Global and Stochastic Analysis
Vol. 8 No. 2 (July-December, 2021)

MUK PUBLICATIONS
Open Access Publisher

INTERVAL VALUED VAGUE IDEALS IN NEARRINGS

NEHA GAHLOT*, NAGARAJU DASARI, MOHIDDIN SHAW SHAIK,
AND BABU PRASAD MUNAGALA

Abstract. In the present paper we propose interval valued (I-V)vague ideals in nearrings, characterizations of I-V vague ideals in nearrings and give some examples related to this. Also study direct product and also talked about normal I-V of vague ideals.

1. Introduction

Dr. Zadeh introduced fuzzy set in 1965[4]. In this he talked about true membership value of a function. Gau and Buehrer introduced the concept of vague theory in [15]. In vague set they talk about membership value of a function as well as non-membership value. To increase the study of vague sets, many authors have considered several extension works of fuzzy sets in [2]. In a vague set, the universal set $\mathfrak{X}$ can be represented by a pair of functions ($\tau_{\mathfrak{X}}$, $\sigma_{\mathfrak{X}}$). In this $\tau_{\mathfrak{X}}$ and $\sigma_{\mathfrak{X}} u$ are the functions from $\mathfrak{X}$ to [0,1] such as $\tau_{\mathfrak{X}}(i) + \sigma_{\mathfrak{X}}(i) \leq 1$. Here $\tau_{\mathfrak{X}}$ gives membership value and $\sigma_{\mathfrak{X}}$ is non-membership value. In the domain of fuzzy sets, a vague set is referred to as an interval membership function, as opposed to point membership. This notion is used in a variety of fields, including fuzzy control systems, decision making, fault diagnostics, knowledge discovery and many more. Vague ideals in group theory introduced by Biswas [12]. A similar concept of vague set is intuitionistic set is given by many researchers in [16],[7]. Vague ideals in Gamma-Nearrings introduced by Y.Bhargav and S.Ragamayi in [14]. L. Bhaskar given ideal of sum in vague ideal of nearring in [6].
Vague set is interval membership function but it is not I-V function because in vague set first value represented membership value and second value represented non-membership value, but in I-V function whole interval represent either membership function or non-membership value's.
Here we introduced I-V vague ideals in nearring and show that vague interval and interval valued vague ideal is different and give some examples related to this.

Also talk about normal interval valued vague ideal and direct product of vague ideals.

2000 *Mathematics Subject Classification.* Primary 16Y30; Secondary 16Y99,03E72.
Key words and phrases. Vague ideals, nearrings, vague ideal in nearrings, I-V ideals, direct product, weakest vague ideals, normal vague ideals.
This research is supported by faculty members of Manipal University Jaipur.

For standard definitions and results we refer Book and article [3], [1]. A vague collection $\mathfrak{A}$ in the discourse universe $\mathfrak{X}$ is a pair ($\tau_{\mathfrak{A}}$, $\sigma_{\mathfrak{A}}$) where $\tau_{\mathfrak{A}}$: $\mathfrak{X} \rightarrow [0, 1]$ and $\sigma_{\mathfrak{A}}$: $\mathfrak{X} \rightarrow [0, 1]$ are mappings such as $\tau_{\mathfrak{A}}(\mathrm{i}) + \sigma_{\mathfrak{A}}(\mathrm{i}) \leq 1 \; \forall \; i \in \mathfrak{X}$. The membership function and non-membership function in [0,1] is $\tau_{\mathfrak{A}}$ and $\sigma_{\mathfrak{A}}$ correspondingly. And $[\tau_{\mathfrak{A}}$, 1 - $\sigma_{\mathfrak{A}}]$ is called interval vague value of i in $\mathfrak{A}$ and it is indicated by $\mathfrak{V}_{\mathfrak{A}}(i)$ that is $\mathfrak{V}_{\mathfrak{A}}(i) = [\tau_{\mathfrak{A}}(i)$, 1 - $\sigma_{\mathfrak{A}}(i)]$.

