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ONE CASE OF QUASI-CORRECTNESS OF THE CANONICAL BOUNDARY VALUE PROBLEM OF THE MEMBRANE THEORY OF CONVEX SHELLS

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ABSTRACT. In this paper we find the geometrical criterion of quasi-stability for one class of problems in the membrane theory of convex shells.

1. Introduction

The static boundary problem for a thin elastic spherical shell with a piecewise smooth boundary (*a spherical dome*) was first posed by Vlasov [1] and Goldenweiser [2] in the framework of the momentless technical theory of shells. The mathematical formulation of this problem and its complete solution are given in [3, 4] using the method of generalized analytic functions [5]. As established in [6], the patterns of solvability of the boundary problems for spherical domes and convex domes of general form have essential differences. In the general case, the standard algorithm for calculating the index of the corresponding Riemann–Hilbert boundary condition for the momentless equilibrium equation does not allow one to obtain effective formulas for its calculation. For some special classes of convex domes this problem is solved in [7, 8]. In particular, for *canonical* domes the quasi-correctness of the main boundary value problem is established. In the present paper we introduce new concepts which allow us to give a complete picture of the solvability of the quasi-correct problem for canonical domes and to refine the results [8].

2. The boundary problem R

Let S — be a simply connected surface with piecewise smooth edge $L = \bigcup_{j=1}^{n} L_j$ and corner points p_i $(i = 1,\dots, n)$. We assume that S is the inner part of the surface S_0 of strictly positive Gaussian curvature of regularity class $W^{3,r}$, $r > 2$, and each of the curves L_j belongs to the class $C^{1,\varepsilon}$, $0 < \varepsilon < 1$. We define a piecewise continuous vector field $\boldsymbol{r} = \{\alpha(s), \beta(s)\}$ on S along L, allowing break points of the first kind at p_j, with tangent and normal components $\alpha(s)$, $\beta(s)$ $(\alpha^2 + \beta^2 = 1,\ \beta \geqslant 0)$, where s is a natural parameter, functions $\alpha(s)$, $\beta(s)$ are Hölder ones on every curve L_j.

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We introduce the following notations: Let J be the map of the surface S_0 onto a complex plane $z = x + iy$, determined by the selection of the conformal isometric parametrization (x, y) on S_0, $D = J(S)$ be a simply connected field limited by the complex surface z with boundary $\Gamma = \bigcup\limits_{j=1}^{n} J(L_j)$ and the corner points $q_i = J(p_i)$.

Let us consider the following problem (problem R): find the complex-valued solution $w(z)$ of the equation (the *bending function* of surface S) in the field D

$$w_{\bar{z}}(z) - B(z)\bar{w}(z) = F(z), \quad z \in D, \tag{2.1}$$

by the given Riemann–Hilbert boundary condition

$$\mathrm{Re}\big\{\lambda(\zeta)w(\zeta)\big\} = \gamma(\zeta), \tag{2.2}$$

where

$$\lambda(\zeta) = s(\zeta)\big[\beta(\zeta)t(\zeta) - \alpha(\zeta)s(\zeta)\big], \tag{2.3}$$

$s(\zeta) = s_1(\zeta) + is_2(\zeta)$, $t(\zeta) = t_1(\zeta) + it_2(\zeta)$, $i^2 = -1$, s_i $(i = 1,\ 2)$ — are the coordinates of the unit vector tangent to Γ at the point ζ, t_i $(i = 1,\ 2)$ which is J-image of the tangent direction on the surface in the point $J^{-1}(\zeta)$, the values of the functions $\alpha(\zeta)$, $\beta(\zeta)$ coincide with the values of functions α, β in the corresponding point $c = J^{-1}(\zeta)$, function $\gamma(\zeta)$ is Hölder one on every curve $\Gamma_j = J(L_j)$, $w_{\bar{z}} = \dfrac{1}{2}(w_x + iw_y)$, $B(z)$, $F(z)$ are functions of the class $L_r(D)$, $r > 2$ in the field D. In this case, the $W^{1,r}$-regular solutions $w(z)$ are found that are in the field D and that are continuously extendable to L, with the exception of break points q_i, in the neighbourhood of which the following assessment holds true $\big|w(z)\big| < \mathrm{const}\cdot\big|z - q_i\big|^{-\alpha_j}$, $0 < \alpha_j < 1$ denote the class of such solutions by H^*.

