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A NON-STATIONARY ONE SERVICE STATION QUEUEING MODEL WITH BULK INPUTS

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ABSTRACT. For examining several systems such as networking, shipping, manufacturing, processing, cargo - handling the queueing models are used extensively. Recently much emphasis is shown for development of time dependent queueing models. This paper introduces an innovative queueing model for which both the incoming and servicing processes are non - stationary. The input system following NHCP process and the servicing facility following NHP process. The input and servicing systems are characterised by non - homogeneous Poisson process basing on time. Further we consider the arbitrary customers incoming in each arriving module is followed by a probability distribution. The queue size distribution is developed through Kolmogorov's forward equations and the model characteristics such as queue size, the expected delay time in queue and system, the service area throughput, the variance of queue size, coefficient of variation of the queue size are developed and analysed. Supposing the batch length following uniform distribution the functioning of the model is studied through statistical analysis and a relative report of the model is also presented. For small periods of time the delay in transmission and congestion in queues can be predicted through time - based batch inputs and time supported servicing facility. A few previous models are used as specific cases for investigating this model.

MSC:35Q99,60G65,60-XX, 65Cxx, 91G80, 60H30, 62P25

KEYWORDS: Non - Stationary queueing systems, NHCP process, bulk inputs, uniform distribution.

1.Introduction

1.1 Literature Survey

Queuing models contribute basic framework for studying various real-life systems like data-voice transmission, message networking, shipping systems, cargo handling, traffic control, reservation system, etc., From the past one decade, a lot of work has been narrated regarding load dependent service rates in evaluating more accurately and expecting the performance measures of satellites and telecommunications of the system. (Choi B.D and Choi D.I [7], Srinivasa Rao et. al. [24], Kin. K. Leung [15], Pillai et. al. [28], Suresh Varmaet. al. [20], Padmavathi. G et. al. [19], Suresh Varma et.al. [8]). All the above authors considered the inputs to the system is single and representing a Poisson process.

Usually, for developing queueing models the incoming and manufacturing facilities are followed by Poisson process. But for most of the real-life circumstances, the Poisson process is not followed by incoming pattern. For example, in networking, incoming units have been transferred to packets of arbitrary length, depending upon size of information stored in buffer for communication. Brockmeyer [5] initiated bulk arrival queuing models. Prasad Reddy et.al. [21] examined mutually dependent communication network with batch inputs. AnyueChen [4] developed a modified

Markovian batch input and batch maintenance queue featuring state dependent control. Nageswara Rao et. al. [18] developed communication network with DBA and improved stage type transmission having batch inputs.

A short time ago Abhishek et.al. [1] developed a queue with one server having batch inputs and semi-Markov services. Achutya Rao et.al. [3] studied communication networks with load dependent (dynamic band width allocation) in which the content of queue is representing the customer service facility. Charan jeet Singh et.al. [6] has considered the performance of bulk arrival with different behaviours of customers incoming. Karthikeyan Natarajan et.al. [9] studied the literature of bulk arrivals in queuing models. Banerjee et.al. [12] examined finite-length batch-size-dependent bulk service queue with queue length dependent vacation. Ramya et.al. [16] developed queuing systems with Compound Poisson (CP) Twice Truncated Geometric bulk arrivals and load dependent service rates. All these authors examined that the arrivals are independent of time and possible to characterize by CP Process.

However, numerous real-life circumstances arising at networking or production processes the arrivals may not be independent of time and exhibit time dependent nature. Gusella [22] made a measurement study of diskless workstation traffic on Ethernet. Leland et. al. [30] analysed the self-similar nature of ethernet traffic. Fowler [13] studied the features and implications of local area network (LAN). Feldmann [14] developed the characteristics of TCP connection arrivals. Rakesh Singhai et. al. [23] has exposed the metropolitan area network (MAN) traffic, wide area network (WAN) traffic and variable bit rate (VBR) traffic exhibit time dependent arrival. A NHCP process is followed by time dependent input rate.

Trinatha Rao. et. al. [28] developed and analysed the performance of time dependent interaction with Poisson inputs and DBA. Suhasini et. al. [27] considered a queue system with time-based batch inputs having state related servicing rates. Hemanth Kumar et. al. [14] developed a single server queuing model with NHCP batch inputs with intervened Poisson Distribution. Meeravalli et. al. [17] has developed and analysed a NHP queueing models. All these authors considered time-based inputs and service process is not based on time and follows a CP Process.

Durga Aparajitha et. al. [10] analysed queueing models with NHP servicing facilities. Sita Rama Murthy et. al. [24] studied queuing models with Cargo Dependent Service Rates. Sreelatha et. al. [26] developed and analysed NHP process queueing model having service rate load dependent. In these papers the authors considered the inputs and services are individual and following NHP process.

1.2 Importance of the model Considered

In many real-life situations inputs come in bulk. The probability distribution is followed by an arbitrary incoming module. Not many efforts are reported in writings relating to bulk queuing models with non – homogeneous inputs and manufacturing systems. Hence, we develop a one service area bulk inputs queueing model with time-based inputs and servicing systems. The time based incoming and servicing systems are characterised with NHP process and servicing rate is based on queue length. At this point we consider that the input and servicing rates are linearly dependent on time. For certain values of constraints a few of the previously developed queuing systems are used as special cases.

