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SIGNED INTERPOLATING DEFLATORS AND HAAR UNIQUENESS PROPERTIES

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Abstract. Currently, the development of the theory of Haar interpolations of financial markets with the use of martingale measures continues. The existence of martingale measures of discounted stock prices means that this kind of interpolation can only be used in complete markets. However, real financial markets often contain elements of arbitrage opportunities. Therefore, it is important to develop techniques for interpolating processes that do not admit martingale measures. This work is devoted to just this problem. Here, signed deflators serve as the main interpolation tool. With their help, the Haar interpolation procedure is defined. In the case of the existence of martingale measures, this procedure leads to the process interpolation, which coincides with the martingale interpolation. The paper introduces the concept of an admissible deflator, defines (as when martingale measures exist) the universal Haar uniqueness property and its weakened variants. The main results of the work are related to the so-called special Haar uniqueness property, which leads to the uniqueness of the admissible deflator.

1. Introduction

Consider a stochastic basis $\big(\Omega, F = (\mathcal{F}_k)_{k=0}^K, P\big)$, where Ω be a set, $F = (\mathcal{F}_k)_{k=0}^K$ be a strictly increasing filtration, $\mathcal{F}_0 = \{\Omega, \emptyset\}$, $K \leq \infty$, any $\mathcal{F}_k$ $(0 \leq k < K+1)$ be finite, and P be a probability on $\mathcal{F}_K$ (if $K = \infty$, then $\mathcal{F}_K = \mathcal{F}_\infty$ is the least σ-algebra containing all $\mathcal{F}_k$, $0 \leq k < \infty$). We assume that the probability measure P loads all non-empty subsets from $\mathcal{F}_k$, $0 \leq k < K+1$.

Definition 1.1. Let $Z = (Z_k, \mathcal{F}_k)_{k=0}^K$ be an adabted process that can take any real values. A martingale $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ is said a signed deflator of the process Z if D_0=1 and the process $DZ = (D_k Z_k, \mathcal{F}_k, P)_{k=0}^K$ is a martingale.

We will also consider on $(\Omega, \mathcal{F}_K)$ Haar filtrations (HF)

$$H = (\mathcal{H}_n)_{n=0}^L,\ \mathcal{H}_n \subset \mathcal{F}_K, \tag{1.1}$$

where $\mathcal{H}_0 = \{\Omega, \emptyset\}$ and each σ-algebra $\mathcal{H}_n$ is generated by a partition of the set Ω into exactly $n+1$ atoms $H_0^n, H_1^n, ..., H_n^n$. A Haar filtration is said special

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Haar filtration if at every moment $n > 1$ only those two atoms of $H_0^n, H_1^n, ..., H_n^n$ can be divided that were obtained by division at the previous moment $n-1$. Haar filtration $(\mathcal{H}_n)_{n=0}^L$ from (1.1) is said interpolating Haar filtration (IHF) of $(\mathcal{F}_k)_{k=0}^K$ if there exists an increasing sequence of integers $n_k, 0 \le k < K+1$, such that $\mathcal{H}_{n_k} = \mathcal{F}_k$ (and hence $\mathcal{H}_L = \mathcal{F}_K$). Special interpolating Haar filtration (SIHF) is defined analogically.

Let us fix an IHF $(\mathcal{H}_n)_{n=0}^L$ of $(\mathcal{F}_k)_{k=0}^K$ and let $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ be a signed deflator of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^K$. Denoting $X_{n_k} := D_k Z_k$ and $Y_{n_k} := D_k$, we obtain martingales $(X_{n_k}, \mathcal{H}_{n_k}, P)_{k=0}^K$ and $(Y_{n_k}, \mathcal{H}_{n_k}, P)_{k=0}^K$. Then we can define two martingales $X = (X_n, \mathcal{H}_n, P)_{n=0}^L$ and $Y = (Y_n, \mathcal{H}_n, P)_{n=0}^L$ in the following obvious way: for any $n < L+1$ find $n_k \ge n$ and put

$$X_n := E^P[X_{n_k}|\mathcal{H}_n],\ Y_n := E^P[Y_{n_k}|\mathcal{H}_n]. \tag{1.2}$$

It is clear that such definitions are correct.

