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STOCHASTIC MODELS IN SOCIETY FRACTAL ANALYSIS OF BIOLOGICAL SIGNALS IN A REAL TIME MODE

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Abstract. The human body is a complex system consisting of more than 32 trillion cells. Many systems and mechanisms in the human body interact in order to maintain homeostasis. Homeostasis is the property of a system in which variables are regulated so that internal conditions remain stable and relatively constant. All living organisms depend on maintaining a complex set of interacting metabolic chemical reactions. Many diseases involve a disturbance of homeostasis.

Negative feedback mechanisms consist of reducing the output or activity of any organ or system back to its normal range of functioning. The feedback is important for the healthy functioning of one's body. Complications can arise if any of the two feedbacks are affected or altered in any way.

As the organism ages, the efficiency in its control systems becomes reduced. The inefficiencies gradually result in an unstable internal environment that increases the risk of illness, and leads to the physical changes associated with aging [1].

Over a lifetime, there may be situations when it is necessary to know, whether the state of the body is stable, or it undergoes substantial changes. It is particularly important to recognize the transition of the organism in a critical condition. For example, it is a problem for medicals performing complicated surgical operations under the general anesthesia.

At the same time, the duration of decision making must not exceed five minutes, so a longer stay of a patient in a state of clinical death can lead to irreparable consequences. In such situations there is no time to conduct the comprehensive analysis of many vital parameters. So, it is possible to use only those data that are continuously monitored, for example, the rhythm of hart or the brain activity.

This paper presents the study of the critical states of the organism by analyzing the heart rate variability in real time. These studies were carried out for a number of years at the St. Petersburg Polytechnic University in collaboration with the Russian Academy of Science. The results can be considered as the additional information for medicals to make decision.

1. Introduction

The problem of searching for connection indicators of cardiac rhythm regulating system is a perspective direction in the modern fundamental science.

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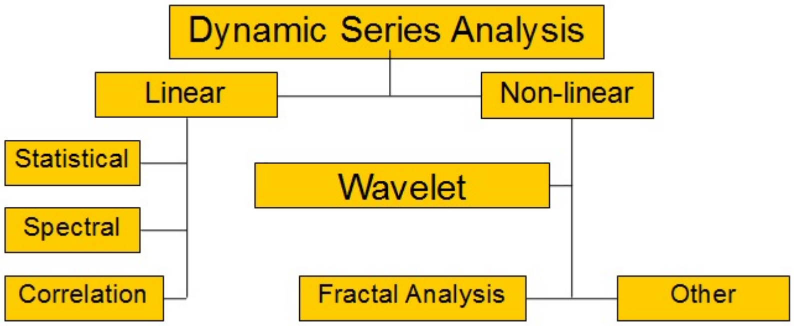


linear dynamic systems [2].

Fig.1. Methods of Dynamic Series Analysis

Recommendations for usage of physiological interpretation of consecutive intervals between QRS-complexes of electrocardiogram (heart rate variability), executed in 1996 by the European Cardiology Society, North American Electrophysiological Community [3] and Committee of Clinic diagnostic Devices and Instruments, Committee of New Medical Technology of the Russian Federation Ministry of Public Health [5] make these time lengths the most perspective in the course of noninvasive diagnostics in the modern practice of scientific investigations.

In spite of the recommended range of variability indexes, for example, indexes of variational pulsometering [2, 5], indexes of correlation rhythmmography [4], indexes of power in various frequency intervals, low and high frequency spectral components, their relations [2, 5], the results, received using them, have specific disadvantages. The majority of calculated components use linear methods of time series analysis (for example, statistic, spectral, correlation and etc.). It simplifies the form evaluation of conformance to considered processes, distorting more their real physiological interpretation. For example, analyzing the same spectral concentration in the course of routine examination of variability in the various frequency bands, the values of power don't have four evident peaks (to which the frequency components are related). The power coefficients are calculated only thanks to connection with the specific preset interval, and not due to highlighting the specified peculiarity.

This situation is explained by nonlinearity, discontinuity, instability and etc. of the cardiac rhythm reference signal, where it is impossible to exclude both contained phenomena of its internal regulation and procedural error of registering abbreviations itself. To solve the existing difficulty, several researchers propose usage of mathematical methods, the majority of which is based on the fractal analysis of non-linear dynamic systems [2].