The queue size distribution is attained through difference – differential equations.The performance measures are determined by originating the terms of the system characteristics such as the average queue size, service area utilization, idealness probability, service station throughput, mean waiting time in queueand system, variance of queue size and variance coefficient ofqueue size. The model sensitivity with respect to the constraints is studied through statistical analysis of the data. The relative analysis of the proposed queueing models with that of homogeneous bulk inputs is produced.

2.Model Description

In this section, a brief discussion for the expansion and study of the queueing model is measured. The following are one service area queueing system assumptions:

- Arrival pattern following heterogeneous Compound Poisson Uniform bulk arrival process with average input rate $\boldsymbol{A(t_p) = \alpha + \beta t_p}$.
- Service pattern following heterogeneous Poisson process having service rate $\boldsymbol{S(t_p) = \gamma + \delta t_p}$.
- The service rate is dependent on the queue size.
- **First – in – First - out** queue discipline is followed.
- The capacity of the queue is infinite.
- The queueing model is represented by a schematic diagram.

Figure1:Pictorial Representation of bulk queueing system

Let the customers size in the queue be m, $\boldsymbol{P_m(t_p)}$ be the probability of 'm' customers in queue at time t_p.

The following are the model difference – differential equations:

$$\left.\begin{aligned}\frac{\partial P_m(t_p)}{\partial t_p} &= -[A(t_p) + mS(t_p)]P_m(t_p) + A(t_p)\sum_{b=1}^{m} P_{m-b}(t_p)C_b + (m+1)S(t_p)P_{m+1}(t_p);\ m > 0 \\ \frac{\partial P_0(t_p)}{\partial t_p} &= -A(t_p)P_0(t_p) + S(t_p)P_1(t_p);\ \ m = 0\end{aligned}\right\} \quad (2.1)$$

Let the probability generating $P(Z, t_p)$ and $C(Z)$ is the probability function of incoming batch size distribution.

$$P(Z, t_p) = \sum_{m=0}^{\infty} P_m(t_p)Z^m\ ;\ C(Z) = \sum_{b=0}^{\infty} c_b Z^b \quad (2.2)$$

Multiplying with Z^mon both sides of equation (1) and adding them for all m-values, we obtain

$$\frac{\partial}{\partial t_p}\left(\sum_{m=0}^{\infty} Z^m P_m(t_p)\right) = -A(t_p)\sum_{m=0}^{\infty} Z^m P_m(t_p) - S(t_p)Z\sum_{m=0}^{\infty} mZ^{m-1}P_m(t_p)$$

$$+A(t_p)\sum_{b=1}^{\infty} c_b Z^b \sum_{m=b}^{\infty} Z^{m-b}P_{m-b}(t_p)\ + S(t_p)\left[\sum_{m=0}^{\infty}(m+1)\, Z^m P_{m+1}(t_p)\right]$$

$$\frac{\partial}{\partial t_p}P(Z, t_p) + S(t_p)(Z-1)\frac{\partial}{\partial Z}P(Z, t_p) = A(t_p)[C(Z) - 1]P(Z, t_p) \quad (2.3)$$

Using Lagrangian method for solving equation (2.3), the auxiliary equation is

$$\frac{\partial t_p}{1} = \frac{\partial Z}{S(t_p)(Z-1)} = \frac{\partial P(Z,t_p)}{A(t_p)[C(Z)-1]P(Z,t_p)} \quad (2.4)$$

Let the incoming rate and servicing rate are linear and is of the form

$A(t_p) = \alpha + \beta t_p$ and $S(t_p) = \gamma + \delta t_p$.

Consider $\frac{\partial t_p}{1} = \frac{\partial Z}{S(t_p)(Z-1)}$

Integrating on both sides, we get

$$X = (Z-1)e^{-\int S(t_p)\partial t_p} \quad (2.5)$$

Consider $\frac{\partial t_p}{1} = \frac{\partial P(Z,t_p)}{A(t_p)[C(Z)-1]P(Z,t_p)}$

Integrating on both sides

$$Y = P(Z,t_p)e^{-[\sum_{b=1}^{\infty} c_b Z^b - 1]\int A(t_p)\partial t_p} \quad (2.6)$$

Making use of equation (5) and solving equation (6),we get

$$Y = P(Z,t_p)exp\left\{-\left[\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i}(Z-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p^2}{2}\right)}\right] \int (\alpha + \beta t_p)e^{\left(\gamma t_p + \frac{\delta t_p^2}{2}\right)} dt_p\right\} \quad (2.7)$$

where X,Y are random coefficients.

