

FORMULATION & DISTRIBUTION OF SUPER PRIMES

S. AGARWAL^{1*}, D.K. SHARMA², A.S. UNIYAL³

ABSTRACT. In this article, we have formulated the new primes, called super primes. With the help of super primes, we have defined perfect super primes & non-super primes and found their distribution within a particular interval. Here we have also designed Java program for finding the super primes, perfect super primes and non-super primes within the given interval.

1. Introduction

Prime numbers are more than just numbers that can only be divided by themselves and 1. G. H. Hardy & E. M. Wright (1960) defined that a number P is said to be prime if

- (i) $P > 1$,
- (ii) P has no positive divisors except 1 and P.

They are a mathematical mystery, the secrets of which mathematicians have been trying to uncover ever since Euclid proved that they have no end. Currently there is no known efficient formula for primes. Besides that, prime numbers have great significance in different fields, and their distribution and possible or impossible functional generation among the natural numbers is an ancient dilemma. Granville A. (1995) & Cramer H. (2020) worked upon the distribution of prime numbers. L. Debnath & K. Basu (2015) described the distribution of prime numbers, prime number theorems, Euler's and Riemann's zeta functions and their link with prime numbers and also discovered the Fermat and the Mersenne prime numbers. In the last two decades, many researchers discovered new primes and tried to find their distribution. In this paper we have defined super primes, perfect super primes, non-super primes and found their distribution in a particular interval.

2. Super Prime

A prime P is called a super prime, if for $P = abcd\dots\dots$ having n digits,

$$f(P) = \sum_{i=1}^n S_i = S_1 + S_2 + \dots + S_n \text{ is also a prime.} \quad \dots(2.1)$$

where, $i = 1, 2, 3, \dots, n$ and

$S_1 =$ Sum of all nC_1 digits of number P
 $= a + b + c + d + \dots\dots$

$S_2 =$ Sum of all nC_2 products of two digits of number P taken at a time
 $= a.b + b.c + c.d + d.a + a.c + a.d + b.d + \dots\dots$

$S_3 =$ Sum of all nC_3 products of three digits of number P taken at a time
 $= a.b.c + b.c.d + c.d.a + a.b.d + \dots\dots$

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**Corresponding Author*

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S_n = Sum of all nC_n products of n digits of number P taken at a time
 Also for every super prime P of order n,

$$f(P) = \sum_{i=1}^n S_i = S_1 + S_2 + \dots + S_n \leq P \quad \dots(2.2)$$

2.1 Non-super prime

A prime number which is not a super prime is called a non-super prime number.

2.2 Perfect Super Prime

A super prime P is called a perfect super prime, if

$$f(P) = S_1 + S_2 + \dots + S_n = P. \quad \dots(2.3)$$

3. Digital Order of Super Prime

Number of digits in a super prime is called the digital order of the super prime.

3.1 Super prime of digital order 1

Every prime number of one digit is a super prime of digital order 1. Super primes of digital order 1 are shown in Table-1.

Table-1 Super Primes of digital order 1

One digit prime number (P)	f(P)	Super prime / Not a super prime
2	f(2) = 2 (Prime)	Super prime
3	f(3) = 3 (Prime)	Super prime
5	f(5) = 5 (Prime)	Super prime
7	f(7) = 7 (Prime)	Super prime

2, 3, 5, 7 are Super primes of digital order 1.

From the above table it is clear that,

Number of super primes of digital order 1, $N[f(P) \leq P] = 4$

Number of perfect super primes of digital order 1, $N[f(P) = P] = 4$

Number of non-super primes = 0

3.2 Super prime of digital order 2

Every two digit super prime is called a Super prime of digital order 2. Super primes of digital order 2 are shown in Table-2.

