

## RESULTS ON VANISHING COEFFICIENTS FOR THE CONTINUED FRACTIONS AND ITS RECIPROCAL

DEVADAS ANU RADHA

ABSTRACT. In the present investigation, using general continued fraction identity of Ramanujan the  $q$ -continued fractions of various orders are established. Further, we obtain the vanishing coefficients for the continued fractions of order twenty two and forty four and its reciprocals.

### 1. Introduction

For any complex numbers  $\delta$  and  $q$ , define the  $q$ -product  $(\delta; q)_\infty$  as

$$(\delta; q)_\infty := \prod_{t=0}^{\infty} (1 - \delta q^t), \quad |q| < 1. \quad (1.1)$$

For simplicity, we often write

$$(\delta_1; q)_\infty (\delta_2; q)_\infty \dots (\delta_m; q)_\infty = (\delta_1, \delta_2, \dots, \delta_m; q)_\infty.$$

The Ramanujan's general theta function  $f(x, y)$  [4, pp. 34] is defined as

$$f(x, y) = \sum_{y=-\infty}^{\infty} x^{n(n+1)/2} y^{n(n-1)/2}, \quad |xy| < 1. \quad (1.2)$$

Jacobi's triple product identity [4, pp. 35, Entry 19] in terms of  $f(x, y)$ , can be stated as

$$f(x, y) = (-x; xy)_\infty (-y; xy)_\infty (xy; xy)_\infty = (-x, -y, xy; xy)_\infty. \quad (1.3)$$

The special cases of  $f(x, y)$  are the theta-functions  $\phi(q)$  and  $f(-q)$  [4, pp. 36, Entry 22 (i)-(iii)] are given by

$$\phi(q) := f(q, q) = \sum_{t=-\infty}^{\infty} q^{t^2} = \frac{(-q; -q)_\infty}{(q; -q)_\infty}, \quad (1.4)$$

$$f(-q) := f(-q, -q^2) = \sum_{t=-\infty}^{\infty} (-1)^t q^{t(3t-1)/2} = (q; q)_\infty. \quad (1.5)$$

One of the notable contributions of Ramanujan is found in the field of  $q$ -continued fractions. He recorded a variety of continued fractions in his notebooks, among which the most famous is the Rogers-Ramanujan continued fraction of order 5. In 2017, M. S. Surekha [12], established modular relations and dissections pertaining

---

2000 *Mathematics Subject Classification.* 11A55; 11F27; 11P84.

*Key words and phrases.*  $q$ -continued fraction, theta function and vanishing coefficient.

to continued fractions of order sixteen analogous to Rogers-Ramanujan continued fraction. Subsequently, N. Saikia and S. Rajkhowa [8, 9] further advanced the study by developing continued fractions of various orders that are analogous to the Rogers-Ramanujan continued fractions, along with formulating modular identities for these fractions.

Now we consider the Rogers-Ramanujan continued fraction  $R(q)$  [5] of order 5 defined by

$$R(q) := q^{1/5} \frac{(q, q^4; q^5)_\infty}{(q^2, q^3; q^5)_\infty} = q^{1/5} \frac{f(-q, -q^4)}{f(-q^2, -q^3)} = \frac{q^{1/5}}{1 + \frac{q}{1 + \frac{q^2}{1 + \frac{q^3}{1 + \dots}}}}, \quad |q| < 1. \quad (1.6)$$

The function  $R(q)$  was initially presented by L. J. Rogers [11] in 1894 and later rediscovered by Ramanujan in 1912. As an illustration, Ramanujan documented the subsequent general identity [4, pp. 24, Entry 12] related to continued fractions. Suppose that  $l, n$  and  $q$  are complex numbers with  $|ln| < 1$  and  $|q| < 1$ , or that  $l = n^{2m+1}$  for some integer  $m$ . Then,

$$\frac{(l^2 q^3; q^4)_\infty (n^2 q^3; q^4)_\infty}{(l^2 q; q^4)_\infty (n^2 q; q^4)_\infty} = \frac{1}{(1 - ln) + \frac{(l - nq)(n - lq)}{(1 - ln)(q^2 + 1) + \frac{(l - nq^3)(n - lq^3)}{(1 - ln)(q^4 + 1) + \dots}}}. \quad (1.7)$$