2. Interval valued vague ideals in nearrings

Definition 2.1. A vague collection $\mathfrak{A}$ is I-V vague ideals in nearrings if it is satisfy following properties for i, j, $k \in N$:

(i) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$
(ii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$
(iii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$
its mean that
(iv) $\overline{\tau_{\mathfrak{A}}}(i - j) \geq \min\{\overline{\tau_{\mathfrak{A}}}(i), \overline{\tau_{\mathfrak{A}}}(j)\}$
(v) $\overline{\tau_{\mathfrak{A}}}(j + i - j) \geq \overline{\tau_{\mathfrak{A}}}(i)$
(vi) $\overline{\tau_{\mathfrak{A}}}(i(j + k) - ik) \geq \overline{\tau_{\mathfrak{A}}}(j)$
and
(vii) 1 - $\overline{\sigma_{\mathfrak{A}}}(i - j) \geq \min\{1 - \overline{\sigma_{\mathfrak{A}}}(i), 1 - \overline{\sigma_{\mathfrak{A}}}(j)\}$, or
$\overline{\sigma_{\mathfrak{A}}}(i - j) \leq \max\{\overline{\sigma_{\mathfrak{A}}}(i), \overline{\sigma_{\mathfrak{A}}}(j)\}$
(viii) 1 - $\overline{\sigma_{\mathfrak{A}}}(j + i - j) \geq 1 - \overline{\sigma_{\mathfrak{A}}}(i)$, or
$\overline{\sigma_{\mathfrak{A}}}(j + i - j) \leq \overline{\sigma_{\mathfrak{A}}}(\mathrm{i})$
(ix) 1 - $\overline{\sigma_{\mathfrak{A}}}(i(j + k) - ik) \leq 1 - \overline{\sigma_{\mathfrak{A}}}(j)$, or
$\overline{\sigma_{\mathfrak{A}}}(i(j + k) - ik) \leq \overline{\sigma_{\mathfrak{A}}}(j)$

Example 2.2. Consider a set, $N = \{0^*, i^*, j^*, k^*\}$ be a collection with two operations that are binary (+) and (.) as described below:

+	0^*	i^*	j^*	k^*
0^*	0^*	i^*	j^*	k^*
i^*	i^*	j^*	k^*	0^*
j^*	j^*	k^*	0^*	i^*
k^*	k^*	0^*	i^*	j^*

.	0^*	i^*	j^*	k^*
0^*	0^*	0^*	0^*	0^*
i^*	0^*	j^*	i^*	i^*
j^*	0^*	j^*	0^*	k^*
k^*	0^*	0^*	0^*	k^*

Clearly N is a nearring. Define as $\overline{\mathfrak{V}_{\mathfrak{A}}}(0^*) = [[0.8, 0.9], [0, 0.1]]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(i^*) = [[0.5, 0.5], [0.4, 0.5]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(j^*) = [[0.1, 0.3], [0.6 ,0.7]]$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(k^*) = [[0.6, 0.7], [0.2, 0.3]]$ now by routine calculation we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ is an I-V vague ideals in nearring.

Example 2.3. Write $N = \mathbb{Z}$(set of integers) under operations "+" thus (N, +, .) is a nearring for mapping $\overline{\mathfrak{V}_{\mathfrak{A}}}$: $N \rightarrow [0, 1]$ define by

$$\overline{\tau_{\mathfrak{A}}}(i) = \begin{cases} [0.5, 0.7] & \text{if } i = 2\text{n for some n} \in \mathbb{Z} \\ 0 & \text{if } i \in \text{collection of odd integers} \\ 1 & \text{if } i = 0 \end{cases}$$

$$\overline{\sigma_{\mathfrak{A}}}(i) = \begin{cases} [0.1, 0.3] & \text{if } i = 2\text{n for some n} \in \mathbb{Z} \\ 1 & \text{if } i \in \text{collection of odd integers} \\ 0 & \text{if } i = 0 \end{cases}$$

Now by routine calculation we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal.

Example 2.4. Write $N = \mathbb{Z}_7$(set of integers modulo 7) thus $(N, +, .)$ is nearring then N is defined as follow:

$$\overline{\tau_{\mathfrak{A}}}(i) = \begin{cases} [0.6, 0.9] & \text{if } i \in \mathbb{Z}_7 \\ [0, 0.1] & \text{if } i \notin \mathbb{Z}_7 \end{cases}$$

$$\overline{\sigma_{\mathfrak{A}}}(i) = \begin{cases} [0.3, 0.1] & \text{if } i \in \mathbb{Z}_7 \\ [0.8, 0.9] & \text{if } i \notin Z\mathbb{Z}7 \end{cases}$$

Now by routine calculation we can show that $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ is an I-V vague ideals in nearring.