Table-2 Super Primes of digital order 2

Two digit prime number (P)	f(P)	Super prime / Not a super prime
11	$f(11) = 1 + 1 + 1 = 3$ (Prime)	Super prime
13	$f(13) = 1 + 3 + 3 = 7$ (Prime)	Super prime
17	$f(17) = 1 + 7 + 7 = 15$ (Not a prime)	Not a super prime
19	$f(19) = 1 + 9 + 9 = 19$ (Prime)	Super prime
23	$f(23) = 2 + 3 + 6 = 11$ (Prime)	Super prime
29	$f(29) = 2 + 9 + 18 = 29$ (Prime)	Super prime
31	$f(31) = 3 + 1 + 3 = 7$ (Prime)	Super prime
37	$f(37) = 3 + 7 + 21 = 31$ (Prime)	Super prime
41	$f(41) = 4 + 1 + 4 = 9$ (Not a prime)	Not a super prime
43	$f(43) = 4 + 3 + 12 = 19$ (Prime)	Super prime
47	$f(47) = 4 + 7 + 28 = 39$ (Not a prime)	Not a super prime
53	$f(53) = 5 + 3 + 15 = 23$ (Prime)	Super prime
59	$f(59) = 5 + 9 + 45 = 59$ (Prime)	Super prime
61	$f(61) = 6 + 1 + 6 = 13$ (Prime)	Super prime
67	$f(67) = 6 + 7 + 42 = 55$ (Not a prime)	Not a super prime
71	$f(71) = 7 + 1 + 7 = 15$ (Not a prime)	Not a super prime
73	$f(73) = 7 + 3 + 21 = 31$ (Prime)	Super prime
79	$f(79) = 7 + 9 + 63 = 79$ (Prime)	Super prime
83	$f(83) = 8 + 3 + 24 = 35$ (Not a prime)	Not a super prime
89	$f(89) = 8 + 9 + 72 = 89$ (Prime)	Super prime
97	$f(97) = 9 + 7 + 63 = 79$ (Prime)	Super prime

11, 13, 19, 23, 29, 31, 37, 43, 53, 59, 61, 73, 79, 89, 97 are super primes of digital order 2.

From the above table it is clear that,

Number of super primes of digital order 2, $N[f(P) \leq P] = 15$

Number of perfect super primes of digital order 2, $N[f(P) = P] = 5$

Number of non-super primes = 6

3.3 Super Primes of digital order 3

A three digit super prime is called a super prime of digital order 3. Super primes of digital order 3 are shown in Table-3.

Table-3 Super Primes of digital order 3

Three digit prime number (P)	f(P)	Super prime / Not a super prime
101	$f(101) = 1 + 0 + 1 + 0 + 0 + 1 + 0 = 3$ (Prime)	Super prime
103	$f(103) = 1 + 0 + 3 + 0 + 0 + 3 + 0 = 7$ (Prime)	Super prime
107	$f(107) = 1 + 0 + 7 + 0 + 0 + 7 + 0 = 15$ (Not a prime)	Not a super prime

