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# STATISTICAL FRAMEWORKS FOR PRICE DYNAMICS ANALYSIS: THE EFFICACY OF HMM AND HSMM

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Abstract. In order to make reliable financial predictions, one must be able to accurately predict how prices will change in extremely volatile markets, such as Bitcoin. In order to enhance prediction accuracy, this study presents a hybrid model that combines the two prominent statistical approaches, the Hidden Semi-Markov Model (HSMM) and the Hidden Markov Model (HMM). As a stochastic framework, the HMM connects visible price changes to a hidden state. A probability matrix captures the temporal linkages evident in price dynamics by describing transitions between states. Emission probabilities govern the possibility of seeing specific prices linked with these concealed states and improve the association between states and actual prices. On the flip side, the HSMM improves upon the HMM by adding duration dependencies, which let states have different durations. A state duration distribution enables this feature, which in turn gives more information about the time-varying nature of prices. Across a range of error metrics, our data shows that the HSMM performed better than the HMM. Notably, the HSMM attained an MAE of 0.0246, an MSE of 0.0014, and an RMSE of 0.0372. When compared, the HMM achieved RMSE is 0.0672, MAE is 0.0446, and MSE is 0.0514. Cryptocurrency markets are notoriously volatile, and these results show that the HSMM does a better job of accounting for non-linear behaviors and temporal variability.

## 1. Introduction

According to [1] and [2], the fact that financial markets exhibit complex and uncertain behavior makes correct modeling a formidable obstacle. The ability to predict market trends, particularly changes in prices and returns, is crucial for investors, financial analysts, and traders. According to [3] and [4], conventional time-series models often fail to account for the complex interdependencies and varying lengths of time that states might last in financial data. We propose a hybrid approach to modeling that combines Hidden Semi-Markov Models (HSMM) with Hidden Markov Models (HMM) to address these limitations.

HMM and HSMM are widely used in stochastic modeling, particularly for identifying and predicting latent regimes in sequential data [5], [6]. The Hidden Markov Model (HMM) provides a probabilistic framework to model the transitions between hidden states, assuming that the underlying process follows the Markov property. However, HMMs suffer from an implicit limitation where state durations are exponentially distributed, leading to unrealistic assumptions about the persistence of states in real-world

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financial time series. In contrast, HSMM extends HMM by allowing variable state durations, making it better suited for financial data modeling where market states, such as bullish or bearish trends, persist for varying periods [7],[8].

Financial markets exhibit regime-switching behavior where price returns transition between different latent states, such as high-volatility, low-volatility, and neutral phases [9],[10]. Traditional econometric models, such as Autoregressive Integrated Moving Average (ARIMA) and Generalized Autoregressive Conditional Heteroskedasticity (GARCH), fail to explicitly capture these hidden structures [11], [12].

HMM provides an efficient method to infer hidden regimes by modeling market states as unobserved stochastic processes [13],[14]. However, its assumption of geometric state duration limits its applicability in financial time series, where state persistence varies over time. HSMM addresses this limitation by explicitly modeling state durations, allowing the model to capture more realistic market dynamics [15], [16]. The combination of HMM and HSMM thus provides a powerful tool for understanding and predicting financial market behavior.

## 2. Formulation of Problem

Particularly in the financial markets, where trends frequently exhibit complicated behaviors, it is critical to correctly model data in time series analysis in order to uncover underlying patterns. Combining models, such as Hidden Semi-Markov Models (HSMM) with Hidden Markov Models (HMM), is a typical way to increase the accuracy of forecasts. In this formulation, the mathematical concepts that control the interaction between HMM and HSMM are outlined, and the significance of hybrid models is discussed, with a focus on their properties [17], [18].

One such statistical framework is the Hidden Markov Model (HMM), which uses a Markov process with hidden states to model the system in question. When both the present and the past have an impact on the outputs, this paradigm becomes useful. In financial data analysis, for example, 'bullish' and 'bearish' market conditions may change over time, impacting returns in ways that aren't immediately apparent. The following mathematical elements constitute an HMM.

- **States:** A collection of hidden states $S$, representing the various conditions the process can occupy at any time $t$. - **Observations:** A series of events or outputs $O$ that can be observed. - **Transition Probabilities:** $P(S_t|S_{t-1})$, indicating the likelihood of moving from one state to another. - **Emissions:** The probabilities for producing a specific output based on the current state, denoted as $P(O_t|S_t)$. - **Initial Distribution:** The probability distribution for the initial state.