By defining the values of  $l$  and  $n$ , and selecting an appropriate value for  $q$ , it is possible to derive a  $q$ -continued fraction of a specific order that adheres to theta function identities similar to those associated with  $R(q)$ . The continued fractions of orders twenty-two and forty-four, which are obtained from the general continued fraction, are presented as follows. For  $t = 0, 1, 2, 3$  and  $5$ , we have

$$\begin{aligned} K_t(q) &= q^{(2t+2)/4} \frac{f(-q^{5-t}, -q^{17+t})}{f(-q^{6+t}, -q^{16-t})} \\ &= \frac{q^{(2t+2)/4} (1 - q^{5-t})}{(1 - q^{11/2}) + \frac{q^{11/2} (1 - q^{(2t+1)/2}) (1 - q^{(21-2t)/2})}{(1 - q^{11/2}) (1 + q^{11}) + \dots}} \end{aligned}$$

and

$$\begin{aligned} P_t(q) &= q^{5-t} \frac{f(-q^{2t+1}, -q^{43-2t})}{f(-q^{21-2t}, -q^{23+2t})} \\ &= \frac{q^5 (1 - q^{2t+1})}{(1 - q^{11}) + \frac{q^{11} (1 - q^{10-2t}) (1 - q^{12+2t})}{(1 - q^{11}) (1 + q^{22}) + \dots}} \end{aligned}$$

In 1978, B. Richmond and G. Szekeres [10] established the Hardy-Ramanujan expansions for the quotients of specific infinite products that emerged in the continued fraction expansions of the Rogers-Ramanujan type. And also they have proved that, if

$$\sum_{n=0}^{\infty} \delta_n q^n = \frac{(q^3, q^5; q^8)_{\infty}}{(q, q^7; q^8)_{\infty}} \quad \text{and} \quad \sum_{n=0}^{\infty} \zeta_n q^n = \frac{(q, q^7; q^8)_{\infty}}{(q^3, q^5; q^8)_{\infty}},$$

then the coefficients  $\delta_{4n+3}$  and  $\zeta_{4n+2}$  always vanishes. The occurrence of coefficients that diminish within a category of infinite products was initially investigated by M. D. Hirschhorn and subsequently by D. Tang [13], as well as N. D. Baruah and M. Kaur [3]. The results of K. Alladi and B. Gordon [1], M. C. Laughing [7] on vanishing coefficient used Ramanujan's well known  ${}_1\psi_1$  summation formula. Hirschhorn[6] presented a novel category of infinite  $q$ -products in his paper, characterized by the property that when the product is expressed as a series in  $q$ , the coefficients in one or more arithmetic progressions are equal to zero. One can see [3] and [6] for more details. The purpose of the paper is to prove some vanishing coefficient results for the continued fraction of order twenty two and forty four and their reciprocals which are obtained in the arithmetic progression of  $q$  series.

## 2. Vanishing coefficients in the series expansion

**Theorem 2.1.** *If*

$$\frac{1}{K_1^*(q)} = q^{-1/4} K_1(q) = \frac{(q^6, q^{16}; q^{22})_{\infty}}{(q^5, q^{17}; q^{22})_{\infty}} = \sum_{n=0}^{\infty} \xi'_n q^n,$$

*then we have,*

$$\xi'_{11n+7} = 0.$$

*Proof.* Andrews and Bressoud [2] stated the following  $p$ -dissection formula

$$\frac{(q^u, q^u, q^{v+x}, q^{u-v-x}; q^u)_{\infty}}{(q^x, q^{u-x}, q^v, q^{u-v}; q^u)_{\infty}} = \sum_{j=0}^{p-1} q^{jv} \frac{(q^{pu}, q^{pu}, q^{pv+x+ju}, q^{(p-j)u-pv-x}, q^{pu})_{\infty}}{(q^{ju+x}, q^{(p-j)u-x}, q^{pv}, q^{(u-v)p}, q^{pu})_{\infty}} \quad (2.1)$$

where all of the powers of  $q$  in each of the infinite products on the right hand side must be multiples of  $p$  and the integer  $v$  must satisfy  $\gcd(v, p) = 1$ . Now, by