109	$f(109) = 1 + 0 + 9 + 0 + 0 + 9 + 0 = 19$ (Prime)	Super prime
113	$f(113) = 1 + 1 + 3 + 1 + 3 + 3 + 3 = 15$ (Not a prime)	Not a super prime
127	$f(127) = 1 + 2 + 7 + 2 + 14 + 7 + 14 = 47$ (Prime)	Super prime
131	$f(131) = 1 + 3 + 1 + 3 + 3 + 1 + 3 = 15$ (Not a prime)	Not a super prime
137	$f(137) = 1 + 3 + 7 + 3 + 21 + 7 + 21 = 63$ (Not a prime)	Not a super prime
139	$f(139) = 1 + 3 + 9 + 3 + 27 + 9 + 27 = 79$ (Prime)	Super prime
149	$f(149) = 1 + 4 + 9 + 4 + 36 + 9 + 36 = 99$ (Not a prime)	Not a super prime
151	$f(151) = 1 + 5 + 1 + 5 + 5 + 1 + 5 = 23$ (Prime)	Super prime
157	$f(157) = 1 + 5 + 7 + 5 + 35 + 7 + 35 = 95$ (Not a prime)	Not a super prime
163	$f(163) = 1 + 6 + 3 + 6 + 18 + 3 + 18 = 55$ (Not a prime)	Not a super prime
167	$f(167) = 1 + 6 + 7 + 6 + 42 + 7 + 42 = 111$ (Not a prime)	Not a super prime
173	$f(173) = 1 + 7 + 3 + 7 + 21 + 3 + 21 = 63$ (Not a prime)	Not a super prime
179	$f(179) = 1 + 7 + 9 + 7 + 63 + 9 + 63 = 159$ (Not a prime)	Not a super prime
181	$f(181) = 1 + 8 + 1 + 8 + 8 + 1 + 8 = 35$ (Not a prime)	Not a super prime
191	$f(191) = 1 + 9 + 1 + 9 + 9 + 1 + 9 = 39$ (Not a prime)	Not a super prime
193	$f(193) = 1 + 9 + 3 + 9 + 27 + 3 + 27 = 79$ (Prime)	Super prime
197	$f(197) = 1 + 9 + 7 + 9 + 63 + 7 + 63 = 159$ (Not a prime)	Not a super prime
199	$f(199) = 1 + 9 + 9 + 9 + 81 + 9 + 81 = 199$ (Prime)	Super prime
211	$f(211) = 2 + 1 + 1 + 2 + 1 + 2 + 2 = 11$ (Prime)	Super prime
223	$f(223) = 2 + 2 + 3 + 4 + 6 + 6 + 12 = 35$ (Not a prime)	Not a super prime
227	$f(227) = 2 + 2 + 7 + 4 + 14 + 14 + 28 = 71$ (Prime)	Super prime
229	$f(229) = 2 + 2 + 9 + 4 + 18 + 18 + 36 = 89$ (Prime)	Super prime
233	$f(233) = 2 + 3 + 3 + 6 + 9 + 6 + 18 = 47$ (Prime)	Super prime
239	$f(239) = 2 + 3 + 9 + 6 + 27 + 18 + 54 = 119$ (Not a prime)	Not a super prime
241	$f(241) = 2 + 4 + 1 + 8 + 4 + 2 + 8 = 29$ (Prime)	Super prime
251	$f(251) = 2 + 5 + 1 + 10 + 5 + 2 + 10 = 35$ (Not a prime)	Not a super prime
257	$f(257) = 2 + 5 + 7 + 10 + 35 + 14 + 70 = 143$ (Not a prime)	Not a super prime
263	$f(263) = 2 + 6 + 3 + 12 + 18 + 6 + 36 = 83$ (Prime)	Super prime
269	$f(269) = 2 + 6 + 9 + 12 + 54 + 18 + 108 = 209$ (Not a prime)	Not a super prime
271	$f(271) = 2 + 7 + 1 + 14 + 7 + 2 + 14 = 47$ (Prime)	Super prime
277	$f(277) = 2 + 7 + 7 + 14 + 49 + 14 + 98 = 191$ (Prime)	Super prime
281	$f(281) = 2 + 8 + 1 + 16 + 8 + 2 + 16 = 53$ (Prime)	Super prime
283	$f(283) = 2 + 8 + 3 + 16 + 24 + 6 + 48 = 107$ (Prime)	Super prime
293	$f(293) = 2 + 9 + 3 + 18 + 27 + 6 + 54 = 119$ (Not a prime)	Not a super prime
307	$f(307) = 3 + 0 + 7 + 0 + 0 + 21 + 0 = 31$ (Prime)	Super prime
311	$f(311) = 3 + 1 + 1 + 3 + 1 + 3 + 3 = 15$ (Not a prime)	Not a super prime
313	$f(313) = 3 + 1 + 3 + 3 + 3 + 9 + 9 = 31$ (Prime)	Super prime
317	$f(317) = 3 + 1 + 7 + 3 + 7 + 21 + 21 = 63$ (Not a prime)	Not a super prime
331	$f(331) = 3 + 3 + 1 + 9 + 3 + 3 + 9 = 31$ (Prime)	Super prime
337	$f(337) = 3 + 3 + 7 + 9 + 21 + 21 + 63 = 127$ (Prime)	Super prime
347	$f(347) = 3 + 4 + 7 + 12 + 28 + 21 + 84 = 159$ (Not a prime)	Not a super prime
349	$f(349) = 3 + 4 + 9 + 12 + 36 + 27 + 108 = 199$ (Prime)	Super prime
353	$f(353) = 3 + 5 + 3 + 15 + 15 + 9 + 45 = 95$ (Not a prime)	Not a super prime
359	$f(359) = 3 + 5 + 9 + 15 + 45 + 27 + 135 = 239$ (Prime)	Super prime
367	$f(367) = 3 + 6 + 7 + 18 + 42 + 21 + 126 = 223$ (Prime)	Super prime
373	$f(373) = 3 + 7 + 3 + 21 + 21 + 9 + 63 = 127$ (Prime)	Super prime
379	$f(379) = 3 + 7 + 9 + 21 + 63 + 27 + 189 = 319$ (Not a prime)	Not a super prime
383	$f(383) = 3 + 8 + 3 + 24 + 24 + 9 + 72 = 143$ (Not a prime)	Not a super prime