Proposition 2.5. *If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideals in nearring N satisfies the condition $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i + j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$ then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i + j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j + i)$.*

Proposition 2.6. *Let $\mathfrak{A}$ be a I-V vague ideal of nearring N and if $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)$ then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$.*

Proof. First suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)$ $\forall$ $i, j \in N$ then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(0), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} = \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$ Similarly using $\overline{\mathfrak{V}_{\mathfrak{A}}}(j - u) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$.

Lemma 2.7. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}}$ be an I-V vague ideals in N if $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) < \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$ then $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j - i)$.*

Lemma 2.8. *Let $\Re$ is an I-V vague ideal in N then for any $\overline{e} \leq \overline{f} \subset [0, 1]$ where $\overline{e} + \overline{f} \leq 1$ then $\exists$ an I-V vague ideals $\overline{\mathfrak{V}_{\mathfrak{A}}}$ in N such that $\overline{\mathfrak{V}_{A_{\overline{i},\overline{j}}}} = \Re$. Here $\overline{i}$ and $\overline{j}$ also an intervals.*

Proof. Let consider an I-V vague ideal $\Re$ of N define a function $\overline{\mathfrak{V}_{\mathfrak{A}}}$: $N \to [0, 1]$ by

$$\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \begin{cases} [\overline{e}, \overline{f}] & \text{if } i \in \Re \\ [\overline{0}, \overline{0}] & \text{if } i \notin \Re \end{cases}$$

Where $\overline{e}$, $\overline{f} \subset (0, 1]$. Clearly $\overline{\mathfrak{V}_{A_{\overline{p},\overline{q}}}} = \Re$.

(i) Let $i, j \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = [\overline{e}, \overline{f}] \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$. If there exists atleast one of i and j is not in $\Re$, then $i - j \notin \Re$ thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = [\overline{0}, \overline{0}] \geq \min\{\overline{\mathfrak{V}_{A_1}}(i), \overline{\mathfrak{V}_{A_1}}(j)\}$.

(ii) Let $i \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = [\overline{e}, \overline{f}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$. If there $i \notin \Re$, then $j + i - j \notin \Re$ thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = [\overline{0}, \overline{0}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$.
(iii) Let $i \in \Re$ now $\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) = [\overline{e}, \overline{f}] \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. If there $j \notin \Re$, then $i(j + k) - ik \notin \Re$ thus $\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) = [\overline{0}, \overline{0}] \geq \overline{\mathfrak{V}_{A_1}}(j)$.
Thus $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a I-V vague ideals of N.

Corollary 2.9. *If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N then the collection $N_{\overline{\mathfrak{V}_{\mathfrak{A}}}}(i) = \{i \in N \mid \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}}}(0)\}$ is a ideal of N.*

Proposition 2.10. *If $\overline{\mathcal{P}} \subseteq \overline{Q}$ in I-V vague ideals in nearring N then $\overline{\tau_{\mathcal{P}}} \subseteq \overline{\tau_{Q}}$ and $\overline{\sigma_{Q}} \subseteq \overline{\sigma_{\mathcal{P}}}$.*

Theorem 2.11. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideals of nearrings N iff each level sub collection $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$, $\overline{\mathfrak{b}} \in Im(\overline{\mathfrak{V}_{\mathfrak{A}}})$ is an ideals of N.*

