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AVERAGE DEGREE LABELING OF GRAPHS

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Abstract. Let G be an undirected, simple, connected graph. If a graph G has vertices labeled by degrees and edges labeled by the least of the average of the degrees of their end vertices, then the graph is said to permit average-degree edge labeling. This paper discusses the average degree edge labeling of complete bipartite graphs, the join of the Path graph P_m and the Complete graph K_n, and of various classes of trees. Using the average degree labeling graph, we can determine the shortest path between the vertices. Additionally, we have tried to develop an algorithm for spanning trees of average degree labeled networks, both maximal and minimal.

1. Introduction

S.M. Hedge *et al.* defined the Edge Sum Labeling for the (n,m)-graph G. Motivated by this labeling [4] and the definition of Average Degree Energy in [5], we present a new sort of labeling called Average Degree Edge labeling of a graph. Graph theory is essential in the study of network systems. In particular, biological neural networks discussed in [2] and [6]. A brain network has two properties: minimizing resource costs and maximizing information flow. [1] outlines the network components. Minimum spanning trees have a direct impact on network architecture, including computer networks, electricity grids, telecommunications networks, water supply networks, and transportation networks. The minimal spanning tree (MST) problem is a common and important basic in the planning and administration of communication networks. A spanning tree is an essential data structure that allows you to find connections quickly and within a small search space. It enables efficient intra-connectivity while preserving the minimalist tree structure, making it beneficial in contexts where merging data points is common. We refer to [3] and [7] for all the terminology and the results of the labeling survey.

2. Preliminaries

Definition 2.1. The Bamboo Tree BT(n,m,k) is a tree obtained from k-copies of the Path P_n of length n-1 and $K_{1,m}$ stars. Identify one of the two pendent vertices of the j^{th} path with the center of the j^{th} star. Identify the other pendent vertex of each path with a single vertex, w_0.

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Definition 2.2. The Banana Tree B(m,k) is a tree obtained by connecting one leaf of each of the m-copies of the star S_k with a single vertex that is distinct from all stars.

Definition 2.3. The join of two graphs P_m and K_n is a graph formed from disjoint copies of P_m and K_n by connecting each vertex of P_m to each vertex of K_n, and is denoted by $P_m + K_n$

Definition 2.4. The cycle cactus $C_k^{(n)}$ consisting of n copies of the cycle of length k is a connected separable graph in which every block is a cycle.

Definition 2.5. The shell graph $C_{(n;n-3)}$ is the join of the complete graph K_1 and path graph,P_m, where n = m+1. A subdivided shell graph is a shell graph in which the edges in the path of the shell are subdivided.

Definition 2.6. The windmill graph Wd(n,k) is the graph obtained by taking k copies of complete graph K_n with a vertex in common.

Definition 2.7. A grid graph is a cartesian product of two path graphs, P_m and P_n. It has mn vertices and 2mn-m-n edges.

Definition 2.8. Let G be a simple, connected, undirected graph without self loop and parallel edges having n vertices $v_1, v_2, \ldots, v_n$. We define vertex degree labeling of a graph G by a into function $A : V(G) \longmapsto \{1, 2, ..., n-1\}$ such that

$$A(v_i) = deg(v_i), 1 \leq i \leq n.$$

Definition 2.9. The Average Degree Edge Labeling of a graph G is a into function defined $A^* : E(G) \longmapsto \{1, 2, ..., n-1\}$ by $A^*(v_iv_j) = \lfloor \frac{deg(v_i)+deg(v_j)}{2} \rfloor$, for all $1 \leq i, j \leq n$, where v_iv_j is an edge in G.

Definition 2.10. A Graph G which admits a vertex degree labeling and Average degree edge labeling is called **Average Degree Labeled Graph**. It is denoted by ADL-graph.

3. Main Results

In this section, we discuss the average degree labeling of some class of trees, the cactus graph, the join of P_m and K_n graph.