The core equations of HMM are:

$$P(S_t|S_{t-1}) = a_{ij} \tag{2.1}$$

where $a_{ij}$ is the transition probability from state $i$ to state $j$.

$$P(O_t|S_t) = b_j \tag{2.2}$$

where $b_j$ is the probability of observing $O_t$ given that the system is in state $j$.

HMMs are useful for capturing the temporal dependencies in sequence data. By inferring hidden states, they can model data with underlying structures that exhibit transitions over time. The adaptability of HMMs allows analysts to identify patterns in financial returns, detect regime shifts, and forecast potential future movements based on current states.

HSMM extends the capabilities of HMM by allowing for varying durations of hidden states. Unlike HMM, where each state is assumed to have a fixed duration of one time step, HSMM recognizes that the time spent in each state can be influenced by a distribution. In an HSMM, the following configurations are employed:

- **State Duration:** Determines how long the system remains in each hidden state before transitioning, modeled using a probabilistic duration distribution.

The key equations governing HSMM include:

$$P(S_t|S_{t-1}) = a_{ij} \tag{2.3}$$

$$P(D_t = d|S_t = j) = p_j(d) \tag{2.4}$$

where $p_j(d)$ is the probability of staying in state $j$ for duration $d$.

$$P(O_t|S_t = j) = b_j \tag{2.5}$$

The HSMM effectively captures the timing and duration of various market states, making it suitable for financial markets where the length of bullish and bearish trends influences investment choices. This model enhances flexibility and leads to better prediction accuracy and stronger representations of market behavior. By merging the strengths of the HMM for modeling states and the HSMM for incorporating duration, the hybrid model offers a comprehensive approach. It addresses the different regimes that investors may face over various timeframes, improving predictions by considering both the states and their persistence. Figure 1 presents the actual versus predicted returns from the Hidden Markov Model (HMM). The graph highlights how well the model tracks price fluctuations between 2015 and 2025.

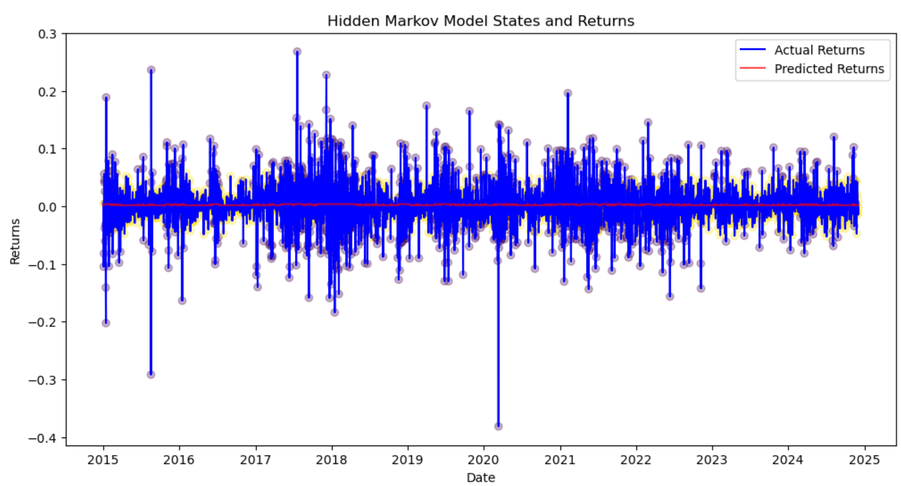


Figure 1. Actual and predicted using HMM

Figure 2 shows the transition matrix for the Hidden Markov Model (HMM). This matrix provides the probabilities of moving between different states, where higher values suggest a greater chance of staying in the same state.