$P_0(0) = 1, P_0(t_p) = 0$ be the primary conditions, making use of them we get $Y = 1$

$$P(Z,t_p) = exp\left\{\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i}(Z-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p^2}{2}\right)} \left[\alpha\int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv + \beta\int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv\right]\right\}; |Z| \le 1 \quad (2.8)$$

3.FEATURES OF QUEUEING MODEL

Let $Z = 0$ in $P(Z,t_p)$ and adding the constant terms, we getthe idleness probability as

$$P_0 = exp\left\{\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i}(-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p^2}{2}\right)} \left[\alpha\int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv + \beta\int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv\right]\right\} \quad (3.1)$$

The average length of the queue is

$$L = e^{-\left(\gamma t_p + \frac{\delta t_p^2}{2}\right)}\left[\alpha\int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv + \beta\int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)}dv\right]E(X) \quad (3.2)$$

where $E(X) = \sum_{b=1}^{\infty} bc_b$ which is average of input batch length.

The servicing area utilization is

$U = 1 - P_0$

$$= 1 - exp\left\{\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i} (-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)} \left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\} \quad (3.3)$$

The service area throughput is

$Thp = S(t_p)U$

$$= (\gamma + \delta t_p)\left\{1 - exp\left\{\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i} (-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)} \left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\}\right\} \quad (3.4)$$

The mean waiting time of queue size is

$$W = \frac{L}{Thp}$$

$$= \frac{e^{-\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)}\left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right] E(X)}{(\gamma + \delta t_p)\left\{1 - exp\left\{\sum_{b=1}^{\infty}\sum_{i=1}^{b} c_b \binom{b}{i} (-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)}\left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\}\right\}} \quad (3.5)$$

The variance of the queue size is

$$V = \sum_{b=1}^{\infty} b c_b \left\{(b-1) e^{-2\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)}\left[\alpha \int_0^{t_p} e^{2\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{2\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right] + e^{-\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)}\left[\alpha \int_0^{t_p} e^{\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\} \quad (3.6)$$

Variance Coefficient in the queue is

$$CV = \frac{\sqrt{V}}{L} \times 100 \quad (3.7)$$

4.PERFORMING TECHNIQUES WITH UNIFORM BULK SIZE ALLOCATION

Let bulk size distribution of incoming style be uniform. Then the probability density function of the bulk length allocation is

$c_b = \frac{1}{s-r+1}; f$or $b = r, r+1, r+2, \dots, s.$ (4.1)

The mean length of the bulk size allocation is $E(X) = \frac{r+s}{2}$ (4.2)

The variance of the bulk size distribution is $V(X) = \frac{1}{12}[(s-r+1)^2 - 1]$ (4.3)

The queue size distributionis

$$P(Z,t) = exp\left\{\sum_{b=r}^{s}\sum_{i=1}^{b} \frac{1}{s-r+1} \binom{b}{i} (Z-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)} \left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\}; |Z| \le 1 \quad (4.4)$$

Let $Z = 0$ in (4.4) and adding the constant expressions, we get idleness probability as

$$P_0 = exp\left\{\sum_{b=r}^{s}\sum_{i=1}^{b} \frac{1}{s-r+1} \binom{b}{i} (-1)^i e^{-i\left(\gamma t_p + \frac{\delta t_p{}^2}{2}\right)} \left[\alpha \int_0^{t_p} e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv + \beta \int_0^{t_p} v e^{i\left(\gamma v + \frac{\delta v^2}{2}\right)} dv\right]\right\} \quad (4.5)$$

The average length of the queue is

$$L=\left[e^{-\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right]\frac{r+s}{2} \tag{4.6}$$

The service area utilization is

$$U=1-P_0$$

$$=1-exp\left\{\sum_{b=r}^{s}\sum_{i=1}^{b}\frac{1}{s-r+1}\binom{b}{i}(-1)^i e^{-i\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right\} \tag{4.7}$$

The service station throughput is

$$Thp=S(t_p)U$$

$$=(\gamma+\delta t_p)\left\{1-exp\left\{\sum_{b=r}^{s}\sum_{i=1}^{b}\frac{1}{s-r+1}\binom{b}{i}(-1)^i e^{-i\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right\}\right\} \tag{4.8}$$

The mean waiting time of queue size is

$$W=\frac{L}{Thp}$$

$$=\frac{\left[e^{-\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right]\frac{r+s}{2}}{(\gamma+\delta t_p)\left\{1-exp\left\{\sum_{b=r}^{s}\sum_{i=1}^{b}\frac{1}{s-r+1}\binom{b}{i}(-1)^i e^{-i\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{i\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right\}\right\}} \tag{4.9}$$

The variance of the queue size is

$$V=\sum_{b=r}^{s}b\left(\frac{1}{s-r+1}\right)\left\{(b-1)e^{-2\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{2\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{2\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right.$$
$$\left.+e^{-\left(\gamma t_p+\frac{\delta t_p{}^2}{2}\right)}\left[\alpha\int_0^{t_p}e^{\left(\gamma v+\frac{\delta v^2}{2}\right)}dv+\beta\int_0^{t_p}ve^{\left(\gamma v+\frac{\delta v^2}{2}\right)}dv\right]\right\} \tag{4.10}$$

Variance Coefficient of the queue size is

$$CV=\frac{\sqrt{V}}{L}\times 100 \tag{4.11}$$

5. STATISTICAL ANALYSIS OF THE MODEL

Here,the statistical analysis of the model is used for studying the system performance. The customers input to queue is following NHCP process with input rate $A(t_p)=\alpha+\beta t_p$ and the servicing facility to queue is following NHP process with servicing rate$S(t_p)=\gamma+\delta t_p$.