Remark 1.2. From the properties of the mathematical expectation it follows the implication:

$$\{D_k = 0\} \subset \{D_k Z_k = 0\}\ (P - a.s.) \Rightarrow \{Y_n = 0\} \subset \{X_n = 0\}\ (P - a.s.). \tag{1.3}$$

Definition 1.3. The process $Z^{int} = (Z_n^{int}, \mathcal{H}_n)_{n=0}^L$ defined by the formula

$$Z_n^{int} = \begin{cases} Z_k,\ if\ n = n_k\ (0 \le k < K+1), \\ \dfrac{X_n}{Y_n},\ if\ n \ne n_k, Y_n \ne 0, \\ 1, \quad if\ n \ne n_k, Y_n = 0, \end{cases} \tag{1.4}$$

will be called H-interpolation of the process Z with the help of the deflator D.

Remark 1.4. Let the process $Z = (Z_k, (\mathcal{F}_k)_{k=0}^K)$ admit a martingale measure Q, equivalent to the physical measure P, i.e. the process $(Z_k, \mathcal{F}_k, Q)_{k=0}^K$ be a martingale. Denote $h := \dfrac{dQ}{dP}$ and $D_k := E^P[h|\mathcal{F}_k]$. It is clear that the process $D = (D_k, \mathcal{F}_k)_{k=0}^K$ is a strictly positive deflator of the process Z. Hence for all $n \le n_k$ $Y_n = E^P[Y_{n_k}|\mathcal{H}_n] = E^P[D_k|\mathcal{H}_n] > 0$ and $Z_n^{int} = \dfrac{X_n}{Y_n}$. Applying the generalized Bayes formula, it is easy to see that the process $(Z_n^{int}, \mathcal{H}_n, Q)_{n=0}^L$ is a martingale. From this fact it follows that H-interpolation of the process Z with the help of deflator D coincides with the Haar interpolation of Z with respect to the martingale measure Q (c.f. [1], [2]).

Proposition 1.5. *The process $Y = (Y_n, \mathcal{H}_n, P)_{n=0}^L$ is a signed deflator of the process $Z^{int} = (Z_n^{int}, \mathcal{H}_n)_{n=0}^L$.*

Proof. The proof follows from the equality (1.4) and from Remark 1.2. □

2. Haar uniqueness properties for deflators

Definition 2.1. We say that a signed deflator $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ satisfies the Haar uniqueness property (HUP) if there exists a Haar interpolation $H = (\mathcal{H}_n)_{n=0}^L$ of the initial filtration F such that the process (1.4) admits only one deflator, namely the deflator $Y = (Y_n, \mathcal{H}_n, P)_{n=0}^L$, defined by (1.2).

Definition 2.2. We say that a signed deflator $D = (D_k, \mathcal{F}_k, P)_{k=0}^K$ satisfies the universal Haar uniqueness property — UHUP (resp., the special Haar uniqueness property — SHUP) if for every interpolating (resp., special interpolating) Haar filtration $H = (\mathcal{H}_n)_{n=0}^L$ of the initial filtration F the process (1.4) admits only one deflator, namely the deflator $Y = (Y_n, \mathcal{H}_n, P)_{n=0}^L$, defined by (1.2).

We use in the sequel the following system of notations. Let A be an atom in $\mathcal{F}_k$, B_i $(i = 1, 2, \dots, m)$ be atoms in $\mathcal{F}_{k+1}$,

$$A = B_1 + B_2 + \cdots + B_m, a := Z_k|_A, b_i := Z_{k+1}|_{B_i}, p_i := P(B_i), d_i := D_{k+1}|_{B_i}.$$

Generally splitting index m of atom A and numbers a, b_i, p_i, d_i depend on A.

Definition 2.3. A signed deflator D of the process Z is said admissible if $\forall 0 \le k < K+1$, for all atom $A \in \mathcal{F}_k$ and for all non-empty subset $I \subset \{1, 2, \dots, m\}$

$$\sum_{i \in I} p_i d_i \neq 0.$$

The aim of this paper is to prove the following theorems.

Theorem 2.4. *Let $\forall k : 0 \le k < K+1$ and for all atom $A \in \mathcal{F}_k$ we have $m \ge 3$. If there exists an admissible signed deflator D satisfying SHUP, then the numbers $a, b_1, \dots, b_m$ are different.*

Theorem 2.5. *Let $\forall k : 0 \le k < K+1$ and for all atom $A \in \mathcal{F}_k$ we have $m \ge 4$ and the numbers $a, b_1, \dots, b_m$ be different. Then there exists an admissible signed deflater D satisfying SHUP.*

The problem of the existence of admissible deflators satisfying UHUP will be considered in subsequent works.