Formulation of the problem

- Is there an integral characteristic of the condition of the body?
- What is a thermodynamic quantity which can be such an index?

- Can we use it to predict changes in the body state in the near-term and long-term perspective and to recognize the critical situation?

It is known that the physical quantity characterizing the chaotic behavior of the system is the entropy. If we consider the living organism as a complex system, its chaotic behavior can be characterized by evaluating the entropy generation.

Dynamic of energy exchange in the organism is characterized by the balance between inner thermodynamic entropy production and its exchange with environment (scattering).

This phenomenon has an irregular character. The irregularity shows itself by the variable heat loss in the process of the organism development, its pathologies and decease.

The feature of irregularity is connected with nonlinear and synergetic effects, which are characterized by the dynamic chaos. These considerations show that on the overall statistical fluctuation the chaos is a vitally necessary part of the normal organism activity.

Method of Investigation. Mathematical Model

Chaos

A commonly used definition of chaos says that, for a dynamical system to be classified as chaotic, it must have these properties:

- It must be sensitive to initial conditions;
- It must be topologically mixing;
- It must have dense periodic orbits.

The Lyapunov exponent measures the sensitivity to initial conditions. Given two starting trajectories in the phase space that are infinitesimally close, with initial separation end up diverging at a rate given by

$$|\delta Z(t)| = e^{\lambda t} \, |\delta Z_0|$$

where t is the time and λ is the Lyapunov exponent.

Topologically mixing (or topological transitivity) means that the system will evolve over time any given region or open set of its phase space.

For a chaotic system to have a dense periodic orbit means that every point in the space is approached arbitrarily closely by periodic orbits [1].

2. Fractal analysis

Fractal analysis is assessing fractal characteristics of data. It consists of several methods to assign a fractal dimension and other fractal characteristics to a dataset, which may be a pattern or signal extracted from phenomena including geometric objects, heart rates, digital images, molecular motion, networks, etc. An important limitation of fractal analysis is that an empirically determined fractal dimension does not necessarily prove that a pattern is fractal; rather, other essential characteristics have to be considered.

The concept of a fractal is associated with geometrical objects satisfying two criteria: self-similarity and fractional dimensionality. Self-similarity means that an object is composed of sub-units and sub-sub-units on multiple levels that resemble the structure of the whole object. Mathematically, this property should hold on all scales. However, in the real world, there are necessarily lower and upper bounds

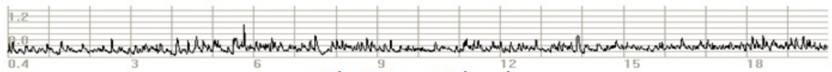


Fig.2. Input Signal

over which such self-similar behavior applies. The second criterion for a fractal object is that it has a fractional dimension. This requirement distinguishes fractals from Euclidean objects, which have integer dimensions [1].

The fractal methods are based on the examination of scaling invariance (scaling) of the process conditions. They may be divided into methods which directly use the idea of fractal as a geometric entity in the multidimensional phase space and transferred to it by manipulating initial time series for setting coordinates, and methods examining the scale invariance of the initial process features. The well-known feature of scaling – fractal (correlation) dimensionality may be related to the first ones. But here, there are a number of difficulties connected first of all with convergence of such feature in the finite-dimensional space of enclosure. Absence of convergence curve saturation of correlation dimensionality are demonstrated in several works appeared in recent years [3, 4, 6]. At the same time, the deterministic rhythms – for example, fetal rhythm for periods of gestation 38-40 weeks and ventricular fibrillation, suggests that the direction of fundamental research vector is correct.

Consider the various definitions of entropy:

Thermal dynamics:$T = \frac{dQ}{dT}, \int \frac{dQ}{dT} = 0$ for reversible process;

Statistical physics: $S{=}k \ln \Gamma$, where k is the Boltzmann constant and Γ is the number of microstates;

In our case, these definitions are difficult to use to achieve the goals

Shannon informational entropy: $H(P) = -k \sum\limits_{i=1}^{n} p_i \ln p_i$;

Renyi entropy: $H_q(p) = \frac{1}{1-q} \sum\limits_{i=1}^{n} p_i \ln p_i^q$

The Renyi entropy tends to Shannon entropy as $q \to 1$.