3. The problem R for canonical domes

Let p be one of the corner points p_i of the boundary L, $\boldsymbol{k}_1$, $\boldsymbol{k}_2$ are the main directions in this point, k_1, k_2 are the main curvatures corresponding to them $(k_1 > k_2)$. The surface S is the *canonical* dome K if the direction of one of the curves converging at each corner point coincides with the principal direction $\boldsymbol{k}_2$, and the values ν_i of the interior angles at points p_i meet the conditions $0 < \nu_i \leqslant \dfrac{\pi}{2}$ $(i = 1,\ 2,\dots,\ n)$. The problem R for the canonical dome K is called canonical if the field direction $\boldsymbol{r}$ at each point p is the direction of the generalized tangent [8] at that point, i. e. $\boldsymbol{r}_1 = \boldsymbol{r}_2$, where $\boldsymbol{r}_i$ $(i = 1,\ 2)$ are the unilateral limits of the vector-function $\boldsymbol{r}$ at the point p. Let us also introduce the notations: δ_i^2 is the ratio of the corresponding main curvatures at point p_i $(0 < \delta_i < 1)$, $p(\nu_i)$ is the corner point p_i with an internal angle ν_i, $T(\nu_i)$ is the set (*sector*) of directions of the *generalized* tangent at this point, T is the set of continuous vector fields $\boldsymbol{r}$ on L, setting the direction of the generalized tangent at each corner point $p(\nu_j)$. According to [8], the canonical problem R is quasi-correct for any field $\boldsymbol{r} \in T$ if $n \geqslant 2$. The following formula for the index κ of the boundary condition (2.2) gives a complete picture of the solvability of the problem R for any field $\boldsymbol{r} \in T$.

4. Properties of the boundary condition (2.2)

The problem R is a family of $R^{\boldsymbol{r}}$ problems (2.1)–(2.3), each of which is given by the choice of vector field $\boldsymbol{r}$. According to I. N. Vekua [5] we call problem $R^{\boldsymbol{r}}$ s-quasi correct in class H^* if it is unconditionally solvable in this class and its solution depends on s real arbitrary constants (s is the *order of quasi-correctness*).

Definition 4.1. The canonical problem R is called quasi-resistant with respect to the field of directions of the generalized tangent if $R^{\boldsymbol{r}}$ problem is s-quasi-correct for any field $\boldsymbol{r} \in T$.

Remark 4.2. By the theorem on solvability of the Riemann-Hilbert problem for generalized analytic functions [5], the problem R is quasi-resistant iff index κ is an invariant of the field $\boldsymbol{r} \in T$.

Remark 4.3. The technique [6, 7] to calculate the index of a boundary condition (2.2) uses the notion [9] of a *singular node* p_i of problem (2.1), (2.2) or a *singular point* $q_i = J(p_i)$ of a discontinuity of boundary condition (2.2), in which $\omega_i = 2\pi k$, where ω_i is a jump of the function argument

$$\Lambda(\zeta) = \overline{\lambda(\zeta)}[\lambda(\zeta)]^{-1} \tag{4.1}$$

at the discontinuity point q_i, taken with an inverse sign, k is an integer. Taking expression (2.3) for $\lambda(\zeta)$ and condition $0 < \nu \leqslant \dfrac{\pi}{2}$, for the *singular node* q of the canonical problem R we have

$$\omega = 2\pi. \tag{4.2}$$

Let $p(\nu)$ be the corner point of the boundary L, $\boldsymbol{r} \in T(\nu)$. We call the direction of the field $\boldsymbol{r}$ a *singular* direction of the generalized tangent at the point $p(\nu)$ if the point $q = J(p)$ of the discontinuity of the boundary condition (2.2) is a *singular node* [9] of problem (2.2), (2.3).