3. Admissible deflators of one-step process Z

For one-step processes we have $A = \Omega$. Let $D = (D_k, \mathcal{F}_k, P)_{k=0}^1$ be an admissible deflator of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^1$, $D_1 = \sum_{i=1}^m d_i I_{B_i}$, $Z_1 = \sum_{i=1}^m b_i I_{B_i}$, $m \ge 3$.

Proposition 3.1. *Let $m = 3$ and numbers b_1, b_2, b_3 are different. All admissible deflators $D = (D_k, \mathcal{F}_k)_{k=0}^1$ of the process Z are given by the equalities:*

$$\begin{cases} d_1 = \dfrac{b_2 - a + p_3(b_3 - b_2)d_3}{p_1(b_2 - b_1)} \\ d_2 = \dfrac{a - b_1 + p_3(b_3 - b_1)d_3}{p_1(b_2 - b_1)}, \end{cases} \tag{3.1}$$

where d_3 can take any real values, except

$$0,\ \frac{1}{p_3},\ -\frac{b_1 - a}{p_3(b_3 - b_2)},\ -\frac{b_2 - a}{p_3(b_3 - b_2)},\ -\frac{b_1 - a}{p_3(b_3 - b_1)},\ -\frac{b_2 - a}{p_3(b_3 - b_1)}.$$

Proof. It is obvious that the formula (3.1) gives all the signed deflators of Z. On the other hand, it is clear that the deflator D is admissible if and only if $d_i \neq 0, d_i \neq \frac{1}{p_i}$ $(i = 1, 2, 3)$. Passing with the help of the formulas (3.1) from inequalities $d_i \neq 0, d_i \neq \frac{1}{p_i}$ $(i = 1, 2)$ to equivalent inequalities for d_3, we obtain what is required. □

Proposition 3.2. *Let $m \geq 4$ and numbers $b_1, \dots, b_m$ are different. Then there exist admissible deflators of the process Z.*

Proof. A process D is a deflator of the process Z iff

$$\begin{cases} p_1d_1 + p_2d_2 + p_3d_3 + \dots + p_md_m = 1 \\ b_1p_1d_1 + b_2p_2d_2 + b_3p_3d_3 + \dots + b_mp_md_m = a. \end{cases} \tag{3.2}$$

This deflator is admissible iff for all non-empty subset $I \subset \{1, 2, \dots, m\}$ we have $\sum_{i \in I} p_i d_i \neq 0$. Let the last sum lack at least two terms. Fix two of them. Without loss of generality, we can assume that these will be p_1d_1 and p_2d_2.

Let us solve system (3.2) with respect to d_1 and d_2 (if the indicated sum did not contain the terms $p_{i_1}d_{i_1}$ and $p_{i_2}d_{i_2}$, we would resolve (3.2) with respect to d_{i_1} and d_{i_2}). We have:

$$\begin{cases} d_1 = \frac{(b_3-b_2)p_3}{(b_2-b_1)p_1}d_3 + \frac{(b_4-b_2)p_4}{(b_2-b_1)p_1}d_4 + \dots + \frac{(b_m-b_2)p_m}{(b_2-b_1)p_1}d_m + b_2 - a \\ d_2 = -\frac{(b_3-b_1)p_3}{(b_2-b_1)p_2}d_3 - \frac{(b_4-b_1)p_4}{(b_2-b_1)p_2}d_4 - \dots - \frac{(b_m-b_1)p_m}{(b_2-b_1)p_2}d_m - (b_1 - a). \end{cases} \tag{3.3}$$

The set of solutions (3.3) of the system (3.2) is a hyperplane in the space R^m. We represent this hyperplane in the parametric form:

$$\begin{cases} d_1 = \frac{(b_3-b_2)p_3}{(b_2-b_1)p_1}t_1 + \frac{(b_4-b_2)p_4}{(b_2-b_1)p_1}t_2 + \dots + \frac{(b_m-b_2)p_m}{(b_2-b_1)p_1}t_{m-2} + b_2 - a \\ d_2 = -\frac{(b_3-b_1)p_3}{(b_2-b_1)p_2}t_1 - \frac{(b_4-b_1)p_4}{(b_2-b_1)p_2}t_2 - \dots - \frac{(b_m-b_1)p_m}{(b_2-b_1)p_2}t_{m-2} - (b_1 - a) \\ d_3 = t_1 \\ \dots\dots\dots \\ d_m = t_{m-2}. \end{cases} \tag{3.4}$$

Denote the hyperplane (3.4) by T. It is clear that the $m-2$ of n-dimensional vectors generating T are linearly independent. Hence T has the dimension $m-2$.