Theorem 3.1. *Every Path Graph P_n, $n \geq 2$ is an ADL graph.*

Proof. Let $G = P_n$ be a Path graph on n vertices, $n \geq 2$. Let $w_1, w_2, \ldots, w_n$ are the vertices of a path graph such that $d(w_1) = d(w_n) = 1, d(w_i) = 2, \forall i = 2, 3, ..., n-1$. Define $A : V(G) \longmapsto \{1, 2, ..., n-1\}$ by $A(w_i) = deg(w_i), 1 \leq i \leq n$ and $A^* : E(G) \longmapsto \{1, 2, ..., n-1\}$ by $A^*(w_iw_j) = \lfloor \frac{deg(w_i)+deg(w_j)}{2} \rfloor, \forall \quad 1 \leq i, j \leq n$.
Therefore, the vertex degree labeling of G is

$$A(w_i) = \begin{cases} 1 & i = 1, n \\ 2 & i = 2, ..., n-1 \end{cases}$$

and the average degree edge labeling of G is

$$A^*(w_iw_j) = \begin{cases} 1 & i = 1, j = 2 \\ 1 & i = n-1, j = n \\ 2 & i = 2, ..., n-2, j = 3, ..., n-1; i < j \end{cases}$$

Thus, the Path graph, P_n is an ADL graph. □

Example 3.2. *The path graph P_6 shown in figure 1 is an ADL graph. The vertex and edge labeling of P_6 is given by,*

$$A(w_i) = \begin{cases} 1 & i = 1, 6 \\ 2 & i = 2, ..., 5 \end{cases}$$

and the average degree edge labeling of P_6 is

$$A^*(w_iw_j) = \begin{cases} 1 & i = 1, j = 2 \\ 1 & i = 5, j = 6 \\ 2 & i = 2, ..., 4, j = 3, ..., 5; i < j \end{cases}$$

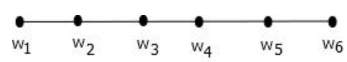

FIGURE 1. Path graph P_6

Theorem 3.3. *Every BT (n,m,k) Bamboo Tree is an ADL graph, for all $n \geq 2$, $k \geq 1$.*

Proof. Let G be a BT (n,m,k) bamboo tree with k(n+m-1)+1 vertices. Let $\mathrm{w}_i^{(j)}$, $\mathrm{v}_s^{(j)}$ be the vertices of Path Graph P_n and star graph $K_{1,m}$ respectively. Let $\mathrm{w}_0^{(0)}$ be the vertex adjacent to $\mathrm{w}_1^{(j)}$, $1 \leq i \leq k$. The vertex degree labeling of G is

$$A(w_i^{(j)}) = \begin{cases} k & i = 0, \quad j = 0 \\ 2 & 1 \leq i \leq n-2, 1 \leq j \leq k \\ m+1 & i = n-1, 1 \leq j \leq k \end{cases}$$

and

$$A(v_s^{(j)}) = 1, \quad 1 \leq s \leq m, \quad 1 \leq j \leq k$$

The edge labeling of G is discussed below with the following cases:
Case 1: When k is even, m is odd

$$A^*(\mathrm{w}_i^{(j)} w_{i+1}^{(j)}) = \begin{cases} \frac{k+2}{2} & i = 0, \quad 0 \leq j \leq k \\ \frac{m+3}{2} & i = n-2, \quad 1 \leq j \leq k \\ 2 & 1 \leq i \leq n-2, \quad 1 \leq j \leq k \end{cases}$$

$$A^*(\mathrm{w}_i^{(j)} v_s^{(j)}) = \lfloor \frac{m+2}{2} \rfloor, i = n-1, 1 \leq j \leq k, 1 \leq s \leq m$$

Case 2: When k is even, m is even

$$A^*(\mathrm{w}_i^{(j)}w_{i+1}^{(j)}) = \begin{cases} \frac{k+2}{2} & i=0, \quad 0 \le j \le k \\ \lfloor \frac{m+3}{2} \rfloor & i=n-2 \quad 1 \le j \le k \\ 2 & 1 \le i \le n-2, 1 \le j \le k \end{cases}$$

$$A^*(\mathrm{w}_i^{(j)}v_s^{(j)}) = \frac{m+2}{2}, \quad i=n-1, 1 \le j \le k, 1 \le s \le m$$

Case 3: When k is odd, m is even

$$A^*(\mathrm{w}_i^{(j)}w_{i+1}^{(j)}) = \begin{cases} \lfloor \frac{k+2}{2} \rfloor & i=0, \quad 0 \le j \le k \\ \lfloor \frac{m+3}{2} \rfloor & i=n-2 \quad 1 \le j \le k \\ 2 & 1 \le i \le n-2, \quad 1 \le j \le k \end{cases}$$

$$A^*(\mathrm{w}_i^{(j)}v_s^{(j)}) = \frac{m+2}{2}, i=n-1, 1 \le j \le k, 1 \le s \le m$$