substituting  $u = 22, x = 11, v = 5$  and  $p = 11$  into (2.1), we obtain

$$\begin{aligned} \frac{(q^{22}, q^{22}, q^{16}, q^6; q^{22})_\infty}{(q^{11}, q^{11}, q^5, q^{17}; q^{22})_\infty} &= \frac{(q^{242}, q^{242}, q^{66}, q^{176}; q^{242})_\infty}{(q^{11}, q^{231}, q^{55}, q^{187}; q^{242})_\infty} \\ &+ q^5 \frac{(q^{242}, q^{242}, q^{88}, q^{154}; q^{242})_\infty}{(q^{33}, q^{209}, q^{55}, q^{187}; q^{242})_\infty} + q^{10} \frac{(q^{242}, q^{242}, q^{132}, q^{110}; q^{242})_\infty}{(q^{55}, q^{187}, q^{55}, q^{187}; q^{242})_\infty} \\ &+ q^{15} \frac{(q^{242}, q^{242}, q^{132}, q^{110}; q^{242})_\infty}{(q^{77}, q^{165}, q^{55}, q^{187}; q^{242})_\infty} + q^{20} \frac{(q^{242}, q^{242}, q^{154}, q^{88}; q^{242})_\infty}{(q^{99}, q^{143}, q^{55}, q^{187}; q^{242})_\infty} \\ &+ q^{25} \frac{(q^{242}, q^{242}, q^{176}, q^{66}; q^{242})_\infty}{(q^{121}, q^{121}, q^{55}, q^{187}; q^{242})_\infty} + q^{30} \frac{(q^{242}, q^{242}, q^{198}, q^{44}; q^{242})_\infty}{(q^{143}, q^{99}, q^{55}, q^{187}; q^{242})_\infty} \\ &+ q^{35} \frac{(q^{242}, q^{242}, q^{220}, q^{22}; q^{242})_\infty}{(q^{165}, q^{77}, q^{187}, q^{165}; q^{242})_\infty} + q^{40} \frac{(q^{242}, q^{242}, q^{242}, q^0; q^{242})_\infty}{(q^{187}, q^{55}, q^{55}, q^{187}; q^{242})_\infty} \\ &+ q^{45} \frac{(q^{242}, q^{242}, q^{264}, q^{-22}; q^{242})_\infty}{(q^{209}, q^{33}, q^{55}, q^{187}; q^{242})_\infty} + q^{50} \frac{(q^{242}, q^{242}, q^{286}, q^{-44}; q^{242})_\infty}{(q^{231}, q^{11}, q^{55}, q^{187}; q^{242})_\infty}. \end{aligned}$$

Multiplying both sides by  $(q^{11}; q^{22})_\infty^2 / (q^{22}; q^{22})_\infty^2$  and then simplifying, we obtain