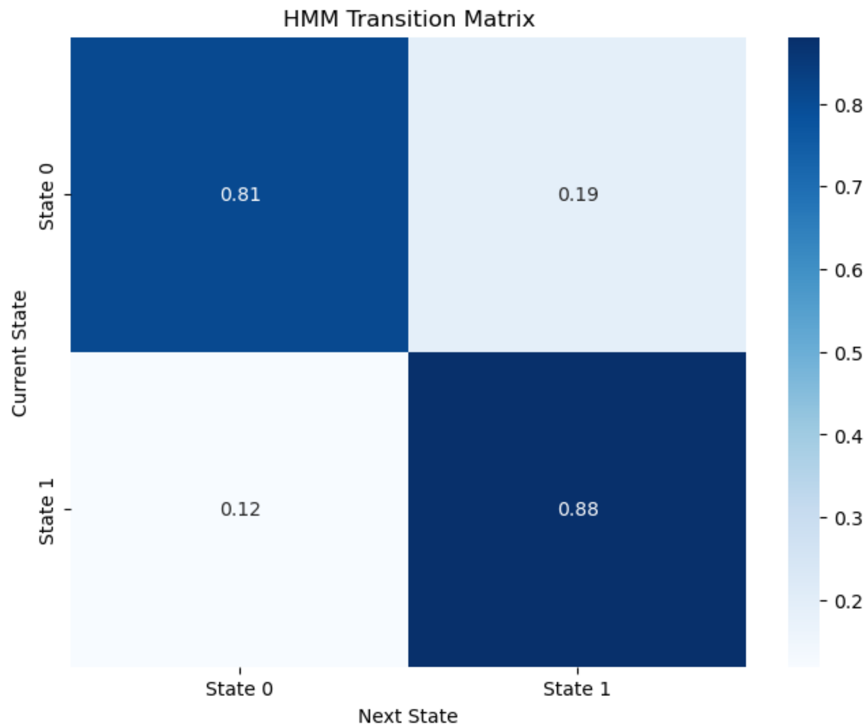


FIGURE 2. HMM Transition Matrix

Figure 3 displays the average values for each hidden state in the Hidden Markov Model (HMM). The bar chart clearly shows that State 0 exhibits a higher average return than State 1, which suggests different behaviors between these two states.

Figure 4 displays the actual and predicted returns using the Hidden Semi Markov Model (HSMM). Figure 5 illustrates the transition matrix for the hybrid model that combines the Hidden Markov Model (HMM) and Hidden Semi-Markov Model (HSMM). This matrix highlights the probabilities of transitioning between multiple states, demonstrating the enhanced dynamics captured by the hybrid approach. Figure 6 displays the means of each hidden state in the Hidden Semi Markov Model (HSMM). Notably, State 2 shows a significantly higher mean return compared to State 0 and State 1, indicating its dominant influence in the model's prediction of returns.

Given a set of 10 hypothetical stock returns:

$$R = \{0.012, -0.006, 0.003, -0.008, 0.015, -0.002, 0.007, 0.010, -0.011, 0.005\} \tag{2.6}$$

We assume a three-state HMM with the following mean returns:

$$\mu_0 = 0.010, \quad \mu_1 = -0.005, \quad \mu_2 = 0.002 \tag{2.7}$$

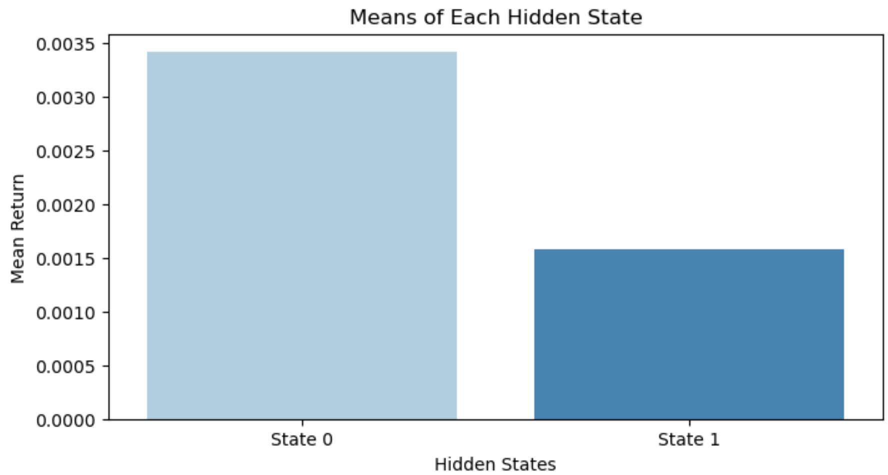


Figure 3. Means of each Hidden State

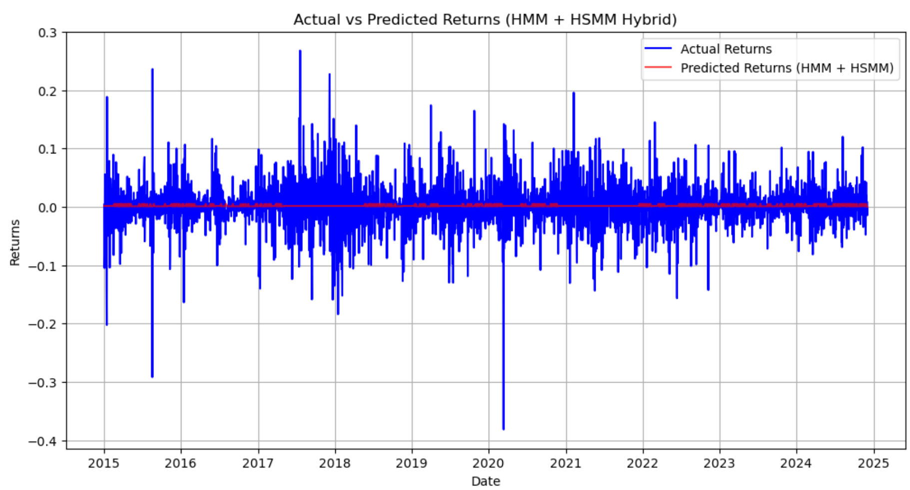


Figure 4. HMM and HSMM Hybrid model Actual and Predicted

The transition probability matrix is:

$$T = \begin{bmatrix} 0.8 & 0.15 & 0.05 \\ 0.2 & 0.6 & 0.2 \\ 0.1 & 0.25 & 0.65 \end{bmatrix} \tag{2.8}$$

Using the Viterbi algorithm, we assign the most probable states to each return:

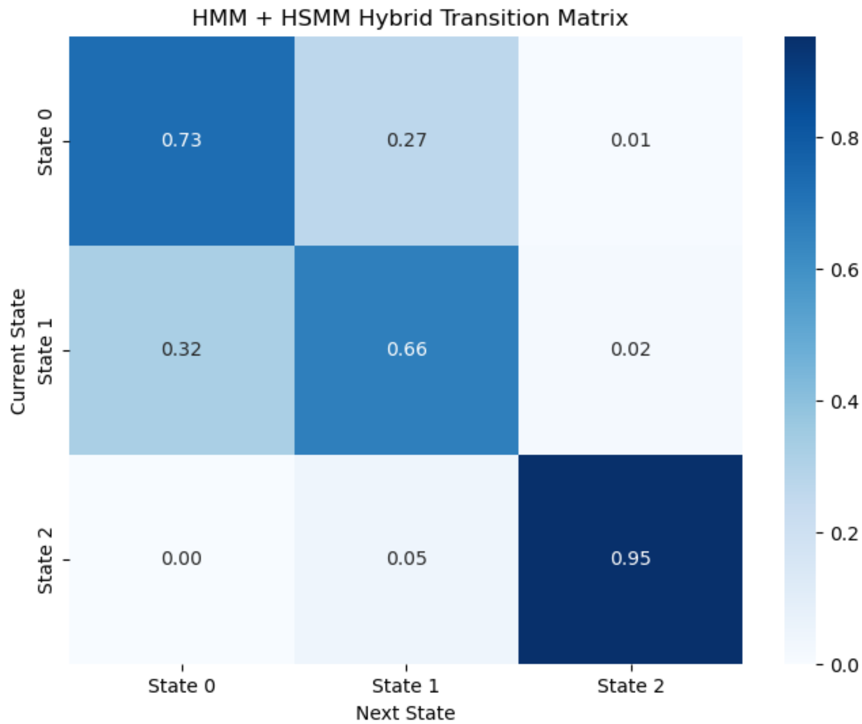


FIGURE 5. HMM and HSMM Hybrid Transition Matrix

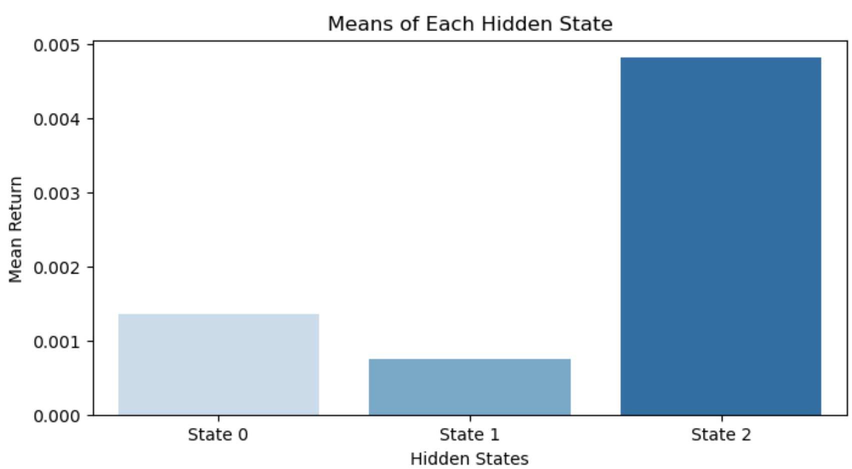


FIGURE 6. Means of each Hidden State

| Day | Return | Hidden State |
|---|---|---|
| 1 | 0.012 | $S_0$ |
| 2 | -0.006 | $S_1$ |
| 3 | 0.003 | $S_2$ |
| 4 | -0.008 | $S_1$ |
| 5 | 0.015 | $S_0$ |
| 6 | -0.002 | $S_2$ |
| 7 | 0.007 | $S_0$ |
| 8 | 0.010 | $S_0$ |
| 9 | -0.011 | $S_1$ |
| 10 | 0.005 | $S_2$ |