Case 4: When k is odd, m is odd

$$A^*(w_i^{(j)}w_{i+1}^{(j)}) = \begin{cases} \lfloor \frac{k+2}{2} \rfloor & i=0, \quad 0 \le j \le k \\ \frac{m+3}{2} & i=n-2, \quad 1 \le j \le k \\ 2 & 1 \le i \le n-2 \quad 1 \le j \le k \end{cases}$$

$$A^*(w_i^{(j)}v_s^{(j)}) = \lfloor \frac{m+2}{2} \rfloor, i=n-1, 1 \le j \le k, 1 \le s \le m$$

Thus, every BT (n,m,k)-Bamboo tree is an ADL graph. □

Example 3.4. *The Bamboo tree BT (7,2,2), shown in Figure 2 is an ADL graph. The vertex and edge labeling of BT (7,2,2) is given by*

$$A(w_i^{(j)}) = \begin{cases} 2 & i=0, \quad j=0 \\ 2 & 1 \le i \le 6, 1 \le j \le 2 \\ 3 & i=6, 1 \le j \le 2 \end{cases}$$

$$A(v_s^{(j)}) = 1, \quad 1 \le s \le 2, \quad 1 \le j \le 3$$

$$A^*(w_i^{(j)}w_{i+1}^{(j)}) = \begin{cases} 2 & i=0, 0 \le j \le 3 \\ 2 & i=6, \quad 1 \le j \le 3 \\ 2 & 1 \le i \le 5 \quad 1 \le j \le 3 \end{cases}$$

$$A^*(w_i^{(j)}v_s^{(j)}) = 2, \quad i=n-7, \quad 1 \le j \le 3, \quad 1 \le s \le 2$$

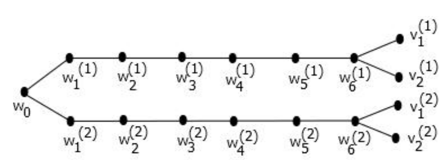

FIGURE 2. BT (7,2,2) - Bamboo tree

Theorem 3.5. *Every B(m,k)-Banana tree with v as a root is an ADL graph, $k \geq 2$, $n \geq k$.*

Proof. Let $w_i^{(j)}$; $1 \leq i \leq k$, $1 \leq j \leq m$ be the vertices of m copies of star graph S_k such that $w_{k-1}^{(j)} \sim v$ and $w_{k-1}^{(j)} \sim w_k^{(j)}$and v be the root of a banana tree $G =$ B(m,k).
The vertex degree labeling of G is,

$$\mathrm{A}(w_i^{(j)}) = \begin{cases} k-1 & i=k, \quad 1 \leq j \leq m \\ 1 & 1 \leq i \leq k-2, \quad 1 \leq j \leq m \\ 2 & i=k-1, \quad 1 \leq j \leq m \end{cases}$$

and A(v) = m
The average degree edge labeling of G is

$$A^*(\mathrm{w}_{k-1}^{(j)}v) = \begin{cases} \lfloor \frac{m+2}{2} \rfloor & 1 \leq j \leq m, \text{m is odd} \\ \frac{m+2}{2} & \text{m is even} \end{cases}$$

$$A^*(\mathrm{w}_i^{(j)} w_{i+1}^{(j)}) = \begin{cases} \lfloor \frac{k+1}{2} \rfloor & w_i^{(j)} \sim w_{i+1}^{(j)}, \text{k is even} \\ \frac{k+1}{2} & w_i^{(j)} \sim w_{i+1}^{(j)}, \text{k is odd} \\ \lfloor \frac{k}{2} \rfloor & \text{otherwise} \end{cases}$$

Thus, every B(m,k)-Banana tree with v as a root is an ADL graph. □

Example 3.6. *The Banana tree B(2,8) is shown in figure 3. The vertex labeling of B(2,8) is given by,*

$$A(w_i^{(j)}) = \begin{cases} 7 & i=8, \quad 1 \leq j \leq 2 \\ 1 & 1 \leq i \leq 6, \quad 1 \leq j \leq 2 \\ 2 & i=7, \quad 1 \leq j \leq 2 \end{cases}$$

