Global and Stochastic Analysis
Vol. 8 No. 2 (July-December, 2021)

MUK PUBLICATIONS
Open Access Publisher

INVESTIGATION OF ONE STOCHASTIC MODEL OF NONLINEAR FILTRATION

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Abstract. The article is devoted to the study of a nonlinear model of fluid filtration based on the Oskolkov stochastic equation. This equation illustrates the dependence of the pressure of a viscoelastic incompressible fluid (for example, oil), filtering in a porous formation, on an external load (for example, the pressure of water injected through the wells into the formation). Sufficient conditions for the existence of solutions of the investigated model with the initial Cauchy condition are constructed.

Introduction

Let $\Omega \subset \mathbb{R}^n$ be a bounded area with a boundary $\partial\Omega$ from the class C^∞. The mathematical model of nonlinear filtration can be described by the Cauchy–Dirichlet problem for a nonclassical partial differential equation:

- phase variable $x \in C^k(0, T; \mathfrak{M})$ is a solution to the nonlinear Oskolkov equation
$$(\lambda - \Delta)x_t - \alpha\Delta x + |x|^{p-2}x = y, \ p \geq \ 2, \tag{0.1}$$
where the function $x = x(s,t)$ corresponds to the pressure of the filtering liquid; the parameters $\alpha \in \mathbb{R}_+$, $\lambda \in \mathbb{R}$, characterize viscous and elastic fluid properties, respectively; the function $x = x(s,t)$ responds to external influences;
- the equation satisfies the initial Cauchy condition
$$x(s,0) = x_0(s), \ s \in \Omega, \tag{0.2}$$
- and the Dirichlet boundary condition
$$x(s,t) = 0, \ (s,t) \in \partial\Omega \times \mathbb{R}_+. \tag{0.3}$$

This model describes the viscoelastic filtration process in case of incompressible fluid (for example, oil) and is explored in the article. For the first time, equation (0.1) was described by A.P. Oskolkov [1]. In general, equation (0.1) illustrates the dependence of the pressure of a viscoelastic incompressible fluid (for example, oil), filtering in a porous formation, on an external load (for example, the pressure of

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Date: Date of Submission April 5, 2021, Date of Acceptance June 18, 2021, Communicated by Yuri E. Gliklikh.

2010 *Mathematics Subject Classification.* 35Q99, 60G65.

This work is supported by RFBR and Chelyabinsk Region, project number 20-41-000001.

water injected through the wells into the formation). Equation (0.1) is not resolved with respect to the highest time derivative, and have the form

$$L\dot{x} = Mx + N(x) + y, \ \ker L \neq \{0\}. \tag{0.4}$$

Such equations are called Sobolev type equations [2, 3, 4]. The paper [5] was the first to propose the phase space method, which is one of the effective methods to study the Cauchy problem for the semilinear Sobolev type equations. A phase space is a closure of the set of all admissible initial values x_0 that are the vectors for which there exists a unique (local) solution to the Cauchy problem.

Often in experiments, "noises" can occur. Then, to study physical models, it is necessary to consider stochastic models. In the stochastic case, the mathematical model of nonlinear filtering has the form

$$\eta(s,t) = 0, \ (s,t) \in \partial\Omega \times \mathbb{R}_+, \tag{0.5}$$

$$(\lambda - \Delta) \overset{o}{\eta} = \alpha\Delta\eta - |\eta|^{p-2}\eta, \ p \geq 2. \tag{0.6}$$

One of the well-known initial problems for the model under consideration is the weakened (in the sense of S.G. Kerin) Cauchy problem