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389	$f(389) = 3 + 8 + 9 + 24 + 72 + 27 + 216 = 359$ (Prime)	Super prime
397	$f(397) = 3 + 9 + 7 + 27 + 63 + 21 + 189 = 319$ (Not a prime)	Not a super prime
401	$f(401) = 4 + 0 + 1 + 0 + 0 + 4 + 0 = 9$ (Not a prime)	Not a super prime
409	$f(409) = 4 + 0 + 9 + 0 + 0 + 36 + 0 = 49$ (Not a prime)	Not a super prime
419	$f(419) = 4 + 1 + 9 + 4 + 9 + 36 + 36 = 99$ (Not a prime)	Not a super prime
421	$f(421) = 4 + 2 + 1 + 8 + 2 + 4 + 8 = 29$ (Prime)	Super prime
431	$f(431) = 4 + 3 + 1 + 12 + 3 + 4 + 12 = 39$ (Not a prime)	Not a super prime
433	$f(433) = 4 + 3 + 3 + 12 + 9 + 12 + 36 = 79$ (Prime)	Super prime
439	$f(439) = 4 + 3 + 9 + 12 + 27 + 36 + 108 = 199$ (Prime)	Super prime
443	$f(443) = 4 + 4 + 3 + 16 + 12 + 12 + 48 = 99$ (Not a prime)	Not a super prime
449	$f(449) = 4 + 4 + 9 + 16 + 36 + 36 + 144 = 249$ (Not a prime)	Not a super prime
457	$f(457) = 4 + 5 + 7 + 20 + 35 + 28 + 140 = 239$ (Prime)	Super prime
461	$f(461) = 4 + 6 + 1 + 24 + 6 + 4 + 24 = 69$ (Not a prime)	Not a super prime
463	$f(463) = 4 + 6 + 3 + 24 + 18 + 12 + 72 = 139$ (Prime)	Super prime
467	$f(467) = 4 + 6 + 7 + 24 + 42 + 28 + 168 = 279$ (Not a prime)	Not a super prime
479	$f(479) = 4 + 7 + 9 + 28 + 63 + 36 + 252 = 399$ (Not a prime)	Not a super prime
487	$f(487) = 4 + 8 + 7 + 32 + 56 + 28 + 224 = 359$ (Prime)	Super prime
491	$f(491) = 4 + 9 + 1 + 36 + 9 + 4 + 36 = 99$ (Not a prime)	Not a super prime
499	$f(499) = 4 + 9 + 9 + 36 + 81 + 36 + 324 = 499$ (Prime)	Super prime
503	$f(503) = 5 + 0 + 3 + 0 + 0 + 15 + 0 = 23$ (Prime)	Super prime
509	$f(509) = 5 + 0 + 9 + 0 + 0 + 45 + 0 = 59$ (Prime)	Super prime
521	$f(521) = 5 + 2 + 1 + 10 + 2 + 5 + 10 = 35$ (Not a prime)	Not a super prime
523	$f(523) = 5 + 2 + 3 + 10 + 6 + 15 + 30 = 71$ (Prime)	Super prime
541	$f(541) = 5 + 4 + 1 + 20 + 4 + 5 + 20 = 59$ (Prime)	Super prime
547	$f(547) = 5 + 4 + 7 + 20 + 28 + 35 + 140 = 239$ (Prime)	Super prime
557	$f(557) = 5 + 5 + 7 + 25 + 35 + 35 + 175 = 287$ (Not a prime)	Not a super prime
563	$f(563) = 5 + 6 + 3 + 30 + 18 + 15 + 90 = 167$ (Prime)	Super prime
569	$f(569) = 5 + 6 + 9 + 30 + 54 + 45 + 270 = 419$ (Prime)	Super prime
571	$f(571) = 5 + 7 + 1 + 35 + 7 + 5 + 35 = 95$ (Not a prime)	Not a super prime
577	$f(577) = 5 + 7 + 7 + 35 + 49 + 35 + 245 = 383$ (Prime)	Super prime
587	$f(587) = 5 + 8 + 7 + 40 + 56 + 35 + 280 = 431$ (Prime)	Super prime
593	$f(593) = 5 + 9 + 3 + 45 + 27 + 15 + 135 = 239$ (Prime)	Super prime
599	$f(599) = 5 + 9 + 9 + 45 + 81 + 45 + 405 = 599$ (Prime)	Super prime
601	$f(601) = 6 + 0 + 1 + 0 + 0 + 6 + 0 = 13$ (Prime)	Super prime
607	$f(607) = 6 + 0 + 7 + 0 + 0 + 42 + 0 = 55$ (Not a prime)	Not a super prime
613	$f(613) = 6 + 1 + 3 + 6 + 3 + 18 + 18 = 55$ (Not a prime)	Not a super prime
617	$f(617) = 6 + 1 + 7 + 6 + 7 + 42 + 42 = 111$ (Not a prime)	Not a super prime
619	$f(619) = 6 + 1 + 9 + 6 + 9 + 54 + 54 = 139$ (Prime)	Super prime
631	$f(631) = 6 + 3 + 1 + 18 + 3 + 6 + 18 = 55$ (Not a prime)	Not a super prime
641	$f(641) = 6 + 4 + 1 + 24 + 4 + 6 + 24 = 69$ (Not a prime)	Not a super prime
643	$f(643) = 6 + 4 + 3 + 24 + 12 + 18 + 72 = 139$ (Prime)	Super prime
647	$f(647) = 6 + 4 + 7 + 24 + 28 + 42 + 168 = 279$ (Not a prime)	Not a super prime
653	$f(653) = 6 + 5 + 3 + 30 + 15 + 18 + 90 = 167$ (Prime)	Super prime
659	$f(659) = 6 + 5 + 9 + 30 + 45 + 54 + 270 = 419$ (Prime)	Super prime
661	$f(661) = 6 + 6 + 1 + 36 + 6 + 6 + 36 = 97$ (Prime)	Super prime
673	$f(673) = 6 + 7 + 3 + 42 + 21 + 18 + 126 = 223$ (Prime)	Super prime
677	$f(677) = 6 + 7 + 7 + 42 + 49 + 42 + 294 = 447$ (Not a prime)	Not a super prime
683	$f(683) = 6 + 8 + 3 + 48 + 24 + 18 + 144 = 251$ (Prime)	Super prime