3. Structure of the Input Signal

As an input to the analysis of the integrated state of the organism the electrocardiogram has been used. For the evaluation of body condition stability heart rate variation is selected. To operate in real-time, moving window method is used.

$R_T = \{r_i(t_i)\}_{i=1}^{k}, t = \sum\limits_{j=1}^{i} r_i, T = t_k$

The average value of the two processes is the same. However, the first process is a sharp transition from one state to another, and the second shows a stable behavior.

Heart Rate Variability (HRV)

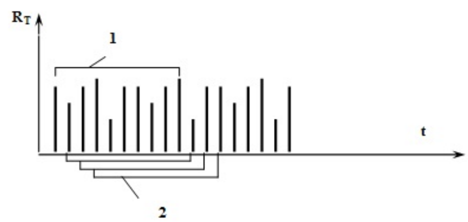


Fig.3. Moving Window

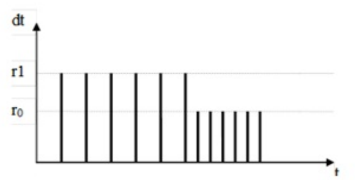


Fig.4. Sharp change of rhythm

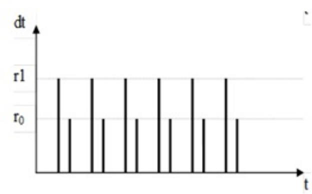


Fig.5. Rhythm stability

4. Embedding Theory

There is a big family of fractal dimensions. The most convenient for our purposes is the Renyi dimension. The Rényi's dimensions are defined from the notion of generalized entropy. The principle is to weight the probability of the most often visited cubes according to the order of the dimension.

The general expression is measured in discrete dimension Renyi:

$$D_{Rq} = \lim_{\varepsilon \longleftrightarrow 0} \lim_{\varepsilon \to 0} \lim_{m \to \infty} \left[\frac{1}{1-q} \frac{\ln I_{Rq}}{\ln(1/\varepsilon)} \right]$$

Here $I_{_{Rq}}\ (q,\varepsilon) = \left[\sum_{i=1}^{M(\varepsilon)} P_i^q(\varepsilon) \right]$ −the generalized Renyi entropy of order q;

$M(\varepsilon)$ - the minimum number of cubes with an edge ε, covering the attractor in the n-dimensional phase space embedding; pi - the probability of visiting the i-th cube phase trajectory; m - number of points used to estimate the dimension. Embedding process has to precede any estimation of fractals from a data series. Theoretically, according to the Takens theorem, any state variable can be used to calculate the invariants of the dynamics. But practically, it is not a calculation that is made, but only an estimation performed by an algorithm. This raises the problem of convergence and it is directly related to the more or less good conditioning of the useful information into the variable [7, 8].

5. The Main Practical Task

The choice of the embedding variable and embedding delay After having chosen the embedding variable, the next step is the determination of the right embedding delay. It can

be estimated by the first zero crossing of the autocorrelation function or, better, by the first local minimum of the mutual information. Measurement accuracy