Time dependent qualities of queueing system are extremely sensitive, the momentary behaviour of the model is studied by calculating the performing measures with the following sets of values of model parameters. Let $t_p=0.05,0.06,0.07,0.08,0.09$;

$r=1,2,3,4,5,6$; $s=10,11,12,13,14,15$;

$\alpha=0.5,1,1.5,2,2.5,3$; $\beta=1.5,2,2.5,3,3.5,4$;

$\gamma=15,16,17,18,19,20$; $\delta=9,11,13,15,17,19$.

The idleness probability, the average queue length, the service area utilization, the service area throughput, the variance of queue length and the variance coefficient of the queue length are calculated for distinct values of the parameters $t_p, r, s, \alpha, \beta, \gamma, \delta$ and placed in the Table-1.

t_p	r	s	α	β	γ	δ	P_0	L	U	Thp	W	V	CV
0.05	1	10	0.5	1.5	15	9	0.9748	0.1042	0.0252	0.3899	0.2672	0.5657	721.8921
0.06							0.9698	0.1189	0.0302	0.4694	0.2533	0.6228	663.6336
0.07							0.9649	0.1322	0.0351	0.5484	0.2411	0.6707	619.4817
0.08							0.9602	0.1442	0.0398	0.6263	0.2303	0.7114	584.8721
0.09							0.9556	0.1551	0.0444	0.7026	0.2208	0.7466	557.0533
	2						0.9537	0.1692	0.0463	0.7316	0.2313	0.8264	537.236
	3						0.9527	0.1833	0.0473	0.7484	0.245	0.9182	522.72
	4						0.952	0.1974	0.048	0.759	0.2601	1.0219	512.0642
	5						0.9515	0.2115	0.0485	0.7662	0.2761	1.1375	504.2497
	6						0.9512	0.2256	0.0488	0.7711	0.2926	1.2652	498.5461
		11					0.9511	0.2397	0.0489	0.7731	0.3101	1.4346	499.6528
		12					0.951	0.2538	0.049	0.7746	0.3277	1.616	500.8388
		13					0.9509	0.2679	0.0491	0.7759	0.3453	1.8093	502.0601
		14					0.9509	0.282	0.0491	0.777	0.3629	2.0146	503.2877
		15					0.9508	0.2961	0.0492	0.7779	0.3807	2.2318	504.5024
			1				0.9095	0.5502	0.0905	1.4309	0.3845	4.1114	368.5114
			1.5				0.87	0.8043	0.13	2.0554	0.3913	5.991	304.3051
			2				0.8322	1.0585	0.1678	2.6529	0.399	7.8706	265.0527
			2.5				0.7961	1.3126	0.2039	3.2243	0.4071	9.7502	237.8953
			3				0.7615	1.5667	0.2385	3.771	0.4155	11.6298	217.674
				2			0.7599	1.5807	0.2401	3.7953	0.4165	11.7473	216.8321
				2.5			0.7584	1.5947	0.2416	3.8195	0.4175	11.8647	215.9995
				3			0.7569	1.6087	0.2431	3.8436	0.4185	11.9821	215.1762
				3.5			0.7554	1.6227	0.2446	3.8677	0.4195	12.0995	214.362
				4			0.7538	1.6367	0.2462	3.8918	0.4206	12.2169	213.5567
					16		0.7547	1.5822	0.2453	4.1243	0.3836	11.6396	215.6228
					17		0.7556	1.5305	0.2444	4.3527	0.3516	11.108	217.7591
					18		0.7567	1.4814	0.2433	4.5763	0.3237	10.6175	219.9608
					19		0.758	1.4346	0.242	4.7948	0.2992	10.164	222.2235
					20		0.7594	1.3902	0.2406	5.0075	0.2776	9.744	224.5428
						11	0.7595	1.3845	0.2405	5.0475	0.2743	9.6876	224.8066
						13	0.7597	1.3789	0.2403	5.0874	0.271	9.6317	225.0713
						15	0.7599	1.3733	0.2401	5.1271	0.2679	9.5764	225.337
						17	0.76	1.3677	0.24	5.1668	0.2647	9.5215	225.6037
						19	0.7602	1.3622	0.2398	5.2064	0.2616	9.4671	225.8714

Table-1: Values of P_0, L, U, Thp, W, V, CV for distinct values of Parameters

From **Table-1**, we observe as time (tp) increases from .05 to .09,the idleness probability reduces to .9556 from .9748, the average queue length raises from .1042 to

.1551, the service area utilisation raises from .0252 to .0444, the service area throughput raises from .3899 to .7026, the mean waiting time of queue size reduces to .2208 from .2672, the variance of the queue length raises from .5657 to .7466 and variance coefficient of queue size reduces to 557.0533 from 721.8921, when the remaining parameters are kept constant.