$$\lim_{t\to 0+} (\eta(t) - \eta_0) = 0. \tag{0.7}$$

Problem (0.5), (0.6) can be reduced to the stochastic equation

$$L \overset{o}{\eta} = M\eta + N(\eta) \tag{0.8}$$

endowed with weakened Cauchy condition (0.7). Solution (0.8) $\eta = \eta(t)$ is a stochastic K-process. Stochastic K-processes $\eta = \eta(t)$ and $\zeta = \zeta(t)$ are considered to be equal, if almost surely each trajectory of one of the processes coincides with a trajectory of other process. Moreover, this derivative coincides with the classical derivative if η is a function. The notion of the Nelson–Gliklikh derivative $\overset{o}{\eta}(t)$ was introduced in the monograph [6]. Also, note that the Nelson–Gliklikh derivative is based on the concept of the derivative in the mean introduced by E. Nelson [7]. In order to study the Cauchy problem, we construct the spaces of K-"noises", i.e. the spaces of stochastic K-processes that are almost surely differentiable in the sense of Nelson–Gliklikh. This approach is based on the paper [8]. Note that this approach allows to transfer on the stochastic case the methods of functional analysis that are applied in the deterministic case [9, 10, 11]. The work [12] considers various mathematical models based on semilinear equations in evolutionary and dynamic form in the deterministic case. The work [13] is the first attempt to study the evolutionary model in the stochastic case. The dynamic model in the stochastic case considers in this article. It should be noted that the main research methods of this model are transformed from the deterministic case [12].

1. Stochastic K-processes. Phase space

Consider a complete probability space $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ and the set of real numbers $\mathbf{R}$ endowed with a Borel σ-algebra. According to [8], a measurable mapping $\xi : \Omega \to \mathbf{R}$ is called a *random variable*. The set of Gaussian random variables having zero expectations (i.e. $\mathbf{E}\xi = 0$) and finite variance forms Hilbert space $\mathbf{L_2}$

(i.e. $\mathbf{D}\xi < +\infty$) with the inner product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$, where $\mathbf{E}$, $\mathbf{D}$ are the expectation and variance of the random variable, respectively.

Consider a set $\mathcal{I} \subset \mathbf{R}$ and the following two mappings. The first one, $f : \mathcal{I} \to \mathbf{L_2}$ associates each $t \in \mathcal{I}$ with the random variable $\xi \in \mathbf{L_2}$. The second one, $g : \mathbf{L_2} \times \Omega \to \mathbf{R}$, associates each pair (ξ, ω) with the point $\xi(\omega) \in \mathbf{R}$. The mapping $\eta : \mathbf{R} \times \Omega \to \mathbf{R}$ of the form $\eta = \eta(t, \omega) = g(f(t), \omega)$, where f and g are defined above, is called a *stochastic process*. According to [8], a random process η is called *continuous*, if almost surely all its trajectories are continuous. The set of continuous stochastic processes forms a Banach space, which is denoted by $\mathbf{C}(\mathcal{I}, \mathbf{L}_2)$. Fix $\eta \in \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$ and $t \in \mathcal{I}$ and denote by $\mathcal{N}_t^\eta$ the σ-algebra generated by the random variable $\eta(t)$. Denote $\mathbf{E}_t^\eta = \mathbf{E}(\cdot|\mathcal{N}_t^\eta)$.

Definition 1.1. Suppose that $\eta \in \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$. The derivative

$$\overset{o}{\eta} = D_S\eta = \frac{1}{2}\left(D + D_*\right)\eta = D\eta\left(t, \cdot\right) + D_*\eta\left(t, \cdot\right) =$$

$$= \lim_{\triangle t \to 0+} \mathbf{E}_t^\eta \left(\frac{\eta\left(t + \triangle t, \cdot\right) - \eta(t, \cdot)}{\triangle t}\right) + \lim_{\triangle t \to 0+} \mathbf{E}_t^\eta \left(\frac{\eta\left(t, \cdot\right) - \eta\left(t - \triangle t, \cdot\right)}{\triangle t}\right)$$

is called the symmetric mean derivative, where the element of Nelson–Gliklikh derivative $D\eta(t, \cdot)$ is a *derivative on the right* (*on the left* $D_*\eta(t, \cdot)$) *of a random process* η *at the point* $t \in (\varepsilon, \tau)$, if the limit exists in the sense of a uniform metric on $\mathbf{R}$.

Denote the l-th Nelson–Gliklikh derivative of the random process η by $\overset{o}{\eta}{}^{(l)}$, $l \in \mathbf{N}$. Note that the Nelson–Gliklikh derivative coincides with the classical derivative, if $\eta(t)$ is a deterministic function. Consider the space of "noises" $\mathbf{C}^l(\mathcal{I}, \mathbf{L}_2)$, $l \in \mathbf{N}$, i.e. the space of random processes from $\mathbf{C}(\mathcal{I}, \mathbf{L}_2)$, whose trajectories are almost surely differentiable by Nelson–Gliklikh on $\mathcal{I}$ up to the order l inclusively.