$A(v) \quad = \quad 2$
and the average degree edge labeling of B(2,8) is

$$A^*(w_7^{(j)}v) = 2, \quad 1 \leq j \leq 2; \quad \textit{m is even}$$

$$A^*(w_i^{(j)} w_{i+1}^{(j)}) = \begin{cases} 4 & i=7, j=1,2; \quad \textit{k is even,} \\ 4 & \textit{otherwise} \end{cases}$$

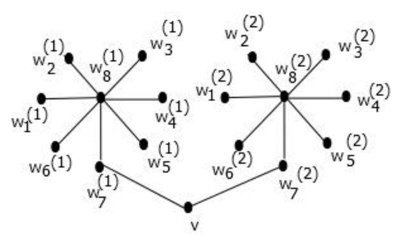

FIGURE 3. B(2,8) - Banana tree

Theorem 3.7. *Every full Binary tree is an ADL graph.*

Proof. Let w_i, i=1,2,...,n are the vertices of a binary tree G. The vertex labeling of$G$ is given by

$$\mathrm{A}(w_i) = \begin{cases} 1 & \text{if } w_i \text{ is a pendent vertex of G} \\ 2 & \text{if } w_i \text{ is a root} \\ 3 & \text{otherwise} \end{cases}$$

The average degree edge labeling of G is

$$A^*(\mathrm{w}_i w_j) = \begin{cases} \lfloor \frac{5}{2} \rfloor & \text{if d}(w_i)\text{=2,} \quad \text{d}(w_j)\text{=3} \\ 2 & \text{if d}(w_i)\text{=1,} \quad \text{d}(w_j)\text{=3} \\ 3 & \text{if d}(w_i) = \text{d}(w_j)\text{=3} \end{cases}$$

Thus, every full Binary tree is an ADL graph. □

Theorem 3.8. *Every graph $G = P_m + K_n$ is an ADL graph.*

Proof. Let G be the join of path graph P_m and complete graph K_n.
Let $w_1, w_2, ..., w_m$ and $w_1^{'}, w_2^{'}, ..., w_n^{'}$ be the vertices of P_m and K_n respectively. The vertex degree labeling of G is

$$\mathrm{A}(w_i) = \begin{cases} n+1 & i = 1, m \\ n+2 & 2 \leq i \leq m-1 \end{cases}$$

and $\quad A(w_j^{'}) = n + m - 1, 1 \leq j \leq n$

The average degree edge labeling of$G$ is

$$A^*(\mathrm{w}_i w_j{'}) = \begin{cases} \lfloor \frac{m+2n}{2} \rfloor & i = 1, m; 1 \leq j \leq n, \\ \lfloor \frac{2n+m+1}{2} \rfloor & 2 \leq i \leq m-1; 1 \leq j \leq n, \end{cases}$$

$A^*(\mathrm{w}_j^{'} w_k^{'}) = n + m - 1, \quad 1 \leq j, k \leq n$

$$A^*(\mathrm{w}_i w_{i+1}) = \begin{cases} \lfloor \frac{2n+3}{2} \rfloor & i = 1, m-1 \\ n+2 & 2 \leq i \leq m-2 \end{cases}$$

Thus, every $G = P_m + K_n$ graph is an ADL graph. □

Theorem 3.9. *Every cycle cactus $C_n^{(m)}$ is an ADL graph for all $n \geq 3$, m>1.*

Proof. Let $w_1^{(j)}, w_2^{(j)}, ..., w_{n-1}^{(j)}, w_n^{(j)} = v$ be the vertices of $G = C_n^{(m)}, 1 \leq j \leq m$, with $\mathrm{d}(w_1^{(j)}) = 2$ and d(v)=2m, $1 \leq i \leq n-1$.
The vertex degree labeling of G is defined by,

$$A(w_1^{(j)}) = 2, \quad 1 \leq i \leq n-1, 1 \leq j \leq m$$

and A(v) = 2m.
The average Degree edge labeling of G is given by

$$A^*(\mathrm{w}_i^{(j)} w_{i+1}^{(j)}) = 2, \quad 1 \leq i \leq n-1, 1 \leq j \leq m$$

$$A^*(w_i^{(j)}v) = m+1, \quad i = 1, n-1, 1 \le j \le m$$

Thus, every cycle cactus $C_n^{(m)}$ is an ADL graph for all $n \ge 3$, m>1. □

Theorem 3.10. *Every Complete bipartite graph $K_{m,n}$ is an ADL graph for all m,n ≥ 2.*