691	$f(691) = 6 + 9 + 1 + 54 + 9 + 6 + 54 = 139$ (Prime)	Super prime
701	$f(701) = 7 + 0 + 1 + 0 + 0 + 7 + 0 = 15$ (Not a prime)	Not a super prime
709	$f(709) = 7 + 0 + 9 + 0 + 0 + 63 + 0 = 79$ (Prime)	Super prime
719	$f(719) = 7 + 1 + 9 + 7 + 9 + 63 + 63 = 159$ (Not a prime)	Not a super prime
727	$f(727) = 7 + 2 + 7 + 14 + 14 + 49 + 98 = 191$ (Prime)	Super prime
733	$f(733) = 7 + 3 + 3 + 21 + 9 + 21 + 63 = 127$ (Prime)	Super prime
739	$f(739) = 7 + 3 + 9 + 21 + 27 + 63 + 189 = 319$ (Not a prime)	Not a super prime
743	$f(743) = 7 + 4 + 3 + 28 + 12 + 21 + 84 = 159$ (Not a prime)	Not a super prime
751	$f(751) = 7 + 5 + 1 + 35 + 5 + 7 + 35 = 95$ (Not a prime)	Not a super prime
757	$f(757) = 7 + 5 + 7 + 35 + 35 + 49 + 245 = 383$ (Prime)	Super prime
761	$f(761) = 7 + 6 + 1 + 42 + 6 + 7 + 42 = 111$ (Not a prime)	Not a super prime
769	$f(769) = 7 + 6 + 9 + 42 + 54 + 63 + 378 = 559$ (Not a prime)	Not a super prime
773	$f(773) = 7 + 7 + 3 + 49 + 21 + 21 + 147 = 255$ (Not a prime)	Not a super prime
787	$f(787) = 7 + 8 + 7 + 56 + 56 + 49 + 392 = 575$ (Not a prime)	Not a super prime
797	$f(797) = 7 + 9 + 7 + 63 + 63 + 49 + 441 = 639$ (Not a prime)	Not a super prime
809	$f(809) = 8 + 0 + 9 + 0 + 0 + 72 + 0 = 89$ (Prime)	Super prime
811	$f(811) = 8 + 1 + 1 + 8 + 1 + 8 + 8 = 35$ (Not a prime)	Not a super prime
821	$f(821) = 8 + 2 + 1 + 16 + 2 + 8 + 16 = 53$ (Prime)	Super prime
823	$f(823) = 8 + 2 + 3 + 16 + 6 + 24 + 48 = 107$ (Prime)	Super prime
827	$f(827) = 8 + 2 + 7 + 16 + 14 + 56 + 112 = 215$ (Not a prime)	Not a super prime
829	$f(829) = 8 + 2 + 9 + 16 + 18 + 72 + 144 = 269$ (Prime)	Super prime
839	$f(839) = 8 + 3 + 9 + 24 + 27 + 72 + 216 = 359$ (Prime)	Super prime
853	$f(853) = 8 + 5 + 3 + 40 + 15 + 24 + 120 = 215$ (Not a prime)	Not a super prime
857	$f(857) = 8 + 5 + 7 + 40 + 35 + 56 + 280 = 431$ (Prime)	Super prime
859	$f(859) = 8 + 5 + 9 + 40 + 45 + 72 + 360 = 539$ (Not a prime)	Not a super prime
863	$f(863) = 8 + 6 + 3 + 48 + 18 + 24 + 144 = 251$ (Prime)	Super prime
877	$f(877) = 8 + 7 + 7 + 56 + 49 + 56 + 392 = 575$ (Not a prime)	Not a super prime
881	$f(881) = 8 + 8 + 1 + 64 + 8 + 8 + 64 = 161$ (Not a prime)	Not a super prime
883	$f(883) = 8 + 8 + 3 + 64 + 24 + 24 + 192 = 323$ (Not a prime)	Not a super prime
887	$f(887) = 8 + 8 + 7 + 64 + 56 + 56 + 448 = 647$ (Prime)	Super prime
907	$f(907) = 9 + 0 + 7 + 0 + 0 + 63 + 0 = 79$ (Prime)	Super prime
911	$f(911) = 9 + 1 + 1 + 9 + 1 + 9 + 9 = 39$ (Not a prime)	Not a super prime
919	$f(919) = 9 + 1 + 9 + 9 + 9 + 81 + 81 = 199$ (Prime)	Super prime
929	$f(929) = 9 + 2 + 9 + 18 + 18 + 81 + 162 = 299$ (Not a prime)	Not a super prime
937	$f(937) = 9 + 3 + 7 + 27 + 21 + 63 + 189 = 319$ (Not a prime)	Not a super prime
941	$f(941) = 9 + 4 + 1 + 36 + 4 + 9 + 36 = 99$ (Not a prime)	Not a super prime
947	$f(947) = 9 + 4 + 7 + 36 + 28 + 63 + 252 = 399$ (Not a prime)	Not a super prime
953	$f(953) = 9 + 5 + 3 + 45 + 15 + 27 + 135 = 239$ (Prime)	Super prime
967	$f(967) = 9 + 6 + 7 + 54 + 42 + 63 + 378 = 559$ (Not a prime)	Not a super prime
971	$f(971) = 9 + 7 + 1 + 63 + 7 + 9 + 63 = 159$ (Not a prime)	Not a super prime
977	$f(977) = 9 + 7 + 7 + 63 + 49 + 63 + 441 = 639$ (Not a prime)	Not a super prime
983	$f(983) = 9 + 8 + 3 + 72 + 24 + 27 + 216 = 359$ (Prime)	Super prime
991	$f(991) = 9 + 9 + 1 + 81 + 9 + 9 + 81 = 199$ (Prime)	Super prime
997	$f(997) = 9 + 9 + 7 + 81 + 63 + 63 + 567 = 799$ (Not a prime)	Not a super prime