It is seen that as parameter (r) increases from 2 to 6, the idleness probability reduces to .9512 from .9537, the average queue length raises from .1692 to .2256, the service area utilisation raises from .0463 to .0488, the service area throughput raises from .7316 to .7711, the mean waiting time of queue size raises from .2313 to .2926, the variance of the queue length raises from .8264 to 1.2652 and variance coefficient of queue size reduces to 498.5461 from 537.236, when the remaining parameters are kept constant.

It is seen that as parameter (s) increases from 11 to 15, the idleness probability reduces to .9508 from .9511, the average queue length raises from .2397 to .2961, the service area utilisation raises from .0489 to .0492, the service area throughput raises from .7731 to .7779, the mean waiting time of queue size raises from .3101 to .3807,the variance of the queue length raises from 1.4346 to 2.2318 and variance coefficient of queue size raises from 499.6528 to 504.5024, when the remaining parameters are kept constant.

It is seen that as parameter (α) increases from 1 to 3, the idleness probability reduces to .7615 from .9095, the average queue length raises from .5502 to 1.5667, the service area utilisation raises from .0905 to .2385, the service area throughput raises from 1.4309 to 3.771, the mean waiting time of queue size raises from .3845 to .4155, the variance of the queue length raises from 4.1114 to 11.6298 and variance coefficient of queue size reduces to 217.674 from 368.5114, when the remaining parameters are kept constant.

It is seen that as parameter (β) increases from 2 to 4, the idleness probability reduces to .7538 from .7599, the average queue length raises from 1.5807 to1.6367, the service area utilisation raises from .2401 to .2462, the service area throughput raises from 3.7953 to 3.8918, the mean waiting time of queue size raises from .4165 to .4206, the variance of the queue length raises from 11.7473 to 12.2196 and variance coefficient of queue size reduces to 213.5567 from 216.8321, when the remaining parameters are kept constant.

It is seen that as parameter (γ)increases from 16 to 20, the idleness probability raises from .7547 to .7594, the average queue length reduces to 1.3902 from 1.5822, the service area utilisation reduces to .2406from .2453, the service area throughput raises from 4.1243 to 5.0075, the mean waiting time of queue size reduces to .2776 from .3836,the variance of the queue length reduces to 9.744 from 11.6396 and variance coefficient of queue size raises from 215.6228 to 224.5428, when the remaining parameters are kept constant.

It is seen that as parameter (δ) increases from 11 to 19, the idleness probability raises from .7595 to .7602, the average queue length reduces to 1.3622 from1.3845, the service area utilisation reduces to .2398 from .2405, the service area throughput raises from 5.0475 to 5.2046, the mean waiting time of queue size reduces to .2616 from .2743, the variance of the queue length reduces to 9.4671 from 9.6876 and variance coefficient of queue sizeraises from 224.8066 to 225.8714, when the remaining parameters are kept constant.

From the fig: 2a, 2b, 2c we observe the relations between performing methods and constraints.