Without loss of generality, we will assume that under the sign of the sum in the inequality $\sum_{i \in I} p_i d_i \neq 0$, there is a term p_3d_3. Using the notation of the parameters in the formula (3.4) and turning the inequality under consideration into an equality, we express t_1 in terms of the remaining parameters, included in this equality. Thus, (3.4) turns into a parametric equation of a hyperplane T', contained in T and having a dimension strictly less than $m-2$.

Now concider the case when the sum in the inequality $\sum_{i \in I} p_i d_i \neq 0$ does not contain only one term. Without loss of generality, we can suppose that it is p_3d_3. Then this inequality is equivalent to the inequality $d_3 \neq \frac{1}{p_3}$. Substituting $t_1 = \frac{1}{p_3}$ in (3.4), we again get a hyperplane $T'' \subset T$ with a dimention strictly less than $m-2$.

It follows from what has been said that all the inequalities characterizing the admissible deflators are satisfied at the points of the hyperplane T after removing from it a finite number of hyperplanes of the type T' and T'' of dimensions, strictly less than the dimension of T. It is obvious that the set of all such points is not empty.

4. Proof of the theorems

Remark that the proofs of Theorems 2.4 and 2.5 for dinamic models can be reduced to static models by the standart way (c.f. [1], [2]). Therefore, we will carry out proofs only for one-step processes.

Lemma 4.1. *Let $D = (D_k, \mathcal{F}_k, P)_{k=0}^{1}$ be an admissible deflator of the process $Z = (Z_k, \mathcal{F}_k)_{k=0}^{1}$, $D_1 = \sum_{i=1}^{m} d_i I_{B_i}$, $Z_1 = \sum_{i=1}^{m} b_i I_{B_i}$, $m \geq 3$. Consider SIHF $H = (\mathcal{H}_n)_{n=0}^{m-1}$ of the form:*

$\mathcal{H}_0 = \mathcal{F}_0 = \{\Omega, \emptyset\}$,
$\mathcal{H}_1 = \sigma\{B_1\}$,
$\mathcal{H}_2 = \sigma\{B_1, B_2\}$,
....
$\mathcal{H}_{m-1} = \sigma\{B_1, B_2, \ldots, B_{m-1}\} = \mathcal{F}_1$.

If $Z^{int} = (Z_n^{int}, \mathcal{H}_n)_{n=0}^{m-1}$ is H-interpolation of the process Z with the help of the deflator D (see Definition 1.3), then any deflator $\tilde{Y} = (\tilde{Y}_n, \mathcal{H}_n)_{n=0}^{m-1}$ of the process Z^{int} can be obtained by the formula $\tilde{Y}_{m-1} = \sum_{i=1}^{m} x_i I_{B_i}$ from the system

$$\begin{cases} \sum\limits_{i=1}^{m} p_i x_i = 1 \\ \sum\limits_{i=1}^{m} b_i p_i x_i = a \\ \sum\limits_{i=2}^{m} p_i (b_i - c_1) x_i = 0 \\ \sum\limits_{i=3}^{m} p_i (b_i - c_2) x_i = 0 \\ \ldots\ldots\ldots\ldots\ldots\ldots \\ \sum\limits_{i=m-1}^{m} p_i (b_i - c_{m-2}) x_i = 0, \end{cases} \tag{4.1}$$

where

$$c_s = \frac{\sum\limits_{j=s+1}^{m} b_j p_j d_j}{\sum\limits_{j=s+1}^{m} p_j d_j}, \; s = 1, 2, \ldots, m-2.$$

Proof. The scheme of the proof of the lemma is as follows. First, using the formulas (1.2) and (1.4), we calculate Z^{int}. Then we calculate the martingale $\tilde{Y} = (\tilde{Y}_n, \mathcal{H}_n)_{n=0}^{m-1}$ by solving the probabilistic Dirichlet problem with the boundary value $\tilde{Y}_{m-1} = \sum_{i=1}^{m} x_i I_{B_i}$ and multiply this martingale by the process Z^{int}. And, finally, complementing the first two equations of the system (4.1) with martingale equalities for the process $\tilde{Y} Z^{int}$, we obtain the system (4.1). □

Lemma 4.2. *The main determinant of the system* (4.1) *is calculated by the formula*

$$\Delta_{\{1,2,\ldots,m\}} = \prod_{i=1}^{m} p_i \cdot \prod_{i=1}^{m-2} (c_i - b_i) \cdot (b_m - b_{m-1}). \tag{4.2}$$

Proof. It is enough to apply a number of elementary transformations to the main determinant of the system (4.1).