Proof. Let $w_1^{(1)}, w_2^{(1)}, ..., w_m^{(1)}$ and $w_1^{(2)}, w_2^{(2)}, ..., w_n^{(2)}$ be the vertices of Bipartite sets V_1 and V_2 respectively.
Vertex degree labeling $A : V(G) \longmapsto \{1, 2, ..., max(m, n)\}$ is defined by

$$A(w_i^{(j)}) = \begin{cases} n & if \quad w_i^{(j)} \in V_1 \\ m & if \quad w_i^{(j)} \in V_2 \end{cases}$$

and Average Degree edge labeling is given by

$$A^*(\text{w}_i^{(1)} w_j^{(2)}) = \begin{cases} \lfloor \frac{n+m}{2} \rfloor & 1 \le i \le m, \quad 1 \le j \le n \end{cases}$$

Thus, $K_{m,n}$ is an ADL graph. □

Theorem 3.11. *Every subdivided shell graph is an ADL graph.*

Proof. Let G be a subdivided shell graph obtained by subdividing the edges of P_m. It has 2m vertices and (3m-2) edges. Let v and $w_i^{(1)}$ be the vertices of K_1 and P_m respectively, $1 \le i \le m$; and $w_i^{(2)}$ be the vertex between $w_i^{(1)}$ and $w_{i+1}^{(1)}$ after removing the edge $w_i^{(1)} w_{i+1}^{(1)}$ of P_m, $1 \le i \le m-1$.
The vertex degree labeling of G is

$$A(v) = m$$

$$A(w_i^{(1)}) = \begin{cases} 2 & i = 1, m \\ 3 & 2 \le i \le m-1 \end{cases}$$

$$A(w_i^{(2)}) = 2, \quad 1 \le i \le m-1$$

The average degree edge labeling of G is

$$A^*(vw_i^{(1)}) = \begin{cases} \lfloor \frac{m+2}{2} \rfloor & i = 1, m \\ \lfloor \frac{m+3}{2} \rfloor & 2 \le i \le m-1 \end{cases}$$

$$A^*(w_i^{(1)} w_j^{(2)}) = \begin{cases} 2 & i = 1, m, \quad j = 1, m-1 \\ \lfloor \frac{5}{2} \rfloor & 2 \le i \le m-1, 2 \le j \le m-2 \end{cases}$$

Thus, every subdivided shell graph is an ADL graph. □

Theorem 3.12. *Every windmill graph Wd(n,k) is an ADL graph, $n \ge 2$, $k \ge 2$.*

Proof. Let G be a Wd(n,k) windmill graph obtained by taking k copies of complete graph K_n with a vertex in common. Let w_0 be the vertex in common and $w_1^{(j)}, w_2^{(j)}, ..., w_{n-1}^{(j)}$, $1 \le j \le k$, be the vertices of copies of K_n.
The vertex degree labeling of G is

$$A(w_0) = k(n-1)$$

$$A(w_i^{(j)}) = n-1; \quad 1 \le i \le n-1, \quad 1 \le j \le k$$

and Average Degree edge labeling is given by

$$A^*(w_i^{(j)}w_l^{(j)}) = \lfloor n-1 \rfloor; \quad w_i^{(j)} \sim w_l^{(j)},$$

$$1 \leq i, l \leq n-1, 1 \leq j \leq k$$

$$A^*(w_i^{(j)}w_0) = \lfloor \frac{(n-1)(k+1)}{2} \rfloor; \quad 1 \leq i \leq n, \quad 1 \leq j \leq k$$

Thus, every windmill graph Wd(n,k) is an ADL graph. □

Theorem 3.13. *Every grid graph $G = P_m \times P_n$ is an ADL graph.*

Proof. Let $w_1^{(1)}, w_2^{(1)}, ..., w_m^{(1)},\ w_1^{(2)}, ..., w_m^{(2)}, ..., w_1^{(n)},\ w_2^{(n)}, ..., w_m^{(n)}$ be the vertices of G such that

$$deg(w_1^{(1)}) = deg(w_1^{(n)}) = deg(w_m^{(1)}) = deg(w_m^{(n)}) = 2$$

$$deg(w_1^{(2)}) = deg(w_1^{(3)}) = ... = deg(w_1^{(n-1)}) = 3$$

$$deg(w_2^{(1)}) = deg(w_3^{(1)}) = ... = deg(w_{m-1}^{(1)}) = 3$$

$$deg(w_m^{(2)}) = deg(w_m^{(3)}) = ... = deg(w_m^{(n-1)}) = 3$$

$$deg(w_2^{(n)}) = deg(w_3^{(n)}) = ... = deg(w_{m-1}^{(n)}) = 3$$

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$$deg(w_3^{(3)}) = ... = deg(w_{m-1}^{(3)} = ... = deg(w_2^{(n-1)}) =$$