Definition 4.4. Let the corner point $p(\nu)$ be called the unstable point of the problem R if the sector $T(\nu)$ contains a *singular* direction.

Let us introduce the notation: $\boldsymbol{\nu}$, $\boldsymbol{\sigma}$ are unilateral limits at the corner point $p(\nu)$ of the tangent singular vector to L, with the vector $\boldsymbol{\sigma}$ defining the principal direction $\boldsymbol{k}_2$ on the surface at p, and the interior angle ν is defined by $(-\boldsymbol{\nu}, \boldsymbol{\sigma})$. The following is true

Lemma 4.5. *If the direction of vector $\boldsymbol{r}$ at the point $p(\nu)$ coincides with the direction of vector $\boldsymbol{\nu}$, then the point $q = J(p)$ is a singular node of the boundary condition* (2.2) *iff*

$$\nu = \arccos \frac{1}{1+\delta}. \tag{4.3}$$

Proof. For the proof, consider the vector-function $\boldsymbol{\rho} = \{\rho_1(\zeta), \rho_2(\zeta)\}$, where $\rho_1(\zeta) + i\rho_2(\zeta) = \beta(\zeta)t(\zeta) - \alpha(\zeta)s(\zeta)$, denoting its unilateral limit by $\boldsymbol{\rho}^{(k)}$ ($k = 1$, 2) at the point q. Let $\boldsymbol{s}_1 = J(\boldsymbol{\nu})$, $\boldsymbol{s}_2 = J(\boldsymbol{\sigma})$, φ and ψ be the values of the angles between the vectors of the pairs $\boldsymbol{s}_1$, $\boldsymbol{s}_2$ and $\boldsymbol{\rho}^{(1)}$, $\boldsymbol{\rho}^{(2)}$ respectively. Thus $0 < \varphi < \pi$, $0 < \psi < 2\pi$, and the value of ψ depends on the choice of the direction of the vector $\boldsymbol{r}$ at the point p. Let us represent the jump ω at point q of the function $\arg \Lambda(\zeta)$

given by equality (4.1) as $\omega = \varphi + \psi$. Denote the value of the interior angle by θ at the corner point $q = J(p)$ of the boundary Γ. From the conditions of the lemma it follows that $\boldsymbol{\rho}^{(1)} = -\boldsymbol{s}_1$, $\varphi = \nu$, and by (4.2) the point q as a singular node is defined by the equality $\nu + \psi = 2\pi$. Using the known properties [5] of the mapping J and evident geometric considerations we write this equality as follows:

$$2\theta + \nu = \pi \tag{4.4}$$

or $\sin 2\theta = \sin\nu$.

From the well-known relation ([5, Ch. 2]) $\sin\theta = \sqrt{\dfrac{\mathcal{K}}{k_1 \cdot k_s}} \cdot \sin\nu$, where $\mathcal{K}$ is the Gaussian curvature of the surface at the point p, k_s is the normal curvature of the surface in the direction of vector $\boldsymbol{s}_1$, as well as the Euler formula for the normal curvature, we obtain

$$(1-\delta)\cos^2\nu + 2\sqrt{\delta}\cos\nu - 1 = 0, \tag{4.5}$$

where (4.2) follows from.

Let $\boldsymbol{r} \in T(\nu)$, τ be the value between vector $\boldsymbol{r}$ and vector $\boldsymbol{\nu}$ $(0 < \tau \leqslant \nu)$, $\omega \equiv \omega(\nu, \tau)$ be the jump of the function argument Λ at node $q = J(p)$ given by the direction $\boldsymbol{r}$. The following is true

Lemma 4.6. *If* $0 < \nu < \arccos\dfrac{1}{1+\delta}$ *then* $2\pi < \omega(\nu, 0) < 3\pi$.