$$\begin{aligned} \sum_{n=0}^{\infty} \xi_n q^n &= \frac{(q^{11}, q^{55}, q^{187}, q^{231}; q^{242})_\infty (q^{33}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{209}; q^{242})_\infty^2}{(q^{66}, q^{176}; q^{242})_\infty (q^{22}, q^{44}, q^{88}, q^{110}, q^{132}, q^{154}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^5 \frac{(q^{33}, q^{55}, q^{187}, q^{209}; q^{242})_\infty (q^{11}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{231}; q^{242})_\infty^2}{(q^{88}, q^{154}; q^{242})_\infty (q^{22}, q^{44}, q^{66}, q^{110}, q^{132}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{10} \frac{(q^{11}, q^{33}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{110}, q^{132}; q^{242})_\infty (q^{22}, q^{44}, q^{66}, q^{88}, q^{154}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{15} \frac{(q^{55}, q^{77}, q^{165}, q^{187}; q^{242})_\infty (q^{11}, q^{33}, q^{99}, q^{121}, q^{143}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{88}, q^{154}; q^{242})_\infty (q^{22}, q^{44}, q^{66}, q^{110}, q^{132}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{20} \frac{(q^{55}, q^{99}, q^{143}, q^{187}; q^{242})_\infty (q^{11}, q^{33}, q^{77}, q^{121}, q^{165}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{88}, q^{154}; q^{242})_\infty (q^{22}, q^{44}, q^{66}, q^{110}, q^{132}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{25} \frac{(q^{55}, q^{187}; q^{242})_\infty (q^{11}, q^{33}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{66}, q^{176}; q^{242})_\infty (q^{22}, q^{44}, q^{88}, q^{110}, q^{132}, q^{154}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{30} \frac{(q^{55}, q^{99}, q^{143}, q^{187}; q^{242})_\infty (q^{11}, q^{33}, q^{77}, q^{121}, q^{165}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{44}, q^{198}; q^{242})_\infty (q^{22}, q^{66}, q^{88}, q^{110}, q^{132}, q^{176}, q^{220}; q^{242})_\infty^2} \\ &+ q^{35} \frac{(q^{55}, q^{77}, q^{165}, q^{187}; q^{242})_\infty (q^{11}, q^{33}, q^{99}, q^{121}, q^{143}, q^{209}, q^{231}; q^{242})_\infty^2}{(q^{22}, q^{220}; q^{242})_\infty (q^{44}, q^{66}, q^{88}, q^{110}, q^{132}, q^{154}, q^{176}, q^{198}; q^{242})_\infty^2} \\ &+ q^{45} \frac{(q^{-22}, q^{33}, q^{55}, q^{187}, q^{209}; q^{242})_\infty (q^{11}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{231}; q^{242})_\infty^2}{(q^{22}, q^{44}, q^{66}, q^{88}, q^{110}, q^{132}, q^{154}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2} \\ &+ q^{50} \frac{(q^{-44}, q^{11}, q^{55}, q^{187}, q^{231}; q^{242})_\infty (q^{33}, q^{77}, q^{99}, q^{121}, q^{143}, q^{165}, q^{209}; q^{242})_\infty^2}{(q^{22}, q^{44}, q^{66}, q^{88}, q^{110}, q^{132}, q^{154}, q^{176}, q^{198}, q^{220}; q^{242})_\infty^2}. \end{aligned}$$

where we used the result  $f(-1, a) = 0$  from [4, pp. 34, Entry 8(iii)]. Since the right hand side of the above equation does not contain any term involving  $q^{11n+7}$ . So extracting the terms involving  $q^{11n+7}$ , we arrive at the result.  $\square$

**Theorem 2.2.** *If*

$$P_1^*(q) = q^{-5}P_1(q) = \frac{(q, q^{43}; q^{44})}{(q^{21}, q^{23}; q^{44})} = \sum_{n=0}^{\infty} \delta_n q^n$$

then we have,

$$\delta_{22n+11} = 0.$$

*Proof.* Now, setting  $u = 44, x = 22, v = 21$  and  $p = 22$  in (2.1), we obtain