$$deg(w_3^{(n-1)}) = ... = deg(w_{m-1}^{(n-1)}) = 4$$

The vertex degree labeling of G is

$$A(w_i^{(j)}) = \begin{cases} 2 & i = 1, m, \quad j = 1, n \\ 3 & i = 1, m, \quad 2 \leq j \leq n-1 \\ & 2 \leq i \leq m-1, \quad j = 1, n \\ 4 & 2 \leq i \leq m-1, \quad 2 \leq j \leq n-1 \end{cases}$$

The average degree edge labeling of G is

$$A^*(\mathrm{w}_i^{(j)}w_{i+1}^{(j)}) = \begin{cases} \lfloor \frac{5}{2} \rfloor & i = 1, m-1; \quad j = 1, n \\ 3 & 2 \leq i \leq m-1; \quad j = 1, n \\ 4 & 2 \leq i \leq m-1; \quad 2 \leq j \leq n-1 \\ \lfloor \frac{7}{2} \rfloor & i = 1, m-1; \quad 2 \leq j \leq n-1 \end{cases}$$

$$A^*(\mathrm{w}_i^{(j)}w_i^{(j+1)}) = \begin{cases} \lfloor \frac{5}{2} \rfloor & i = 1, m; \quad j = 1, n-1 \\ 3 & i = 1, m; \quad 2 \leq j \leq n-1 \\ 4 & 2 \leq i \leq m-1; \quad 2 \leq j \leq n-1 \\ \lfloor \frac{7}{2} \rfloor & 2 \leq i \leq m-1; \quad j = 1, n-1 \end{cases}$$

□

4. Algorithms

The maximal and minimal spanning tree based on average degree labeling:
The small world brain network is discussed in [1] and, inspired by their work, we attempted to identify the maximal and minimal spanning trees of an ADL graph. This finding may be valuable in future research on network theory. Let G be an undirected, average degree labeled graph on 'n' vertices.

Algorithm 4.1. *Below are the steps used in the algorithm to find a Maximal spanning tree shown in flowchart, Figure 4.*

Step 1: Let G be an ADL graph and we call edge labeling values as weights of G.
Step 2: Write the degrees of vertices and edges having weights in descending order.
Step 3: Start with a vertex having maximum degree and add the edges having maximum weight from that vertex in T. If the graph has same vertex degree, then continue in a similar manner.
Step 4: Omit the edges which forms a cycle and continue the process till we get (n-1) edges in T.

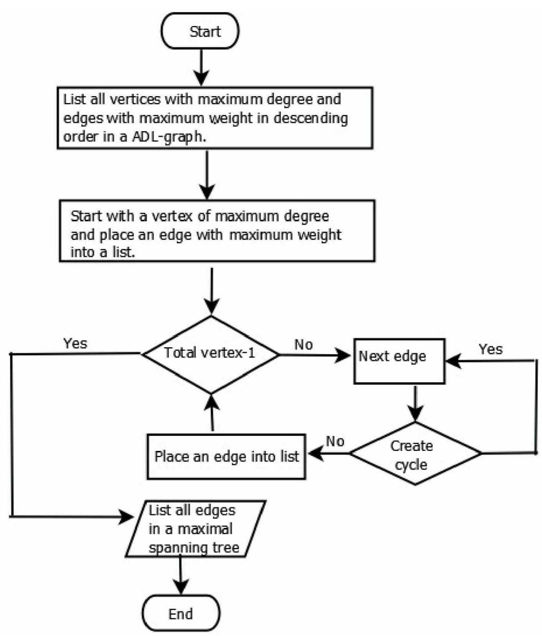

FIGURE 4. Flow chart for maximal spanning tree for ADL graph

Algorithm 4.2. *Below are the steps used in the algorithm to find a Minimal spanning tree shown in flowchart, Figure 5.*

Step 1: Let G be an ADL graph and we call edge labeling values as weights of G.
Step 2: Write the degrees of vertices and edges having weights in increasing order.