Proof. The proof repeats the proof of Lemma 4.5 after replacing the equality (4.2) by the inequality $\omega > 2\pi$.

The consequence of lemma 4.6 is

Lemma 4.7. *If* $0 < \nu < \arccos\dfrac{1}{1+\delta}$ *then* $2\pi < \omega(\nu, \tau) < 3\pi$ *for any* $\tau \in (0; \nu)$.

Proof. For the proof, consider the corner point $p(\nu)$ and its corresponding value $\omega = \varphi + \psi$ for the given $\boldsymbol{r} \in T(\nu)$. Obviously, $\varphi = \pi - \theta$ is a function of the argument ν, and $\psi \equiv \chi(\nu, \tau)$, where τ is the angle between vector $\boldsymbol{\nu}$ and vector $\boldsymbol{r}$, $0 \leqslant \tau \leqslant \nu$. So $\omega = \omega(\nu, \tau)$, where $\omega(\nu, 0) = \pi - \theta + \chi(\nu, 0) > 2\pi$. Due to the known properties [5] of the mapping J for $\delta < 1$ and obvious geometric considerations, the function $\chi(\nu, \tau)$ increases monotonically with respect to the argument τ, which proves the lemma.

Consider the corner point $p(\nu)$ and a vector $\boldsymbol{\sigma} \in T(\nu)$ that defines the principal direction $\boldsymbol{k}_2$ at point p of the surface S. The following is true

Lemma 4.8. *If the direction of vector* $\boldsymbol{r}$ *at point* $p(\nu)$ *coincides with the direction of vector* $\boldsymbol{\sigma}$*, then point* $q = J(p)$ *is a singular node of the boundary condition* (2.2) *iff*

$$\nu = \operatorname{arcctg}\sqrt{t}, \tag{4.6}$$

where t *is the only positive root of the equation*

$$2\sqrt{\frac{1+\delta^2 t}{\delta^2 + t}} + \frac{1+\delta^2 t}{\delta^2 + t} - 4\sqrt{\frac{E}{\mathcal{K}(1+t)^2 + 4Et}} = \frac{1}{t}. \tag{4.7}$$

Here E, $\mathcal{K}$ are the Eulerian difference and the Gaussian curvature of the surface S at the point p.

Proof. On the surface S at point $p(\nu)$ let us consider the vectors $\boldsymbol{\nu}$, $\boldsymbol{\sigma}$ given above and orthogonal unilateral vectors $\boldsymbol{\mu}$, $\boldsymbol{\tau}$ ($\boldsymbol{\mu} \perp \boldsymbol{\nu}$, $\boldsymbol{\tau} \perp \boldsymbol{\sigma}$) directed outside S. Let us introduce the notations: $\boldsymbol{s}_1 = J(\boldsymbol{\nu})$, $\boldsymbol{s}_2 = J(\boldsymbol{\sigma})$, $\boldsymbol{n}_1 = J(\boldsymbol{\mu})$, $\boldsymbol{n}_2 = J(\boldsymbol{\tau})$ are the vectors on the plane z at the point $q = J(p)$, where $\boldsymbol{n}_2 \perp \boldsymbol{s}_2$; $\theta = \pi - (\widehat{\boldsymbol{s}_1\ \boldsymbol{s}_2})$, $\gamma = (\widehat{\boldsymbol{s}_1\ \boldsymbol{n}_2})$ where $(\widehat{\boldsymbol{a}\ \boldsymbol{b}})$ is the angle between $\boldsymbol{a}$ and $\boldsymbol{b}$. Without violating generality we assume that $(\boldsymbol{n}_1, \boldsymbol{n}_2) > 0$, and the vector $\boldsymbol{\rho}^{(1)}$ divides the pair of vectors $\boldsymbol{n}_1$, $-\boldsymbol{n}_2$. Let us denote $\mu = (\widehat{-\boldsymbol{n}_2\ \boldsymbol{\rho}^{(1)}})$. Then for the above introduced values of φ, ψ at the point $q = J(p)$ we have: $\varphi = \dfrac{\pi}{2} + \gamma$, $\psi = \dfrac{3}{2}\pi - \mu$, whence $\omega = 2\pi + (\gamma - \mu)$. Thus, the direction of the vector $\boldsymbol{\sigma}$ at the point $p(\nu)$ is *singular* if