Consider a real separable Hilbert space $(\mathbf{H}, < \cdot, \cdot >)$ identified with its conjugate space with the orthonormal basis $\{\varphi_k\}$. Each element $x \in \mathbf{H}$ can be represented as $x = \sum\limits_{k=1}^{\infty} < x, \varphi_k > \varphi_k$. Next, choose a monotonely decreasing numerical sequence $K = \{\mu_k\}$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 < +\infty$. Consider a sequence of random variables $\{\xi_k\} \subset \mathbf{L}_2$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 \mathbf{D}\xi_k < +\infty$. Denote by $\mathbf{H}_K\mathbf{L_2}$ the Hilbert space of *random K-variables* of the form $\xi = \sum\limits_{k=1}^{\infty} \mu_k\xi_k\varphi_k$. Moreover, there exists a random K-variable $\xi \in \mathbf{H}_K\mathbf{L}_2$, if, for example, $\mathbf{D}\xi_k <$ const $\forall k$. Note that the space $\mathbf{H}_K\mathbf{L}_2$ is a Hilbert space with the scalar product $(\xi^1, \xi^2) = \sum\limits_{k=1}^{\infty} \mu_k^2 \mathbf{E}\xi_k^1\xi_k^2$. Consider a sequence of random processes $\{\eta_k\} \subset \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$ and define the $\mathbf{H}$-*valued continuous stochastic K-process*

$$\eta(t) = \sum_{k=1}^{\infty} \mu_k\eta_k(t)\varphi_k, \tag{1.1}$$

if series (1.1) converges uniformly in the norm $\mathbf{H}_K\mathbf{L}_2$ on any compact set in $\mathcal{I}$. Consider the Nelson–Gliklikh derivatives of the random K-process

$$\overset{o}{\eta}(t) = \sum_{k=1}^{\infty} \mu_k \overset{o}{\eta}_k(t)\varphi_k$$

inclusively in the right-hand side, and all series converge uniformly in the norm $\mathbf{H}_K\mathbf{L}_2$ on any compact from $\mathcal{I}$. Next, consider the space $\mathbf{C}^1(\mathcal{I};\mathbf{H}_K\mathbf{L}_2)$ of continuous stochastic K-processes and the space $\mathbf{C}^1(\mathcal{I};\mathbf{H}_K\mathbf{L}_2)$ of stochastic K-processes whose trajectories are almost surely continuously differentiable by Nelson–Gliklikh.

Consider dual pairs of reflexive Banach spaces $(\mathfrak{H},\mathfrak{H}^*)$ and $(\mathcal{B},\mathcal{B}^*)$ such that the embeddings

$$\mathfrak{H} \hookrightarrow \mathcal{B} \hookrightarrow \mathbf{H} \hookrightarrow \mathcal{B}^* \hookrightarrow \mathfrak{H}^* \tag{1.2}$$

are dense and continuous. Let the operator $L \in \mathcal{L}(\mathfrak{H};\mathfrak{H}^*)$ be linear, continuous, self-adjoint, non-negative defined and Fredholm operator, the operator $M \in \mathcal{L}(\mathfrak{H};\mathfrak{H}^*)$ be linear, continuous, symmetrical and 2-coercive and the operator $N \in C^k(\mathcal{B};\mathcal{B}^*), k \geq 1$, be dissipative. Suppose that $\operatorname{span}\{\varphi_k, \varphi_{k+1}, ..., \varphi_{k+l}\} = \ker L$, and the following condition holds: $\{\varphi_k\} \subset \mathfrak{H}$.

Taking into account that the operator L is self-adjoint and Fredholm, we identify $\mathfrak{H} \supset \ker L \equiv \operatorname{coker} L \subset \mathfrak{H}^*$. We use the subspace $\ker L$ in order to construct the subspace $[\ker L]_K\mathbf{L}_2 \subset \mathbf{H}_K\mathbf{L}_2$ and, similarly, the subspace $[\operatorname{coker} \mathrm{L}]_{\mathrm{K}}\mathbf{L}_2 \subset \mathbf{H}^*_{\mathrm{K}}\mathbf{L}_2$. Taking into account that embeddings (1.2) are continuous and dense, we construct the spaces $\mathfrak{H}^*_K\mathbf{L}_2 = [\operatorname{coker} L]_K\mathbf{L}_2 \oplus [\operatorname{im} L]_K\mathbf{L}_2$ and $\mathcal{B}^*_K\mathbf{L}_2 = [\operatorname{coker} L]_K\mathbf{L}_2 \oplus [\operatorname{im} L \cap \mathcal{B}^*]_K\mathbf{L}_2$.