Step 3: Start with a vertex having minimum degree and add the edges in T having minimum weight. If the graph has same vertex degree, then continue in a similar manner.

Step 4: Omit the edges which forms a cycle and continue the process till we get (n-1) edges in T.

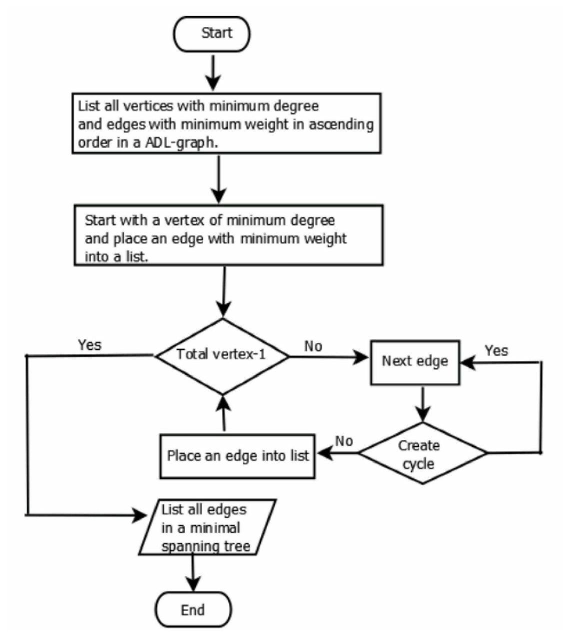

FIGURE 5. Flow chart for minimal spanning tree for ADL graph

Example 4.3. *Consider an ADL graph G with six vertices. Figure 6 shows the maximal and minimal spanning trees for G.*

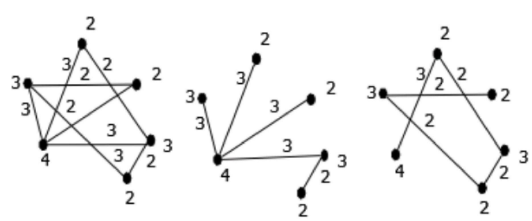

FIGURE 6. Graph G, Maximal and Minimal Spanning trees

The shortest path principle based on average degree labeling is given below:

The shortest path Algorithms play a critical role in network and transportation studies. However, previous evaluations of existing shortest-path methods were mostly based on the distance or weight of an edge in the graph. Most computational tests on shortest path algorithms have used randomly generated networks, which may not accurately represent real road networks, as discussed in [8]. Dijkstra's algorithm can be used to determine the shortest path from one vertex in a graph to every other vertex within the same graph based on distances. Our approach is beneficial in determining the exact shortest path lengths in complex networks. This provides a solid foundation for developing more efficient methods for complicated network research. The approach can be modified to calculate approximate shortest paths and other metrics in complex networks based on average degree labeling.

Algorithm 4.4. *The main hypothesis of this algorithm works in the following way:*

Step 1: Choose a vertex (v) to determine the shortest path to other vertices in an ADL graph (up to a few).

Step 2: Begin with a vertex v and look for an edge adjacent to the vertex w that has the lowest degree. Add it to the path.

Step 3: Choose the next edge linked with vertex w that has the least labeling and place it in the path.

Step 4: Continue from step 3 to add the remaining vertices to the path.

Example 4.5. *Figure 7 shows the shortest path between the vertices v_1 and v_2.*

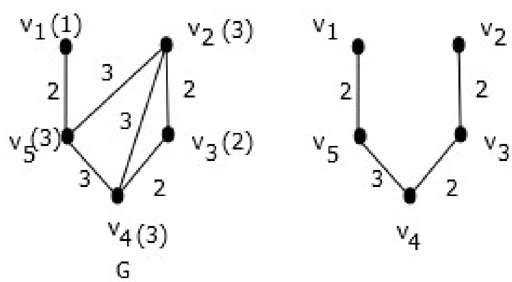

FIGURE 7. Shortest path from v_1 to v_2 via all the vertices in G

Remark 4.6. (a) The regular graph with regularity r on n-vertices has a maximal spanning tree whose weight is r(n-1).

(b) The running time complexity of algorithm is O(n^2) where n is the number of vertices.

5. Conclusion

The average degree labeling of graphs is investigated and shown to be outstanding in the field of network theory analysis. Identifying minimal and maximal spanning trees for undirected graphs is an interesting and difficult issue. This is accomplished in the current study.

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