$$\mu = \gamma. \tag{4.8}$$

Note that $\gamma = \dfrac{\pi}{2} - \theta$ where θ is the value of the internal angle at the corner point q of the boundary Γ. Consequently, the equality (4.8) is equivalent to

$$\cos\mu = \sin\theta. \tag{4.9}$$

Since $\boldsymbol{\rho}^{(1)} = -\boldsymbol{s}_1 \cos\nu + \boldsymbol{n}_1 \sin\nu$ the equality (4.9) takes the form

$$-(\boldsymbol{s}_1, \boldsymbol{n}_2)\cos\nu + (\boldsymbol{n}_1, \boldsymbol{n}_2)\sin\nu = \sqrt{1 - (\boldsymbol{s}_1, \boldsymbol{n}_1)\sin 2\nu} \cdot \sin\theta. \tag{4.10}$$

To find $(\boldsymbol{s}_1, \boldsymbol{n}_2)$, $(\boldsymbol{s}_1, \boldsymbol{n}_1)$, $(\boldsymbol{n}_1, \boldsymbol{n}_2)$ we use the reflection property [5] of J:

$$\sin(\widehat{\boldsymbol{\nu}\ \boldsymbol{\mu}}) = \sqrt{\frac{\mathcal{K}}{k_{\boldsymbol{\nu}} k_{\boldsymbol{\mu}}}}\, \sin(\widehat{\boldsymbol{s}_1\ \boldsymbol{n}_1}), \tag{4.11}$$

where $k_{\boldsymbol{\nu}}$, $k_{\boldsymbol{\mu}}$ are the normal curvatures of the surface S at the point q in the directions $\boldsymbol{\nu}$, $\boldsymbol{\mu}$ respectively, $\boldsymbol{s}_1 = J(\boldsymbol{\nu})$, $\boldsymbol{n}_1 = J(\boldsymbol{\mu})$; $\mathcal{K} = k_1 \cdot k_2$ is the Gaussian curvature of the surface at that point. By Euler's formula [10] for orthogonal directions $\boldsymbol{\nu}$ and $\boldsymbol{\mu}$ we have

$$k_{\boldsymbol{\nu}} = k_1 \sin^2\nu + k_2 \cos^2\nu, \quad k_{\boldsymbol{\mu}} = k_1 \cos^2\nu + k_2 \sin^2\nu,$$

where we get $k_{\boldsymbol{\nu}} \cdot k_{\boldsymbol{\mu}} = \mathcal{K} + E^2 \sin^2 2\nu$ where $E = \dfrac{k_1 - k_2}{2}$ is the Eulerian difference, $\mathcal{K} = k_1 \cdot k_2$ is the Gaussian curvature. As $\boldsymbol{\nu} \perp \boldsymbol{\mu}$, then equality (4.11) takes the form $\sin(\widehat{\boldsymbol{s}_1\ \boldsymbol{n}_1}) = \sqrt{\dfrac{\mathcal{K}}{k_{\boldsymbol{\nu}} k_{\boldsymbol{\mu}}}}$, whence $(\boldsymbol{s}_1, \boldsymbol{n}_1) = -\dfrac{\sqrt{E}\sin 2\nu}{\sqrt{\mathcal{K} + E \sin^2 2\nu}}$. If the equality (4.11) is written for each of the pairs $\boldsymbol{\nu}$, $\boldsymbol{k}_2$ and $\boldsymbol{\mu}$, $\boldsymbol{k}_1$ of directions at the point $p(\nu)$, then using the Euler formula we obtain $(\boldsymbol{s}_1, \boldsymbol{n}_2) = -\dfrac{1}{\sqrt{1 + \alpha\, \mathrm{ctg}^2\nu}}$, $(\boldsymbol{n}_1, \boldsymbol{n}_2) = \dfrac{\mathrm{ctg}\nu}{\sqrt{\alpha + \mathrm{ctg}^2\nu}}$, $\sin\theta = \dfrac{1}{\sqrt{1 + \alpha\, \mathrm{ctg}^2\nu}}$, $\alpha = \dfrac{k_2}{k_1}$. After substituting these expressions into equality (4.10) and cumbersome transformations we obtain equality (4.7), in which $t = \mathrm{ctg}^2\nu$, $\delta^2 = \alpha$.