Now we can prove Theorem 2.4.

Proof. Let there exist an admissible signed deflater D satisfying SHUP. Concider first SIHF as in lemma 4.1. By Proposition 1.5 the process Y from (1.2) satisfies the system (4.1). It follows from Definition 2.2 that this solution is unique, i.e. its main determinant $\Delta_{\{1,2,\dots,m\}}$ is not zero. We get from Lemma 4.2 that $b_{m-1} \neq b_m$. Now calculate the factor $c_1 - b_1$ in (4.2). Putting $x_i = d_i$, $i = 1, 2, \dots, m$, in two first equations of (4.1) we obtain:

$$c_1 - b_1 = \frac{\sum\limits_{j=2}^{m} b_i p_i d_i}{\sum\limits_{j=2}^{m} p_i d_i} - b_1 = \frac{a - b_1 p_1 d_1}{1 - p_1 d_1} - b_1 = \frac{a - b_1}{1 - p_1 d_1}. \tag{4.3}$$

Applying again Lemma 4.2, we get $a \neq b_1$.

Denote $J = \{1, 2, \dots, m\}$. Let $J' = \{j_1, j_2, \dots, j_m\}$ be a permutation of J. Put $\mathcal{H}_0 = \mathcal{F}_0 = \{\Omega, \emptyset\}$, $\mathcal{H}_1 = \sigma\{B_{j_1}\}$, $\mathcal{H}_2 = \sigma\{B_{j_1}, B_{j_2}\}$, $\dots$, $\mathcal{H}_{m-1} = \sigma\{B_{j_1}, B_{j_2}, \dots, B_{j_{m-1}}\} = \mathcal{F}_1$. Lemmas 4.1 and 4.2 and the transformation (4.3) give us in this general situation $b_{j_{m-1}} \neq b_{j_m}$ and $a \neq b_{j_1}$. It means that the numbers $a, b_1, \dots, b_m$ are different. □

Before formulating the next proposition, we note that in the case $m = 3$ UHUP coincides with SHUP.

Proposition 4.3. *Let $m = 3$ and the numbers $a, b_1, \dots, b_m$ are different. All admissible deflators D of the process Z (see Proposition 3.1) satisfy UHUP.*

Proof. This fact follows from Lemmas 4.1 and 4.2 and from the proof of Theorem 2.4. Remark only that in this case in the expression 4.2 $\prod\limits_{i=1}^{m-2}(c_i - b_i) = \frac{a-b_1}{1-p_1 d_1} \neq 0$. □

Now let us prove Theorem 2.5.

Proof. Without loss of generality, we can assume that $b_i \neq 0$, $i = 1, 2, \dots, m$.

Let $J = \{1, 2, \dots, m\}$ and let $J' = \{j_1, j_2, \dots, j_m\}$ be an arbitrary permutation of J. We proceed as in the second part of the proof of Theorem 2.4. Then determinant (4.2) takes the form:

$$\Delta_{J'} = \prod_{i=1}^{m} p_{j_i} \cdot \prod_{i=1}^{m-2} (c_{j_i} - b_{j_i}) \cdot (b_{j_m} - b_{j_{m-1}}), \tag{4.4}$$

where

$$c_{j_s} = \frac{\sum\limits_{i=s+1}^{m} b_{j_i} p_{j_i} d_{j_i}}{\sum\limits_{j=s+1}^{m} p_{j_i} d_{j_i}}, \; s = 1, 2, \dots, m-2.$$

It is clear that a deflator satisfies SHUP iff for any permutation J' of J the inequalities $\sum_{i=s+1}^{m} b_{j_i} p_{j_i} d_{j_i} \neq 0$, $s = 2, \dots, m-2$, are fulfilled. Solving them with respect to $d_{j_{s+1}}$, we get the equivalent inequalities:

$$d_{j_{s+1}} \neq \frac{\sum_{i=s+2}^{m} b_{j_i} p_{j_i} d_{j_i}}{b_{j_{s+1}} p_{j_{s+1}}}, \quad s = 2, \dots, m-2. \tag{4.5}$$