$$\begin{aligned} & \frac{(q^{44}, q^{44}, q^{43}, q; q^{44})_{\infty}}{(q^{22}, q^{22}, q^{21}, q^{23}; q^{44})_{\infty}} = \frac{(q^{968}, q^{968}, q^{484}, q^{484}; q^{968})_{\infty}}{(q^{22}, q^{946}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{21} \frac{(q^{968}, q^{968}, q^{528}, q^{440}; q^{968})_{\infty}}{(q^{66}, q^{902}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{42} \frac{(q^{968}, q^{968}, q^{572}, q^{396}; q^{968})_{\infty}}{(q^{110}, q^{858}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{63} \frac{(q^{968}, q^{968}, q^{616}, q^{352}; q^{968})_{\infty}}{(q^{154}, q^{814}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{84} \frac{(q^{968}, q^{968}, q^{660}, q^{308}; q^{968})_{\infty}}{(q^{198}, q^{770}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{105} \frac{(q^{968}, q^{968}, q^{704}, q^{264}; q^{968})_{\infty}}{(q^{242}, q^{726}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{126} \frac{(q^{968}, q^{968}, q^{748}, q^{220}; q^{968})_{\infty}}{(q^{286}, q^{682}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{147} \frac{(q^{968}, q^{968}, q^{792}, q^{176}; q^{22})_{\infty}}{(q^{330}, q^{638}, q^{506}, q^{165}; q^{968})_{\infty}} + q^{168} \frac{(q^{968}, q^{968}, q^{836}, q^{132}; q^{968})_{\infty}}{(q^{374}, q^{594}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{189} \frac{(q^{968}, q^{968}, q^{880}, q^{88}; q^{968})_{\infty}}{(q^{418}, q^{550}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{210} \frac{(q^{968}, q^{968}, q^{924}, q^{44}; q^{968})_{\infty}}{(q^{462}, q^{506}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{231} \frac{(q^{968}, q^{968}, q^{968}, q^0; q^{968})_{\infty}}{(q^{462}, q^{506}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{252} \frac{(q^{968}, q^{968}, q^{1012}, q^{-44}; q^{968})_{\infty}}{(q^{550}, q^{418}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{273} \frac{(q^{968}, q^{968}, q^{1056}, q^{-88}; q^{968})_{\infty}}{(q^{594}, q^{374}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{294} \frac{(q^{968}, q^{968}, q^{1100}, q^{-132}; q^{968})_{\infty}}{(q^{638}, q^{330}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{315} \frac{(q^{968}, q^{968}, q^{1144}, q^{-176}; q^{968})_{\infty}}{(q^{682}, q^{286}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{336} \frac{(q^{968}, q^{968}, q^{1188}, q^{-220}; q^{968})_{\infty}}{(q^{726}, q^{242}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{357} \frac{(q^{968}, q^{968}, q^{1232}, q^{-264}; q^{968})_{\infty}}{(q^{770}, q^{198}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{378} \frac{(q^{968}, q^{968}, q^{1276}, q^{-308}; q^{968})_{\infty}}{(q^{814}, q^{154}, q^{462}, q^{506}; q^{968})_{\infty}} \\ & + q^{399} \frac{(q^{968}, q^{968}, q^{1320}, q^{-352}; q^{968})_{\infty}}{(q^{858}, q^{110}, q^{462}, q^{506}; q^{968})_{\infty}} + q^{420} \frac{(q^{968}, q^{968}, q^{1364}, q^{-396}; q^{968})_{\infty}}{(q^{906}, q^{66}, q^{462}, q^{506}; q^{968})_{\infty}}. \end{aligned}$$

Multiplying  $(q^{22}; q^{44})_{\infty}^2 / (q^{44}; q^{44})_{\infty}^2$  on both sides and using  $f(-1, a) = 0$ , then extracting the terms involving  $q^{22n+11}$ , we arrive at the result.  $\square$

**Remark** The following table represent the remaining vanishing coefficients in  $q$ -series for the continued fraction.

$q$ -series/continued fractions	vanishing coefficients
$K_2^*(q) = q^{-3/4}K_2(q) = \frac{(q^4, q^{18}; q^{22})}{(q^7, q^{15}; q^{22})} = \sum_{n=0}^{\infty} \varrho_n q^n$	$\varrho_{11n+5} = 0$

$\frac{1}{K_3^*(q)} = q^{-5/4} K_3(q) = \frac{(q^8, q^{14}; q^{22})}{(q^3, q^{19}; q^{22})} = \sum_{n=0}^{\infty} \Omega'_n q^n$	$\Omega'_{11n+5} = 0$
$K_4^*(q) = q^{-7/4} K_4(q) = \frac{(q^2, q^{20}; q^{22})}{(q^9, q^{13}; q^{22})} = \sum_{n=0}^{\infty} \nu_n q^n$	$\nu_{11n+10} = 0$
$\frac{1}{K_5^*(q)} = q^{-9/4} K_5(q) = \frac{(q, q^{21}; q^{22})}{(q^{10}, q^{12}; q^{22})} = \sum_{n=0}^{\infty} \varsigma'_n q^n$	$\varsigma'_{11n+10} = 0$
$\frac{1}{P_1^*(q)} = \sum_{n=0}^{\infty} \xi'_n q^n$	$\xi'_{22n+21} = 0$
$P_2^*(q) = q^{-4} P_2(q) = \frac{(q^3, q^{41}; q^{44})}{(q^{19}, q^{25}; q^{44})} = \sum_{n=0}^{\infty} \omega_n q^n$	$\omega_{22n+8} = 0$
$\frac{1}{P_2^*(q)} = \sum_{n=0}^{\infty} \omega'_n q^n$	$\omega'_{22n+16} = 0$
$P_3^*(q) = q^{-3} P_2(q) = \frac{(q^5, q^{39}; q^{44})}{(q^{17}, q^{27}; q^{44})} = \sum_{n=0}^{\infty} \alpha_n q^n$	$\alpha_{22n+1} = 0$
$\frac{1}{P_3^*(q)} = \sum_{n=0}^{\infty} \alpha'_n q^n$	$\alpha'_{22n+7} = 0$
$P_4^*(q) = q^{-2} P_4(q) = \frac{(q^7, q^{37}; q^{44})}{(q^{15}, q^{29}; q^{44})} = \sum_{n=0}^{\infty} \eta_n q^n$	$\eta_{22n+12} = 0$
$\frac{1}{P_4^*(q)} = \sum_{n=0}^{\infty} \eta'_n q^n$	$\eta'_{22n+16} = 0$
$P_5^*(q) = q^{-1} P_5(q) = \frac{(q^9, q^{35}; q^{44})}{(q^{13}, q^{31}; q^{44})} = \sum_{n=0}^{\infty} \delta_n q^n$	$\delta_{22n+19} = 0$
$\frac{1}{P_5^*(q)} = \sum_{n=0}^{\infty} \delta'_n q^n$	$\delta'_{22n+21} = 0$