Let $\boldsymbol{r} \in T(\nu)$, $\tau = (\widehat{\boldsymbol{r}\ \boldsymbol{k}_2})$, $\omega \equiv \omega(\nu, \tau)$ be the jump of the function argument $\Lambda(\zeta)$ at node $q = J(p)$. The following is true

Lemma 4.9. *If* $\operatorname{arcctg}\sqrt{t} < \nu \leqslant \frac{\pi}{2}$ *where t is the root of equation* (4.7)*, then* $\pi < \omega(\nu, 0) < 2\pi$*.*

Proof. The proof repeats the proof of Lemma 4.8 after replacing the equality (4.8) by the inequality $\gamma < \mu$.

Lemma 4.10. *If* $\operatorname{arcctg}\sqrt{t} < \nu \leqslant \frac{\pi}{2}$ *then* $\pi < \omega(\nu, \tau) < 2\pi$ *for any* $\tau \in (0; \nu)$*.*

Proof. The proof repeats the proof of Lemma 4.8 with the difference that the function $\chi(\nu, \tau)$ in the representation for ψ is monotonically decreasing.

Remark 4.11. If the values γ_1 and γ_2 are given by lemma 4.5 and lemma 4.9 respectively at the corner point p of the canonical dome K, then $\gamma_1 < \gamma_2$. This can easily be seen using obvious geometric considerations by comparing the equalities (4.2) and (4.4) under the condition $\frac{k_2}{k_1} < 1$.

The following is true

Lemma 4.12. *If* $p(\nu)$ *is an angular point of the boundary* L*,* $\gamma_1 < \nu < \gamma_2$*, then there is a single singular direction* $\boldsymbol{\gamma} \in T(\nu)$ *of the generalized tangent. For the jump* ω *at the point* $q = J(p)$ *given by the vector* $\boldsymbol{r} \in T(\nu)$*, one of the following conditions holds:*

1) $\pi < \omega < 2\pi$, $(\widehat{\boldsymbol{r}\,\boldsymbol{\nu}}) < (\widehat{\boldsymbol{\gamma}\,\boldsymbol{\nu}})$;
2) $2\pi < \omega < 3\pi$, $(\widehat{\boldsymbol{r}\,\boldsymbol{\sigma}}) < (\widehat{\boldsymbol{\gamma}\,\boldsymbol{\sigma}})$,

where the direction $\boldsymbol{\sigma}$ *gives the main direction to* $\boldsymbol{k}_2$ *at the point* p*.*

Proof. The existence of the specified direction $\boldsymbol{\gamma} \in T(\nu)$ follows from Lemmas 4.5–4.10 and the representation of the jump ω at the point $q = J(p)$. The singularity is a consequence of the monotonicity of the function $\chi(\nu, \tau)$ with respect to the argument τ (Lemma 4.7).

