24.2075	23.5488	16.322
0.06	23.7955	22.792	22.2843	14.2208
0.07	22.3097	21.4874	21.1027	12.5806
0.08	20.9723	20.2966	20.012	11.2672
0.09	19.7696	19.2133	19.0105	10.1935

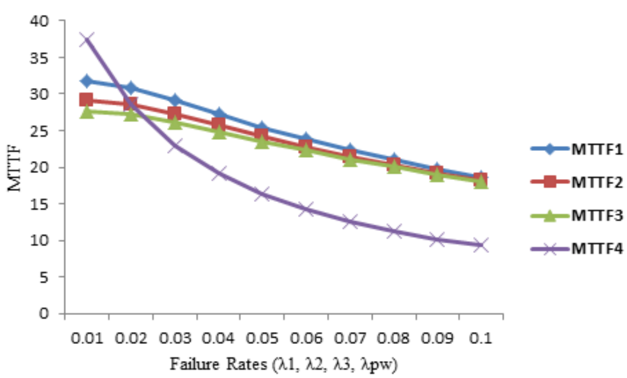

FIGURE 6
VARIATION IN MTTF AS A FUNCTION OF FALIURE RATES

5.4 Cost analysis

A cost analysis is an empirical method for assessing a proposed action by calculating its net worth. If the service facility is always open, the system's expected benefit for the interval [0, t) is given by

$$E_p(t) = K_1 \int_0^t P_{up}(t)dt - K_2 t \qquad (56)$$

The revenue per unit time and service cost per unit time, respectively, are K_1 and K_2. One can get the result as in (57) by using the same set of parameters as in (44). As such,

$$\begin{aligned} E_p(t) = k_1 \Big[& 0.00325165 - 0.00296305\, e^{-2.74054\, t} \\ & + 0.000411614\, e^{-1.47688\, t} - 0.0000145953\, e^{-1.33273\, t} \\ & - 0.0000189959\, e^{-1.2657\, t} - 0.000608992\, e^{-0.945719\, t} \\ & - 0.0000363892\, e^{-0.841054\, t} - 0.0000256124\, e^{-0.809664\, t} \\ & - 6.73833\times10^{-12} e^{-0.102\, t} + 4.12328\times10^{-11} e^{-0.082\, t)} \\ & - 0.0000121094\, e^{-0.062\, t} + 0.0000164861\, e^{-0.062\, t} \\ & - 1.10575\times10^{-10}\, e^{-0.052\, t} + 1.15916\times10^{-11}\, e^{-0.042\, t} \\ & + 0.991817\, t \Big] - k_2 t \end{aligned} \qquad (57)$$

With K_1= 1and K_2= 0.1, 0.2, 0.3, 0.4, and 0.5, and t = 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10 units of time, the expected profit can be seen in Table 7, which is represented by Figure 7.

TABLE 7
TIME-DEPENDENT VARIATION IN EXPECTED BENEFIT

Time	Expected profit $E_p(t)$				
	K_2=0.1	K_2=0.2	K_2=0.3	K_2=0.4	K_2=0.5
0	0	0	0	0	0
1	0.894702	0.794702	0.694702	0.594702	0.494702
2	1.78679	1.58679	1.38697	1.18679	0.986792
3	2.67867	2.37867	2.07867	1.77867	1.47867
4	3.57051	3.17051	2.77051	2.37051	1.97051
5	4.46233	3.96233	3.46233	2.96233	2.46233
6	5.35415	4.75415	4.15415	3.55415	2.95415
7	6.24597	5.54597	4.84597	4.14597	3.44597
8	7.13779	6.33779	5.53779	4.73779	3.93779
9	8.0296	7.1296	6.2296	5.3296	4.4296
10	8.92142	7.92142	6.92142	5.92142	4.92142

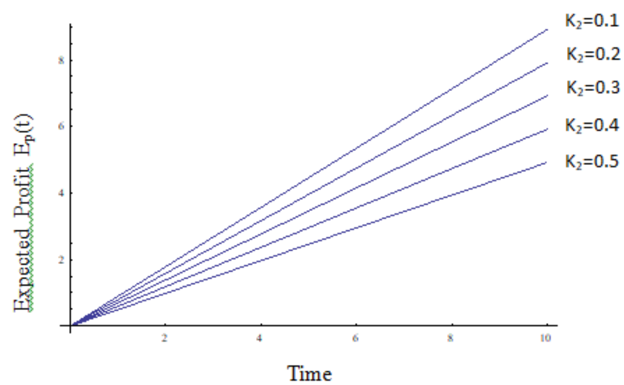

FIGURE7
EXPECTED PROFIT OVER TIME

6. CONCLUSION

To display, evaluate, and make system reliability metrics for various failure and repair rates. The study of availability in two separate cases is shown in Table 2 and the corresponding Figure 2. When failure rates are fixed at different values, we find that the availability of the origin system varies over time in case I. In case II, we use the reduction approach to increase the availability of the origin system by reducing the failure rates of the system's units by a factor ρ such that, $0<\rho<1$. When comparing the availability of the original system in case I to the availability of the system using the reduction method in case II, it can be seen that using the reduction method improves the availability of the original system. As a result, we can conclude that using the reduction approach is a viable alternative. In case III, Table 3 and Figure 3 show a comparison of the improved system's availability vs. time for various values of ρ, where $0<\rho<1$ and failure rates are the same. It shows that as the value of the factor ρ decreases, the value of the system's availability increases. In both cases, it can be shown from Figures 2 and 3 that the system's availability decreases as time passes. The system's reliability is measured in three separate cases and shown in Table 4, Figure 4, Table 5, and Figure 5, much like its availability. It is concluded that, when failure rates are set at various values, the reliability of the origin system decreases with time, as in case I, and that, in case II, we increase the reliability of the origin system by using a reduction method with a factor ρ such that $0<\rho<1$, to reduce the failure rates of the system's units. When the reliability of the original system in case I is compared to the reliability of the system using the reduction method in case II, it can be shown that the original system's reliability is improved using the reduction method, as shown in Table 4 and Figure 4. Finally, case III shows a comparison of the improved system's reliability for various values of ρ vs. time, where $0<\rho<1$, and failure rates are set at various levels. As shown in Table 5 and Figure 5, decreasing the value of the factor ρ increases the value of the system's reliability. Figures 2 and 4 show that reliability values are lower than availability for the same values of failure rates, indicating that repair plays an important role in improving the efficiency of repairable systems. Table 6 and Figure 6 show that as the value of $\lambda_1, \lambda_2, \lambda_3$ and λ_{pw} increases, the complex system's mean-time-to-failure (*MTTF*) decreases. In addition, we can see that *MTTF* w.r.t. λ_1 > *MTTF* w.r.t. λ_2 > *MTTF* w.r.t. λ_3. After 0.03, the system's *MTTF* with respect to λ_{pw} is the lowest failure rate variation value. We can deduce from Table 7 and Figure 7 that expected profit declines as service cost rises K_2 over time. From 0.5 to 0.1, the estimated expected profit for K_2 is highest at K_2 = 0.1 and lowest at K_2 = 0.6. As a result, the profit is higher for low service costs than for high service costs.

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