101, 103, 109, 127, 139, 151, 193, 199, 211, 227, 229, 233, 241, 263, 271, 277, 281, 283, 307, 313, 331, 337, 349, 359, 367, 373, 389, 421, 433, 439, 457, 463, 487, 499, 503, 509, 523, 541, 547, 563, 569, 577, 587, 593, 599, 601, 619, 643, 653, 659, 661,

673, 683, 691, 709, 727, 733, 757, 809, 821, 823, 829, 839, 857, 863, 887, 907, 919, 953, 983, 991 are super primes of digital order 3.

From the above table it is clear that,

Number of super primes of digital order 3, $N[f(P) \leq P] = 71$

Number of perfect super primes of digital order 3, $N[f(P) = P] = 3$

Number of non-super primes = 72

4. Java Program to find Super Primes, Perfect Super Primes & Non-super Primes within a Particular Interval

```
import java.util.*;
public class superprime
{
    static int sum=0;
    static void printCombinations(int[] sequence, int N)
    {
        int[] data = new int[N];
        for (int r = 0; r <= sequence.length; r++)
        {
            combinations(sequence, data, 0, N - 1, 0, r);
        }
    }
    static void combinations(int[] sequence, int[] data, int start, int end, int index,
int r)
    {
        if (index == r)
        {
            int num=1;
            for (int j = 0; j < r; j++)
            {
                num=num*data[j];
            }
            sum=sum+num;
        }
        for (int i = start; i <= end && ((end - i + 1) >= (r - index)); i++)
        {
            data[index] = sequence[i];
            combinations(sequence, data, i + 1, end, index + 1, r);
        }
    }
    public static void main(String args[])
    {
        Scanner in=new Scanner(System.in);
        long n,a,dig,min,max;
```

```

inti,c=0,count1=0,count2=0,count3=0;
boolean flag1,flag2;
int perfect[]=new int[500];
System.out.println("Enter minimum value");
min=in.nextLong();
System.out.println("Enter maximum value");
max=in.nextLong();
System.out.println("\n The Super Primes in the given range are: \n");
for(n=min;n<=max;n++)
{
    sum=0;
    flag1=true;
    if(n==1)
    flag1=false;
    for(i=2;i<=n/2;i++)
    {
        if(n%i==0)
        {
            flag1=false;
            break;
        }
    }
    if(flag1==true)
    {
        count3++;
        a=n;
        c=0;
        while(a>0)
        {
            c++;
            a=a/10;
        }
        a=n;
        int sequence[]=new int[c];
        i=c-1;
        while(a>0)
        {
            dig=a%10;
            sequence[i]=(int)dig;
            i--;
            a=a/10;
        }
        printCombinations(sequence, sequence.length);
        sum--;
        flag2=true;
    }
}