We use the subspace $\operatorname{coim} L \subset \mathfrak{H}$ such that the subspace $[\operatorname{coim} L]_K\mathbf{L}_2 \equiv \mathfrak{H}^1_K\mathbf{L}_2$ and $[\ker L]_K\mathbf{L}_2 \equiv \mathfrak{H}^0_K\mathbf{L}_2$, then $\mathfrak{H}_K\mathbf{L}_2 = \mathfrak{H}^0_K\mathbf{L}_2 \oplus \mathfrak{H}^1_K\mathbf{L}_2$ and $\mathcal{B}_K\mathbf{L}_2 = [\ker L]_K\mathbf{L}_2 \oplus \overline{[\operatorname{coim} L]}_K\mathbf{L}_2$. The following lemma is correct, since the operator L is self-adjoint and Fredholm.

Lemma 1.1. (i) Let $L \in \mathcal{L}(\mathfrak{H};\mathfrak{H}^*)$ be a linear, continuous, self-adjoint, non-negatively defined and Fredholm operator, then $L \in \mathcal{L}(\mathfrak{H}_K\mathbf{L}_2;\mathfrak{H}^*_K\mathbf{L}_2)$ is a linear, continuous, self-adjoint, non-negatively defined and Fredholm operator, and

$$\mathfrak{H}_K\mathbf{L}_2 \supset [\ker L]_K\mathbf{L}_2 \equiv [\operatorname{coker} L]_K\mathbf{L}_2 \subset \mathfrak{H}^*_K\mathbf{L}_2$$

if

$$\mathfrak{H} \supset \ker L \equiv \operatorname{coker} L \subset \mathfrak{H}^*.$$

(ii) There exists a projector Q of the space $\mathfrak{H}^*_K\mathbf{L}_2$ on $[\operatorname{coim} L]_K\mathbf{L}_2$ along $[\operatorname{coker} L]_K\mathbf{L}_2$.

(iii) There exists a projector P of the space $\mathfrak{H}_K\mathbf{L}_2$ on $[\ker L]_K\mathbf{L}_2$ along $[\operatorname{coim} L]_K\mathbf{L}_2$.

Proof. By virtue of the construction of spaces and operators, the proof is based on the idea of the proof for the deterministic case presented in [13]. □

Suppose that $\mathcal{I} \equiv (0,+\infty)$. We use the space $\mathbf{H}$ in order to construct the spaces of K-"noises", spaces $\mathbf{C}^k(\mathcal{I};\mathbf{H}_K\mathbf{L}_2)$ and $\mathbf{C}^k(\mathcal{I};\mathfrak{H}_K\mathbf{L}_2)$, $k \in \mathbf{N}$. Consider stochastic Sobolev type equation (0.8).

Fix $\omega \in \Omega$. Let $\eta = \eta(t), t \in \mathcal{I}$ be a solution to equation (0.8), then η belongs to the set

$$\mathfrak{M} = \begin{cases} \{\eta \in \mathfrak{H}_K\mathbf{L}_2 : (\mathbf{I} - Q)\Big(M\eta + N(\eta)\Big) = 0\}, & \text{if } \ker L \neq \{0\}; \\ \mathfrak{H}_K\mathbf{L}_2, \quad \text{if} \quad \ker L = \{0\}. \end{cases} \tag{1.3}$$

Definition 1.2. A stochastic K-process $\eta \in \mathbf{C}^k(\mathcal{I}; \mathfrak{H}_K\mathbf{L}_2)$ is called a *solution to equation* (0.8), if almost surely all trajectories of η satisfy equation (0.8) for all $t \in \mathcal{I}$. A solution $\eta = \eta(t)$ to equation (0.8) that satisfies initial value condition (0.7) is called a *solution to Cauchy problem* (0.7), (0.8), if the solution satisfies condition (0.7) for some random K-variable $\eta_0 \in \mathfrak{H}\mathbf{L}_2$.