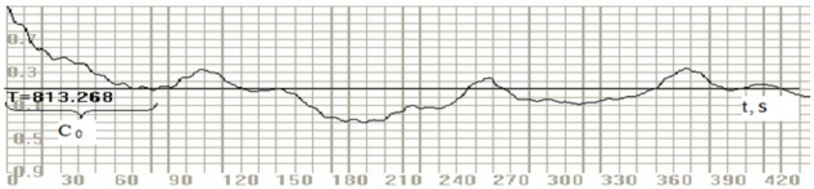


Fig.6. Autocorrelation function

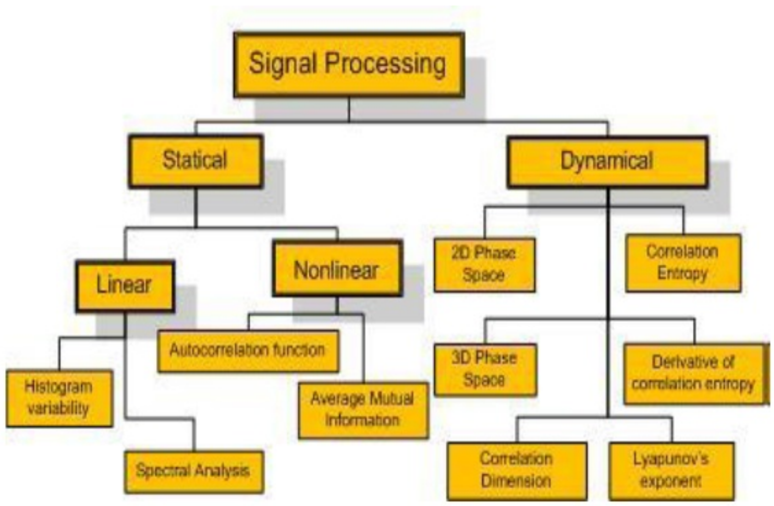


Fig.7. Structure of hardware and software

and speed of analysis is achieved by using the power spectrum of the autocorrelation function of the signal. This provides accounting the latent periodicity of the spectral properties of the signal, resistance to the selection of the discretization; effectively solve problems with additive noise.

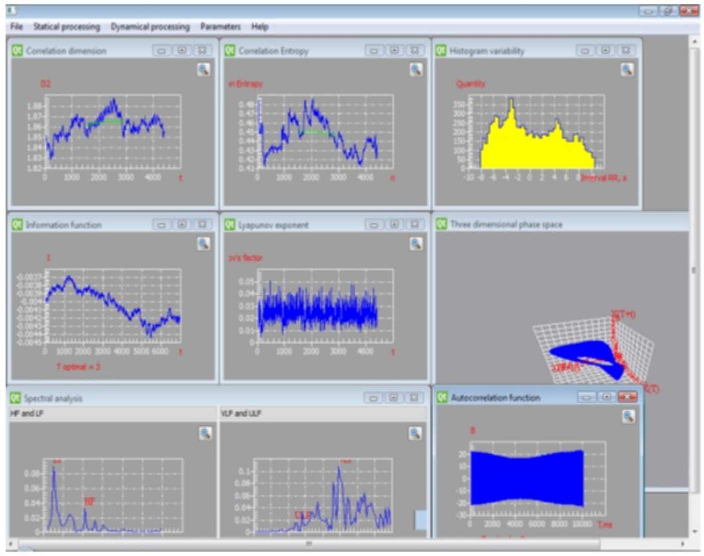
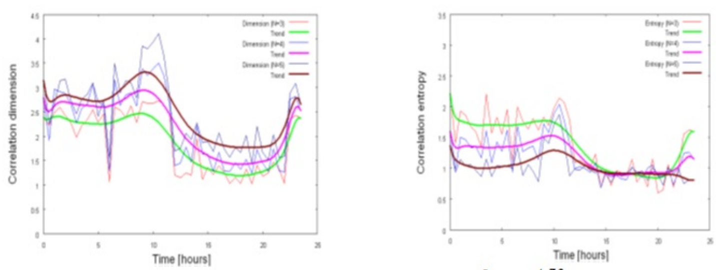


Fig.8. Possibilities of the complex



1. *A 4 years child*

2. *A 79 years woman*

Fig.9. Trends in the correlation dimension of the attractor of cardiac rhythm in various dimensions of the phase space during the daily monitoring

6. Model analysis

The analysis of fractal components of daily ECG obtained during clinical studies of the complex has shown the high efficiency of fractal - entropic characteristics of electrocardiogram in pathological and age-related changes, as well as the impact of various external and internal loads.

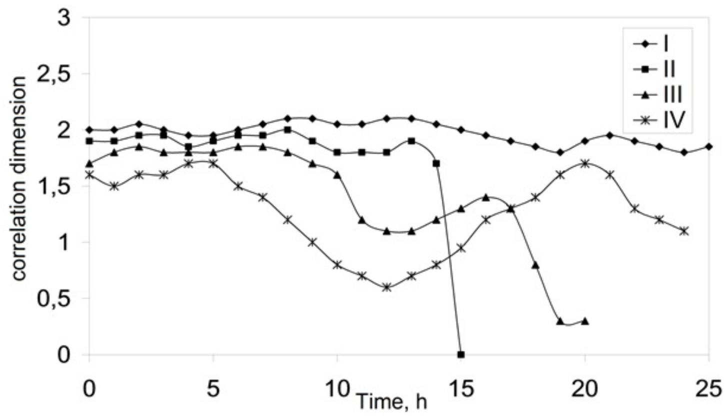


Fig.10. Correlation dimension of the entropy attractor of heart rate in diseased organisms with recovery phase trajectories

Seriously ill: I- woman aged 49 in a vegetative state after undergoing clinical death and resuscitation, II-man, 57 years, sudden cardiac arrest with fatal outcome, III-a man, 88 years, heart rhythm disturbance of life-threatening; IV-woman 69 years, the impact of anesthesia for surgery to remove part of the colon under general anesthesia.