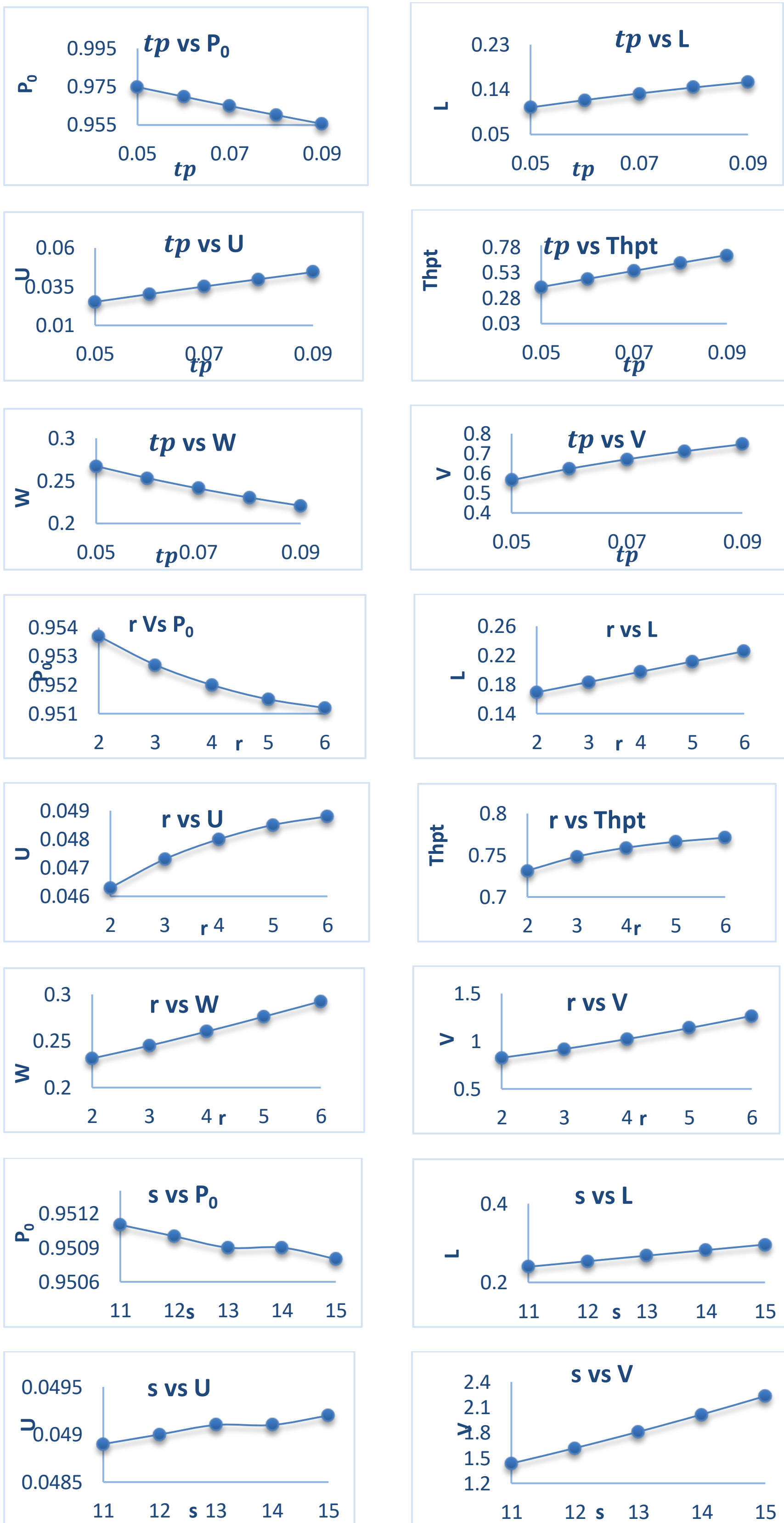

Fig 2a: The relations between performing methods and constraints

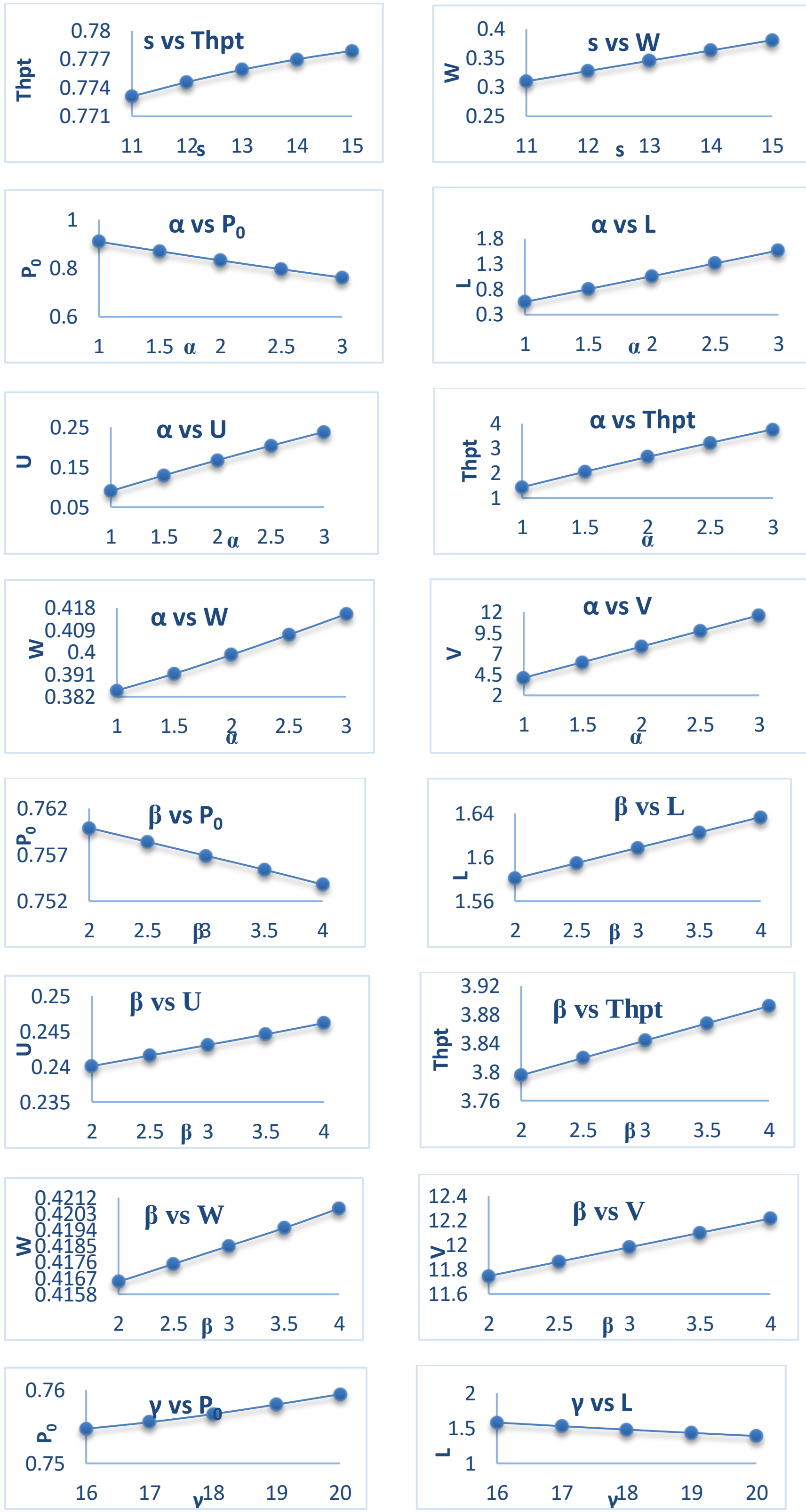

Fig 2b: The relations between performing methods and constraints

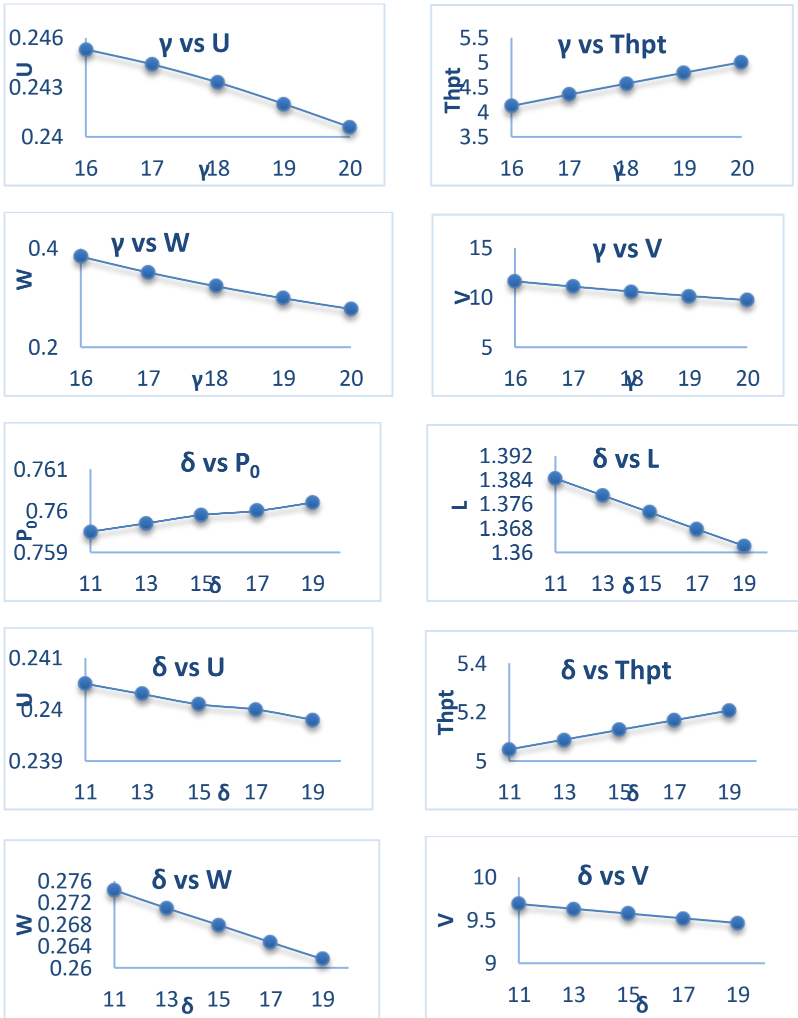

Fig 2c: The relations between performing methods and constraints

6. MODEL SENSITIVITY

The model sensitivity is made with respect to the values of time (tp), incoming parameters α and β, servicing parameters γ and δ and parameters altogether on average queue length(L), utilization (U), mean waiting time (W) and throughput (Thp), are calculated and given in the **Table-3** with variation of -15%, -10% , -5%, 0%, +5%, +10%, +15% on model constraints.

		-15%	-10%	-5%	0%	5%	10%	15%
t_p=0.07	L	0.1182	0.123	0.1277	0.1322	0.1365	0.1407	0.1448
	U	0.03	0.0317	0.0334	0.0351	0.0368	0.0384	0.0401
	Thpt	0.4655	0.4932	0.5208	0.5484	0.5758	0.603	0.6301
	W	0.254	0.2495	0.2452	0.2411	0.2371	0.2334	0.2298
λ1=0.5	L	0.1145	0.1204	0.1263	0.1322	0.1381	0.144	0.1498
	U	0.0304	0.032	0.0335	0.0351	0.0366	0.0382	0.0397
	Thpt	0.4753	0.4997	0.524	0.5484	0.5726	0.5969	0.6211