Theorem 1.1. Suppose that the set $\mathfrak{M}$ is a simple Banach C^k-manifold at the point $\eta_0 \in \mathfrak{M}$. Then, there exists a solution $\eta \in \mathbf{C}^1(\mathcal{I}; \mathfrak{H}_K\mathbf{L}_2)$ to Cauchy problem (0.7), (0.8).

Proof. The proof is similar to the proof of the classical theorem on the existence of a solution to the Cauchy problem [14]. Since equation (0.8) is an equation of Sobolev type, the set $\mathfrak{M}$ constructed earlier is a simple Banach C^1-manifold everywhere, with the possible exception of the zero point. The idea of the proof is based on the properties of the operators and the implicit function theorem. Let us describe the main steps of the proof.

1. Let $\mathfrak{H}\mathbf{L}_2 = \mathfrak{H}^0\mathbf{L}_2 \oplus \mathfrak{H}^1\mathbf{L}_2$. Due to representation of the space $\mathfrak{H}$ and properties of the operator L, we have $L\eta = L(\eta^1 + \eta^0) = L_1\eta^1$, where L_1 is restriction of the operator to the subspace $\mathfrak{H}^1\mathbf{L}_2$.

2. Since the set $\mathfrak{M}$ is a simple Banach C^1-manifold at the point η_0, there exists a diffeomorphism $\delta \in C^1(\mathfrak{O}_0^1; \mathfrak{O}_0^{\mathfrak{M}})$, where $\mathfrak{O}_0^{\mathfrak{M}}$ and $\mathfrak{O}_0^1$ are neighborhoods of the points $\eta_0 \in \mathfrak{M}$ and $\eta_0^1 \in P\eta_0$, respectively.

3. In the neighborhood of $\mathfrak{O}_0^0$, equation(0.8) is reduced to the regular equation $\dot{\eta} = F(\eta)$, where $F = \delta'_{\eta^1} L_1^{-1} Q(M + N) : \eta \to T_\eta\mathfrak{M}$, then $F \in C^1(\mathfrak{O}_0^{\mathfrak{M}}; T_0\mathfrak{M})$, where $T_0\mathfrak{M}$ is the restriction of the tangent bundle $T\mathfrak{M}$ to $\mathfrak{O}_0^{\mathfrak{M}}$. By virtue of the classical theorem on the existence of a unique solution to Cauchy problem [14], we obtain the unique local solvability of problem (0.7), (0.8).

4. In $\operatorname{coim} L$, introduce the norm $|\eta|^2 = \langle L\eta, \eta\rangle$. Applying the dissipativity property of operators, we obtain

$$\frac{1}{2}\frac{d}{dt}|\eta|^2 = \frac{1}{2}\frac{d}{dt}|\eta - v|^2 \leq \langle M\eta + N(\eta) - (Mv + N(v)), \eta - v\rangle \leq 0,$$

where $v = v(s)$ is a solution to the stationary equation $Mv + N(v) = 0$, which in turn is by a stationary solution to equation (0.8), $\eta = \eta(t)$ is a solution to equation (0.8). Hence, by virtue of the uniqueness of the solution to Cauchy problem (0.7), (0.8), we obtain extension of the solution to the interval $(0, +\infty)$. □

2. Stochastic mathematical model of nonlinear filtration

Consider Dirichlet problem (0.5) for Oskolkov stochastic equation (0.6) with weakened (in the sense of S.G. Kerin) Cauchy problem (0.7). Consider the functional spaces $\mathfrak{N} = \overset{\circ}{W}{}_2^1(\Omega)$, $\mathfrak{B} = L_p(\Omega)$, $\mathbf{H} = L_2(\Omega)$ defined in the domain Ω. The operators L, M and N are defined as follows:

$$\langle L\eta, z\rangle = \mathbf{E}\int_\Omega (\lambda\eta z + \nabla\eta\cdot\nabla z)\ ds\ \forall\ \eta, z \in \mathfrak{H}\mathbf{L}_2,$$