5. Formulation of results

Let p_i $(i = 1, \ldots, n)$ be the corner points of the boundary L of the canonical dome K, δ_i $(0 < \delta_i < 1)$ be the ratio of the principal curvatures of the surface at these points. The following is true

Theorem 5.1. *The corner point* $p(\nu_i)$ *is an unstable point of the problem* R *iff*

$$\arccos\frac{1}{1+\delta_i} \leqslant \nu_i \leqslant \operatorname{arcctg}\sqrt{t_i},$$

where t_i *is the only positive root of the equation* (4.7)*, where* $\delta = \delta_i$*,* $\mathcal{K} = \mathcal{K}_i$*,* $E = E_i$ *are Gaussian curvature and Eulerian difference of surface* S *at point* p_i $(i = 1, \ldots, n)$*.*

Proof. The statement of the theorem is the consequence of Lemmas 4.5, 4.8, and 4.12. To simplify the formulation, we will assume that the number n of corner points of the boundary satisfies the condition $n \geqslant 2$.

Theorem 5.2. *The canonical problem* R *is quasi-resistant in the class* H^* *with respect to the vector field* $\boldsymbol{r} \in T$ *iff the boundary* L *contains no instability points.*

Proof. If the point $p(\nu)$ is not an instability point, then one of the following conditions is satisfied:

1) $0 < \nu < \dfrac{1}{1+\sqrt{\dfrac{k_2}{k_1}}}$;

2) $\operatorname{arcctg}\sqrt{t} < \nu \leqslant \dfrac{\pi}{2}$, where t is the root of equation (4.7).

Let us consider the formula [4] for the index κ of the boundary condition (2.2) in the class H^*:

$$\kappa = -4 + \sum_{i=1}^{n} \left(1 + \left[\frac{\omega_i}{\pi}\right]\right), \tag{5.1}$$

where the jump ω_i at the point $q_i = J(p_i)$ is defined above. Let m and r be the number of points ($r+m=n$), where conditions 1) and 2) are satisfied, respectively. Then from (5.1) by virtue of Lemmas 4.7–4.10 we obtain:

$$\kappa = 3m + 2r - 4.$$

In such a case the number $s = 3m + 2r - 3$ is the order of quasi-correctness of the problem R.

Let $p(\nu_i)$ be the unstable point of the problem R, $\boldsymbol{\gamma}_i$ be the *singular* direction of the generalized tangent at the point q_i, $\boldsymbol{\gamma}_i \in T(\nu_i)$). Following Lemma 4.12, we partition the set of non-special directions of the sector $T(\nu_i)$ into two non-intersecting classes $T^{(1)}(\nu_i)$ $T^{(2)}(\nu_i)$ as follows: $\boldsymbol{r} \in T^{(1)}(\nu_i)$ in case $(\widehat{\boldsymbol{r}\,\boldsymbol{\nu}_i}) < (\widehat{\boldsymbol{\gamma}_i\,\boldsymbol{\nu}_i})$; $\boldsymbol{r} \in T^{(2)}(\nu_i)$ in case $(\widehat{\boldsymbol{r}\,\boldsymbol{\sigma}}) < (\widehat{\boldsymbol{\gamma}_i\,\boldsymbol{\sigma}})$. Let $q_{i_1}, \ldots, q_{i_m}$ be the unstable points of the problem R. Let us denote such a subset of the set T of directions of the generalized tangent by $T^{(1)}_{i_1,\ldots,i_s} \circ T^{(2)}_{j_1,\ldots,j_k}$ that $\boldsymbol{r} \in T^{(1)}(\nu_{i_m})$ at each point q_{i_m} ($m = 1, \ldots, s$) and $\boldsymbol{r} \in T^{(2)}(\nu_{j_m})$ at each point q_{j_m} ($m = 1, \ldots, k$, $q_{i_m} \neq q_{j_m}$).

Theorem 5.3. *The canonical problem R is quasi-resistant in the class H^* with respect to a vector field $\boldsymbol{r} \in T^{(1)}_{i_1,\ldots,i_s} \circ T^{(2)}_{j_1,\ldots,j_k}$ ($i_m \neq j_r$, $m = 1, \ldots, s$, $r = 1, \ldots, k$), $s + k = n$.*

The statement of the theorem follows from the formula (5.1) for the index κ and conditions 1), 2) of lemma 4.12.

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