	W	0.241	0.241	0.241	0.2411	0.2411	0.2412	0.2413
λ2=1.5	L	0.13	0.1307	0.1315	0.1322	0.1329	0.1336	0.1344
	U	0.0346	0.0347	0.0349	0.0351	0.0353	0.0354	0.0356
	Thpt	0.5404	0.5431	0.5457	0.5484	0.551	0.5536	0.5563
	W	0.2406	0.2407	0.2409	0.2411	0.2412	0.2414	0.2416
μ1=15	L	0.1409	0.1379	0.135	0.1322	0.1295	0.1268	0.1243
	U	0.0356	0.0355	0.0353	0.0351	0.0349	0.0347	0.0345
	Thpt	0.4768	0.501	0.5249	0.5484	0.5714	0.5941	0.6164
	W	0.2954	0.2752	0.2572	0.2411	0.2266	0.2135	0.2016
μ2=9	L	0.1324	0.1324	0.1323	0.1322	0.1321	0.132	0.132
	U	0.0351	0.0351	0.0351	0.0351	0.0351	0.0351	0.0351
	Thpt	0.5453	0.5463	0.5473	0.5484	0.5494	0.5504	0.5514
	W	0.2429	0.2423	0.2417	0.2411	0.2405	0.2399	0.2393

Table-2: Model Sensitivity at tp=0.07, λ_1=0.5, λ_2=1.5, μ_1=15, μ_2=9

The measures of performance are highly affected when the variation in time (tp), from -15% to +15% the average queue length raises from .1182 to .1448, service area utilization raises from .03 to .0401, the service area throughput raises from .4655 to .6301 and the average waiting time queue length is reduced to .2298 from .254. For variation in incoming constraint α from -15% to +15% the expected queue length raises from .1145 to .1498, service area utilization raises from .0304 to .0397, the service area throughput raises from .4753 to .6211 and average waiting time of queue length raises from .241 to .2413 and for variation in incoming constraint β from -15% to +15% the average queue length raises from .13 to .1344, service area utilization raises from .0346 to .0356, the service area throughput raises from .5405 to .5563 and average waiting time of queue length raises from .2406 to .2416 . For variation in servicing constraint γ from -15% to +15%, average queue length reduces to .1243 from .1409, service area utilization reduces to .0354 from .0356, service area throughput raises from.4768 to .6164 and the average waiting time is reduced to .2016from .2954 and for variation in servicing constraint δ from -15% to +15% the expected queue length reduces to .132from .1324, service area utilization remains constant at .0351, service area throughput raises from .5453 to .5514 and the waiting time is reduced to .2393 from .2429.

The analysis with all the parameters reflects the strategy of load dependence, reduces the crowding in queues, the lag in communication and improves the condition of servicing.

7. RELATIVE ANALYSIS OF THE MODEL

In this session, the relative analysis or survey is made between the heterogeneous and homogeneous Poisson input and service rates.

tp	Performance Ration	Time-BasedInput and Servicing Facility	Time IndependentInput and ServicingFacility	Difference	Percentage Variation
0.05	L	0.10419	0.09673	0.00746	7.712188566
	U	0.02524	0.0235	0.00174	7.404255319

	W	0.2672	0.27446	0.00726	2.6451942
0.06	L	0.11892	0.1088	0.01012	9.301470588
	U	0.03021	0.02775	0.00246	8.864864865
	W	0.25334	0.26142	0.00808	3.090811721
0.07	L	0.1322	0.11918	0.01302	10.92465179
	U	0.03508	0.0318	0.00328	10.31446541
	W	0.24108	0.24984	0.00876	3.506243996
0.08	L	0.14421	0.12811	0.0161	12.56732496
	U	0.03984	0.03565	0.00419	11.75315568
	W	0.23026	0.2396	0.00934	3.898163606
0.09	L	0.15511	0.13581	0.0193	14.21103012
	U	0.04444	0.03926	0.00518	13.19409068
	W	0.22075	0.23058	0.00983	4.26316246

Table:3 Relative analysis of model with Time-Based and Time Independent Poisson Input and Service rates

This methodinvolves the following queueing systems:

- M/M/1 in which the servicing rate is state dependent when $\beta = 0, \delta = 0$.
- One Service Facility queueing system with NHP inputs and Poisson servicing process in which service rate is state dependent given by the Trinatha Rao et. al. (2012).
- One Service Facility queueing system having NHCP batch inputs in which service rate dependent on state is given by Suhasini et. al. (2012, 2013).
- One Service Facility queueing system with Poisson inputs and time-based servicing facility with load dependent servicing rate given by Durga Aparajitha et. al. (2014).
- Single Server Non – homogeneous Poisson queueing model with service rateload dependent by Sreelatha et. al. (2020).

8. Conclusion

This articleis based on queueing system in which the inputs and servicing facilities are time-based and inputs are in bulk having load dependent service rate. Here the input method is categorised by NHCP process and servicing facility is categorised by NHP process. Making use of Chapman – Kolmogrov's equations the queue size distribution is developed. Assuming the distribution of the arrival size in a module follow a Uniform distribution. The average queue length, expected lag time of queue and system, idleness probability, servicing area utilization ad throughput are the characteristics of the queueing model. Through statistical analysis the sensitivity of system characteristics with respect to incoming constraints are also analysed. The system performance can be predicted much closer to the real – life situations for short periods of time through the non - homogeneous environment of incoming and servicing methods. Hence this model is used in examining the communication network of self-similarity such as LAN, WAN and MAN in reducing the burstiness of buffers and improving quality of transmission. Some of the previous models are considered as certain cases for particular values of constraints.

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