$$\langle M\eta, z\rangle = -\mathbf{E}\int_\Omega \alpha\nabla\eta\cdot\nabla z\ ds\ \forall\ \eta, z \in \mathfrak{H}\mathbf{L}_2,$$

$$\langle N(\eta), z\rangle = -\mathbf{E}\int_\Omega |\eta|^{p-2}\eta z\ ds\ \forall\ \eta, z \in \mathfrak{B}\mathbf{L}_2,$$

where $\langle\cdot,\cdot\rangle$ is the dot product in $\mathbf{HL}_2$. We denote by $\{\varphi_k\}$ the sequence of eigenfunctions of the homogeneous Dirichlet problem for the Laplace operator $(-\Delta)$ in the domain Ω, and denote by $\{\lambda_k\}$ the corresponding sequence of eigenvalues numbered in non-decreasing order taking into account the multiplicity.

Lemma 2.1. [13] (i) *For all $\lambda \geq -\lambda_1$, the operator $L \in \mathcal{L}(\mathfrak{H}\mathbf{L}_2;\mathfrak{H}^*\mathbf{L}_2)$ is self-adjoint, Fredholm and non-negative definite.*

(ii) *For all $\alpha \in \mathbb{R}_+$, the operator $M \in \mathcal{L}(\mathfrak{H}\mathbf{L}_2;\mathfrak{H}^*\mathbf{L}_2)$ is symmetric and the operator $(-M)$ is 2-coercive.*

(iii) *The operator $N \in C^1(\mathfrak{B}\mathbf{L}_2;\mathfrak{B}^*\mathbf{L}_2)$ is dissipative and the operator $(-N)$ is p-coercive.*

Let $\lambda \geq -\lambda_1$

$$\ker L = \begin{cases} \{0\}, \text{ if } \lambda > -\lambda_1; \\ \operatorname{span}\{\varphi_1\}, \text{ if } \lambda = -\lambda_1. \end{cases}$$

Then

$$\operatorname{im} L = \begin{cases} \mathfrak{H}^*\mathbf{L}_2, \quad \lambda > -\lambda_1; \\ \{\eta \in \mathfrak{H}^*\mathbf{L}_2 : \langle\eta,\varphi_1\rangle = 0\}, \text{ if } \lambda = -\lambda_1, \end{cases}$$

$$\operatorname{coim} L = \begin{cases} \mathfrak{H}\mathbf{L}_2, \text{ if } \lambda > -\lambda_1; \\ \{\eta \in \mathfrak{H}\mathbf{L}_2 : \langle\eta,\varphi_1\rangle = 0\}, \text{ if } \lambda = -\lambda_1. \end{cases}$$

Hence the projectors

$$P = Q = \begin{cases} \mathbb{I}, \text{ if } \lambda > -\lambda_1; \\ \mathbb{I} - \langle\cdot,\varphi_1\rangle, \text{ if } \lambda = -\lambda_1. \end{cases}$$

Let us construct the set

$$\mathfrak{M} = \begin{cases} \mathfrak{H}\mathbf{L}_2, \quad \lambda > -\lambda_1; \\ \{\eta \in \mathfrak{H}\mathbf{L}_2 : \mathbf{E}\int_\Omega(-\alpha\Delta\eta + |\eta|^{p-2}\eta)\varphi_1\ ds = 0, \quad \lambda = -\lambda_1, \end{cases}$$

Theorem 2.2. *Let $\lambda \geq -\lambda_1$, $\alpha \in \mathbb{R}_+$, $n = 2$, $\forall p$ or $n \geq 3$, $2 \leq p \leq 2 + \frac{4}{n-2}$, then*
(i) the set $\mathfrak{M}$ is a simple Banach C^1-manifold modelled by the space $\operatorname{coim} L$;

(ii) for any $\eta_0 \in \mathfrak{M}$, $T \in \mathbb{R}_+$, *there exists a unique solution* $\eta \in \mathbf{C}^k(\mathcal{I}; \mathfrak{H}_K\mathbf{L}_2)$ *to problem* (0.5) – (0.7).

Proof. Statement (i) was obtained in [13]. Statement (ii) is a consequence of Theorem 1.1 and Lemma 2.1.

Acknowledgments. This work was funded by RFBR and Chelyabinsk Region, project number 20-41-000001.

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