```


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```

        for(i=2;i<=sum/2;i++)
        {
            if(sum%i==0)
            {
                flag2=false;
                break;
            }
        }
        if(flag2==true)
        {
            System.out.print(n+"\t");
            count1++;
            if(n==sum)
            {
                perfect[count2++]=n;
            }
        }
    }
}
System.out.println("\n The Perfect Super Primes in the given range are:\n");
for(i=0;i<count2;i++)
System.out.print(perfect[i)+"\t");
System.out.println();
System.out.println("Total number of Super Primes are: "+count1);
System.out.println("Total number of Perfect Super Primes are: "+count2);
System.out.println("Total number of Non-Super Primes are: "+(count3-count1));
}
}

```

5. Distribution of Super Primes, Prefect Super Primes & Non-Super Primes

From the Java program the results are obtained for the distribution of super primes, prefect super primes & non-Super primes, which are shown in Table-4.

Table-4 Distribution of Super primes, Prefect super primes & Non-Super primes

Interval	No. of Super primes	No. of Perfect super primes	No. of Non-super primes
[1-1000]	90	12	78
[1001-2000]	51	1	84
[2001-3000]	77	1	50
[3001-4000]	49	0	71
[4001-5000]	53	1	66
[5001-6000]	62	0	52
[6001-7000]	44	0	73
[7001-8000]	34	0	73

[8001-9000]	43	1	67
[9001-10000]	48	0	64
[10001-11000]	39	0	67
[11001-12000]	30	0	73
[12001-13000]	59	0	50
[13001-14000]	29	0	76
[14001-15000]	36	0	66
[15001-16000]	45	0	63
[16001-17000]	28	0	70
[17001-18000]	28	0	76
[18001-19000]	42	0	52
[19001-20000]	32	0	72

Similarly, Super primes, Prefect Super primes & Non-Super primes within any interval can be determined.

6. Graphical Representation of Distribution of Super Primes, Prefect Super Primes & Non-Super Primes

The graphical representation of distribution of super primes, prefect super primes & non-super primes is shown in Figure-1.

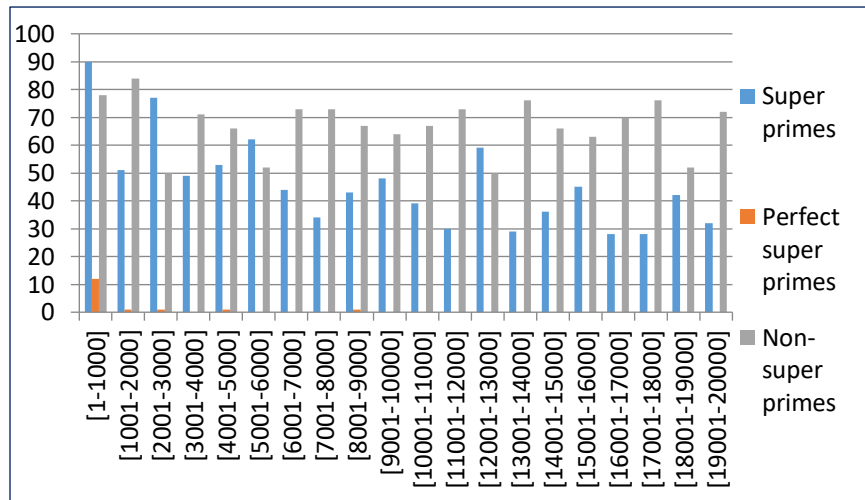


Figure-1 Distribution of Super primes, Prefect Super primes & Non-Super primes

Theorem 6.1: For a prime P , the total number of terms in the expansion of $f(P)$ is equal to $2^n - 1$.

Proof: We know that the function,

$$f(P) = \sum_{i=1}^n S_i = S_1 + S_2 + \dots + S_n$$