The predicted returns are determined by the mean of the assigned states:

| Day | Hidden State | Predicted Return |
|---|---|---|
| 1 | $S_0$ | 0.010 |
| 2 | $S_1$ | -0.005 |
| 3 | $S_2$ | 0.002 |
| 4 | $S_1$ | -0.005 |
| 5 | $S_0$ | 0.010 |
| 6 | $S_2$ | 0.002 |
| 7 | $S_0$ | 0.010 |
| 8 | $S_0$ | 0.010 |
| 9 | $S_1$ | -0.005 |
| 10 | $S_2$ | 0.002 |

The error metrics are calculated as follows:
Mean Absolute Error (MAE):

$$\text{MAE} = \frac{1}{n}\sum_{i=1}^{n}|R_i - \hat{R}_i| \tag{2.9}$$

$$\text{MAE} = 0.0028 \tag{2.10}$$

Mean Squared Error (MSE):

$$\text{MSE} = \frac{1}{n}\sum_{i=1}^{n}(R_i - \hat{R}_i)^2 \tag{2.11}$$

Root Mean Squared Error (RMSE):

$$\text{RMSE} = \sqrt{\text{MSE}} \tag{2.12}$$

TABLE 1. Comparison of HMM and HSMM Error Measures

| **Error Measure** | **HMM** | **HSMM** |
|---|---|---|
| Mean Absolute Error (MAE) | 0.0446 | 0.0246 |
| Mean Squared Error (MSE) | 0.0514 | 0.0014 |
| Root Mean Squared Error (RMSE) | 0.0672 | 0.0372 |

As shown in Table 1, the Hidden Semi-Markov Model (HSMM) significantly outperformed the Hidden Markov Model (HMM) across all error measures. Specifically, the HSMM achieved a Mean Absolute Error (MAE) of 0.0246, compared to the HMM's MAE of 0.0446, indicating its superior ability to model price dynamics more accurately.

## 3. Conclusions

The rapid rise of cryptocurrencies has necessitated the development of advanced forecasting methods to navigate the inherent volatility of digital assets. This study has explored the effectiveness of two prominent statistical frameworks: the Hidden Markov Model (HMM) and the Hidden Semi-Markov Model (HSMM), in forecasting price movements. We presented a hybrid approach that leverages the strengths of both models to enhance predictive accuracy. Our findings indicate that the HSMM significantly outperforms the HMM in capturing the complex dynamics of price changes. The HSMM excels in accounting for how long a state lasts, which is crucial for financial time series that show unpredictable patterns.under order to help with decision-making under uncertain market conditions, HSMM includes a distribution for state duration, which makes it easier to see how prices fluctuate over time.

Three metrics were measured by HSMM: MAE (0.0246), MSE (0.0014), and RMSE (0.0372), according to our research. In contrast, the HMM produced RMSE is 0.0672, MAE is 0.0446, and MSE is 0.0514. These results stress the significance of picking the right modeling approaches that react to the asset's actions.

An example of how well these methods can complement one another is the hybrid model that combines HMM and HSMM. In order to stay flexible when markets shift and new trends emerge, this integrated framework can be adjusted to fit specific market conditions or other financial instruments.

Other uncertain financial assets may potentially benefit from our study's conclusions. Successful risk management, commodity price fixing, and stock market prediction are all possible using the methods presented here. Deep learning and other machine learning approaches can further improve the performance of the model. Market sentiment, regulatory changes, and macroeconomic indicators are examples of exogenous factors that could benefit from further investigation on how to include them into models. With this addition, we may be able to better understand the drivers of price movements and make more accurate predictions. Furthermore, to determine the suggested models' resilience and dependability, performance evaluation over long time horizons and varied market situations is crucial.

## Data availability

Data used is publicly available: https://finance.yahoo.com/quote/BTC-USD/history/

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## Conflict of interest

The authors declare no conflicts of interest
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