Proof. Let for any I-V vague ideal $\overline{\mathfrak{V}_{\mathfrak{A}}}$ of N and let $\overline{\mathfrak{b}} \in \mathrm{Im}(\overline{\mathfrak{V}_{\mathfrak{A}}})$. (i) For any i, j, $k \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$, we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} = \overline{\mathfrak{b}}$ thus $i - j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$. (ii) For any i, $j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{b}}$ thus $j + i - j \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$. (iii) For any i, j, $k \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) \geq \overline{\mathfrak{V}_{\mathfrak{A}}}(j) = \overline{\mathfrak{b}}$ thus $i(j + k) - ik \in \overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$. Hence $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ is a ideal of N. Conversely assume that $\overline{\mathfrak{V}_{\mathfrak{A}_{\overline{\mathfrak{b}}}}}$ is a ideal of N for every $\overline{\mathfrak{b}} \in \mathrm{Im}(\overline{\mathfrak{V}_{\mathfrak{A}}})$ suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) < \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ), \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)\}$ for some $i_\circ$, $j_\circ \in N$ then by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}\,[\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) + \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ), \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)\}]$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ - j_\circ) < \overline{\mathfrak{b}}_\circ$, $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ) > \overline{\mathfrak{b}}_\circ$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ) > \overline{\mathfrak{b}}_\circ$. Hence $(i_\circ - j_\circ) \notin \overline{V_{A_{\overline{\mathfrak{b}}_\circ}}}$, $i_\circ \in \overline{V_{A_{\overline{\mathfrak{b}}_\circ}}}$ and $j_\circ \in \overline{V_{A_{\overline{\mathfrak{b}}_\circ}}}$ this is contradiction. (ii) Suppose that $\overline{V_{\mathfrak{A}}}(j_\circ + i_\circ - j_\circ) < \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ)$ for some $i_\circ$, $j_\circ \in N$, then by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}[\overline{\mathfrak{V}_{\mathfrak{A}}}\,(j_\circ + i_\circ - j_\circ) + \overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ)]$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ + i_\circ - j_\circ) < \overline{\mathfrak{b}}_\circ$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ) > \overline{\mathfrak{b}}_\circ$. Hence $(j_\circ + i_\circ - j_\circ) \notin \overline{V_{A_{\overline{\mathfrak{b}}_\circ}}}$ and $i_\circ \in \overline{V_{A_{\overline{\mathfrak{b}}_\circ}}}$ this is an contradiction. (iii) Suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ(j_\circ + k_\circ) - i_\circ k_\circ) < \overline{\mathfrak{V}_{\mathfrak{A}}}(v_\circ)$ for some $i_\circ$, $j_\circ$, $k_\circ \in N$ then by taking $\overline{\mathfrak{b}}_\circ = \frac{1}{2}[\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ(j_\circ + k_\circ) - i_\circ k_\circ) + \overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ)]$ we have $\overline{\mathfrak{V}_{\mathfrak{A}}}(i_\circ(j_\circ + k_\circ) - i_\circ k_\circ) < \overline{\mathfrak{b}}_\circ$ and $\overline{\mathfrak{V}_{\mathfrak{A}}}(j_\circ) > \overline{\mathfrak{b}}_\circ$. Hence $i_\circ(j_\circ + k_\circ) - i_\circ k_\circ \notin \overline{\mathfrak{V}_{A_{\overline{\mathfrak{b}}_\circ}}}$ and $j_\circ \in \overline{\mathfrak{V}_{A_{\overline{\mathfrak{b}}_\circ}}}$ a contradiction.
Thus $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a I-V vague ideal of N.

Theorem 2.12. *Let $\overline{\mathfrak{V}_{\mathfrak{A}}}$ be an I-V vague sub collection of nearring N then $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\mathfrak{V}_{\mathfrak{A}}}^-, \overline{\mathfrak{V}_{\mathfrak{A}}}^+]$ is an I-V vague ideal of N if and only if $\overline{\mathfrak{V}_{\mathfrak{A}}}^-$, $\overline{\mathfrak{V}_{\mathfrak{A}}}^+$ are I-V vague ideals of N.*

Proof. proof is straight forward.

Theorem 2.13. *If $\{\overline{\mathfrak{V}_{\mathfrak{A}}} \mid \dot{a} \in \Omega\}$ is a family of I-V vague ideals of N, then $\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}$ is also an I-V vague ideals of N, where Ω is any index set.*

Proof. Let $i, j \in N$. Then $\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) \colon \dot{a} \in \Omega\} \geq \inf\{\min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(j)\} : \dot{a} \in \Omega\} = \min\{\inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i) : \dot{a} \in \Omega\}, \inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i) : \dot{a} \in \Omega\}\} = \min\{\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(j)\}$.

$\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j)$= $\inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) : \dot{a} \in \Omega\} \geq \inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i) : \dot{a} \in \Omega\} = \underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(i)$. Now $\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) = \inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) : \dot{a} \in \Omega\} \geq \inf\{\overline{\mathfrak{V}_{\mathfrak{A}}}(j) : \dot{a} \in \Omega\} = \underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. Therefore $\underset{\dot{a}\in\Omega}{\cap}\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of N.