where, $i = 1, 2, 3, \dots, n$ and

$S_1 =$ Sum of all ${}^n C_1$ digits of number P
 $= a + b + c + d + \dots$

$S_2 =$ Sum of all ${}^n C_2$ products of two digits of number P taken at a time
 $= a.b + b.c + c.d + d.a + a.c + a.d + b.d + \dots$

$S_3 =$ Sum of all ${}^n C_3$ products of three digits of number P taken at a time
 $= a.b.c + b.c.d + c.d.a + a.b.d + \dots$

.....

.....

$S_n =$ Sum of all ${}^n C_n$ products of n digits of number P taken at a time

Therefore, $f(P) =$ Sum of $({}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n)$ terms(6.1)

The binomial expansion of term $(1+x)^n$ is given by

$$(1+x)^n = {}^n C_0 x^0 + {}^n C_1 x^1 + {}^n C_2 x^2 + {}^n C_3 x^3 + \dots + {}^n C_n x^n$$

Put $x = 1$, we get

$$(1+1)^n = {}^n C_0 1^0 + {}^n C_1 1^1 + {}^n C_2 1^2 + {}^n C_3 1^3 + \dots + {}^n C_n 1^n$$

$${}^n C_0 + {}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n = 2^n$$

or, $1 + {}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n = 2^n$

or, ${}^n C_1 + {}^n C_2 + {}^n C_3 + \dots + {}^n C_n = 2^n - 1$

Using this value in equation (6.1), we have

$$f(P) = \text{Sum of } (2^n - 1) \text{ terms}$$

Therefore, the total number of terms in the expansion of $f(P)$ is equal to $2^n - 1$.

7. Conclusion

Here in this paper we have defined new primes and found their distribution in a particular interval. The study will definitely provide the further scope of research in the field of number theory and also its applications in different fields.

8. Further Scope of Study

In our upcoming research papers, we will find the application of super primes in cryptography for secure communication. Also we will compare the super primes with the existing primes and try to generate some relationship among them.

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¹S. AGARWAL: ASSOCIATE PROFESSOR, DEPARTMENT OF MATHEMATICS, NEW DELHI INSTITUTE OF MANAGEMENT, NEW DELHI, INDIA, (EMAIL: meshubhamagarwal@gmail.com)

²D.K. SHARMA: RESEARCH SCHOLAR, DEPARTMENT OF MATHEMATICS, M.B. (GOVT.) P.G. COLLEGE, HALDWANI, INDIA, (EMAIL: sharmadeepak0111209@gmail.com)

³A.S. UNIYAL: ASSOCIATE PROFESSOR, DEPARTMENT OF MATHEMATICS, M.B. (GOVT.) P.G. COLLEGE, HALDWANI, INDIA, (EMAIL: asuniyal0111@gmail.com)