**Conclusion** Andrews and Bressoud [2] in their paper defined, consider integers  $i$  and  $k$  such that  $1 \leq i < j$ , where  $i$  and  $j$  are coprime with opposite parity and

$$\frac{(q^i, q^{2j-i}; q^{2j})_{\infty}}{(q^{j-i}, q^{j+i}; q^{2j})_{\infty}} = \sum_{n=0}^{\infty} \phi_n q^n, \quad (2.2)$$

then  $\phi_{jn+i(j-i+1)/2} = 0$ . The results mentioned in this paper can also be obtained using (2.2).

## References

1. K. Alladi, B. Gordon, Vanishing Coefficients in the Expansion of Products of Rogers-Ramanujan Type, *In: Rademacher Legacy in Mathematics (University Park, PA, 1992), Contemporary Mathematics* **166** (1994) 129-139.

2. G. E. Andrews, D. Bressoud, Vanishing coefficients in infinite product expansion, *J. Aust. Math. Soc. Ser. 27* (1979) 199-202.
3. N. D. Baruah, M. Kaur, Some results on vanishing coefficients in infinite product expansions, *Ramanujan J. 53* (2020) 551-568.
4. B. C. Berndt, Ramanujan's Notebooks, Part III, Springer, New York, 1991.
5. B. C. Berndt, Ramanujan's Notebooks, Part V, Springer, New York, 1998.
6. M. D. Hirschhorn, Two remarkable  $q$ -series expansions, *Ramanujan J. 49(2)* (2018) 451-463.
7. Mc. Laughlin, Further results on vanishing coefficients in infinite product expansions, *J. Aust. Math. Soc. Ser. A 98* (2015) 69-77.
8. S. Rajkhowa, N. Saikia, Some results on Ramanujan's continued fractions of order ten and applications, *Indian J. Pure Appl. Math.* <https://doi.org/10.1007/s13226-023-00456-5> (2023).
9. S. Rajkhowa, Saikia, Theta-function identities of Ramanujan's continued fractions of order fourteen and twenty eight, partition identities and vanishing coefficients, *Funct. Approx. Comment. Math. 70(2)* (2024) 233-244.
10. B. Richmond, G. Szekeres, The Taylor coefficients of certain infinite products, *Acta Sci. Math. (Szeged) 40* (1978) 347-369.
11. L. J. Rogers, Second memoir on the expansion of certain infinite products, *Proc. London Math. Soc. 25* (1894) 318-343.
12. M. S. Surekha, On the modular relations and dissections for a continued fraction of order sixteen, *Palestine Journal of Mathematics 6(1)* (2017) 119-132.
13. D. Tang, Vanishing coefficients in some  $q$ -series expansions, *Int. J. Number Theory 15(4)* (2019) 763-773.

DEPARTMENT OF MATHEMATICS, MANIPAL INSTITUTE OF TECHNOLOGY, MANIPAL ACADEMY OF HIGHER EDUCATION, MANIPAL -576 102, INDIA  
 Email address: [anu.radha@manipal.edu](mailto:anu.radha@manipal.edu)