7. Diagnosis of a Pregnant Woman and a Fetus on the Basis of Fractal Analysis

Pregnancy is happiness. But in particular cases it may be associated with the development of various complications for both mother and fetus, including representing a critical threat to their life. Diagnosis of such threats is usually performed by the medical's professional experience. A promising direction for the management of pregnant women is finding the generalized parameter that determines the sustainability of the system " pregnant – fetus". It must include the identification and prediction of critical conditions with life-threatening for both mother and fetus throughout pregnancy, childbirth and the postpartum period. The most common method of evaluating the functional state is to analyze the electric potentials - electrocardiography (ECG). In recent years, the theory of nonlinear dynamical systems and fractal sets is being developed, as well as the application of these methods to the analysis of processes with a chaotic organization, to which belongs heart rate variability (HRV) of mother and fetus.

One of the most successful results in the recent years is the work of our Russian colleagues, which demonstrates that the correlation dimensionality may serve as a test of violation in the regulation mechanisms of fetal heart rhythm. Quantitative evaluation of stochastic processes in the fetal heart rate dynamics was made by the mean value of the index D. As expected, under normal ongoing pregnancy the fetal heart rate demonstrates more pronounced chaotic dynamics [9].

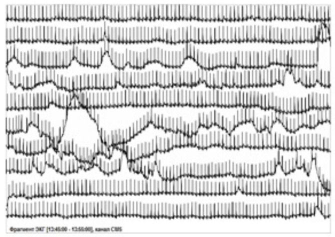


Fig.11. ECG of a Pregnant woman
Over the entire period of observation (1 hour) recorded sinus rhythm with a single episode of group extra systoles

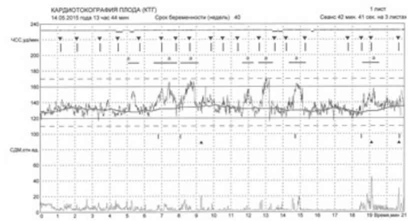


Fig.12. ECG of a Fetus
The heart rate of the fetus is higher than that of the mother. Also clearly seen cardiac arrhythmia

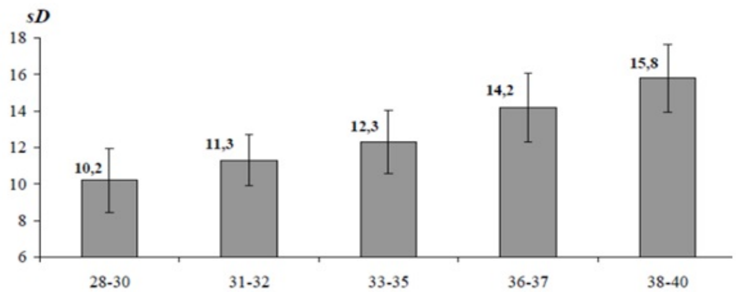


Fig.13. Change in chaotic behavior

Conclusions

The results of this work are:

- A mathematical model for evaluating the state of the body, based on fractal analysis of variation of heart rate in real time.
- The program complex, allowing carrying out a study of time series in static and dynamic modes.
- The evaluation of software tools designed using the classic examples of attractors.
- The results show that normal HRV of fetus demonstrate the character of his behavior with the minimum value of the correlation dimension. The reduction of deterministic chaos and random dynamics in the heart rate variability may be a predictor of fetus's intrauterine suffering.
- This suggests that on the overall statistical fluctuation the chaos is a vitally necessary part of the organism activity.
- The practical value of this approach is to provide doctors the new methodology for evaluating the critical states of pregnancy on the basis of information entropy index ECG signals. This will allow using the pre-emptive therapy, reduce the risk of complications to ensure the adequacy of the implementation and management of pregnancy.

- The analysis of real processes in healthy and sick people. With some confidence we can assert that the analysis of data allows to determine the estimated time of transition of the organism in a critical condition

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