Then we proceed as in Proposition 3.2. Represent the hyperplane T, given by system (3.2), in the form:

$$\begin{cases} d_{j_1} = \frac{(b_{j_3}-b_{j_2})p_{j_3}}{(b_{j_2}-b_{j_1})p_{j_1}} t_1 + \frac{(b_{j_4}-b_{j_2})p_{j_4}}{(b_{j_2}-b_{j_1})p_{j_1}} t_2 + \cdots + \frac{(b_{j_m}-b_{j_2})p_{j_m}}{(b_{j_2}-b_{j_1})p_{j_1}} t_{m-2} + b_{j_2} - a \\ d_{j_2} = -\frac{(b_{j_3}-b_{j_1})p_{j_3}}{(b_{j_2}-b_{j_1})p_{j_2}} t_1 - \frac{(b_{j_4}-b_{j_1})p_{j_4}}{(b_{j_2}-b_{j_1})p_{j_2}} t_2 - \cdots - \frac{(b_{j_m}-b_{j_1})p_{j_m}}{(b_{j_2}-b_{j_1})p_{j_2}} t_{m-2} - (b_{j_1} - a) \\ d_{j_3} = t_1 \\ \dots\dots\dots\dots\dots\dots\dots\dots \\ d_{j_m} = t_{m-2}. \end{cases} \tag{4.6}$$

Putting in the inequalities (4.5) $d_{j_{s+1}} = t_{s-1}$, $d_{j_{s+2}} = t_s, \dots$, $d_{j_m} = t_{m-2}$ and turning these inequalities into equalities, we obtain:

$$t_{s-1} = \frac{\sum_{i=s+2}^{m} b_{j_i} p_{j_i} t_{i-2}}{b_{j_{s+1}} p_{j_{s+1}}}, \quad s = 2, \dots, m-2. \tag{4.7}$$

Now fix s and put (4.7) in (4.6). As a result, we obtain the paramrtric equation of some hyperplane T''' with dimension strictly less than $m-2$. It follows from this that all the admissible deflators satisfying SHUP are the points of the hyperplane T after removing from it a finite number of hyperplanes of the type T', T'' and T''' of dimensions, strictly less than the dimension of T (c.f. the denotations from the proof of Proposition 3.2). It is obvious that the set of all such points is not empty. □

Remark 4.4. In fact, it follows from the proof of Theorem 2.5 that if $m \geq 4$ and numbers $a, b_1, \dots, b_m$ are different, then the set of admissible deflators satisfying SHUP is dense in the set of all signed deflators in the Euclidean metric of the hyperplane T. But unlike the case of $m = 3$, there are admissible deflators that do not satisfy SHUP. Let us consider the case $m = 4$ and $b_1 < b_2 < b_3 < b_4$. It is clear that if there exists an admissible deflator for which $c_2 = b_2$ (see Lemma 4.2), then this deflator does not satisfy SHUP. It is easy to show that such deflators are odtained from the system

$$\begin{cases} d_1 = \frac{(b_2-a)}{(b_2-b_1)p_1} \\ d_2 = \frac{(a-b_1)(b_3-b_2)+(b_3-b_1)(b_4-b_2)p_4 d_4}{(b_2-b_1)(b_3-b_2)p_2} \\ d_3 = -\frac{(b_4-b_2)p_4 d_4}{(b_3-b_2)p_3} \end{cases} \tag{4.8}$$

for all real values of d_4, except for

$$\frac{(b_1-a)(b_3-b_2)}{(b_3-b_1)(b_4-b_2)p_4},\ \frac{1}{p_4},\ -\frac{(b_3-b_2)}{(b_4-b_2)p_4},\ \frac{(b_2-a)(b_3-b_2)}{(b_3-b_1)(b_4-b_2)p_4},\ \frac{(b_1-a)(b_3-b_2)}{[(b_3-b_1)(b_4-b_2)+(b_2-b_1)(b_3-b_2)]p_4},$$
$$-\frac{(b_2-a)}{(b_2-b_1)p_4},\ \frac{(b_1-a)}{(b_4-b_2)p_4},\ \frac{(b_2-a)(b_3-b_2)}{(b_2-b_1)(b_4-b_2)p_4},\ -\frac{(b_2-b_1)(b_3-b_2)}{(b_3-b_1)(b_4-b_2)p_4},\ 0.$$

5. Conclusion

The most important problem in the Haar interpolations topic consists in the following: find the sufficient conditions on parameters of the market, under which various interpolating deflators exist (for interpolating martingale measures see [1]-[3] (the case of finite probability space) and [4]-[11] (the case of countable probability space)). These deflators can be used in investigation not only of financial markets, but of other complex systems (for example, considered in [12]-[15]).

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