Theorem 2.14. *If $\{\overline{\mathfrak{V}_{\mathfrak{A}}}_i(i) \mid \dot{a} \in I\}$ be a family of an is an I-V vague ideal of N, then $\underset{\dot{a}\in I}{\vee}\overline{\mathfrak{V}_{\mathfrak{A}}}_{\dot{a}}$ a also I-V vague ideal of N.*

Proof. proof is similar.

Definition 2.15. Consider two nearrings N and M. A homomorphism is a map ß: $N \rightarrow M$ if it satisfying following conditions ß$(i + j)$ = ß(i) + ß(j) and ß(ij) = ß(i)ß(j) $\forall\ i, j \in N$.

Theorem 2.16. *Let N and M be nearrings and an onto homomorphism ß: $N \rightarrow M$. If is a ß invariant I-V vague ideal of N then ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$) is an I-V vague ideal of M.*

Proof. Suppose that N and M be nearrings, ß: $N \rightarrow M$ be an onto homomorphism, $\overline{\mathfrak{V}_{\mathfrak{A}}}$ be a ß invariant I-V vague ideal of N and $i \in M$. Suppose p $\in M$, t $\in$ ß$^{-1}(i)$, p = ß(t) then p $\in$ ß$^{-1}(i) \Rightarrow$ ß(t) = i = ß(p), since $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a ß invariant, $\overline{\mathfrak{V}_{\mathfrak{A}}}$(t) = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(p) $\Rightarrow$ ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i)$ = $\underset{t\in ß^{-1}(i)}{inf}\ \overline{\mathfrak{V}_{\mathfrak{A}}}$(t) = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(p). Hence ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i)$ = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(p). (i) Let i, $j \in M$. Then $\exists$ p, q $\in N$ such that ß(p) = i, ß(q) = $j \Rightarrow$ ß(p - q) = i - j $\Rightarrow$ ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i - j)$ = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(p - q) $\geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}$(p), $\overline{\mathfrak{V}_{\mathfrak{A}}}(q)\}$ = $\min\{$ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i)$, ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(j)\}$. (ii) Let i, $j \in M$. Then $\exists$ p, q $\in N$ such that ß(p) = i, ß(q) = $j \Rightarrow$ ß(q + p - q) = $j + i - j \Rightarrow$ ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(j + i - j)$ = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(q + p - q) $\geq \overline{\mathfrak{V}_{\mathfrak{A}}}$(p) = ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i)$. (iii) Let i, j, $k \in M$. Then $\exists$ p, q, r $\in N$ such that ß(p) = i, ß(q) = j, ß(r) = $k \Rightarrow$ ß(p(q + r) - qr) = $i(j + k) - ik \Rightarrow$ ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(i(j + k) - ik)$ = $\overline{\mathfrak{V}_{\mathfrak{A}}}$(p(q + r) - qr) $\geq \overline{\mathfrak{V}_{\mathfrak{A}}}$(q) = ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$)$(j)$. Hence ß($\overline{\mathfrak{V}_{\mathfrak{A}}}$) is an I-V vague ideal of M

Theorem 2.17. *Let N and M be two nearrings and an homomorphism mapping ß: $N \rightarrow M$. If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is an I-V vague ideal of M then ß$^{-1}$($\overline{\mathfrak{V}_{\mathfrak{A}}}$) is an I-V vague ideal of N.*

Proof. proof is straight forward.

3. Normal interval valued vague ideal

A I-V vague ideal $\overline{\mathfrak{V}_{\mathfrak{A}}}$ of N is know as normal I-V vague ideal if $\overline{\mathfrak{V}_{\mathfrak{A}}}(0)$ = [1, 1] it means that $\tau_{\mathfrak{A}} = 1$ and 1 - $\sigma_{\mathfrak{A}} = 1$.

Note:- Example(3) is an example of Normal interval valued vague ideal.

Theorem 3.1. *Let t $\in$ (0, 1] be a real number. If $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is a normal I-V vague ideal of N, then $\overline{\mathfrak{V}^t_{\mathfrak{A}}}$ is also a normal vague ideal of N and $N_{\overline{\mathfrak{V}^t_{\mathfrak{A}}}} = N_{\overline{\mathfrak{V}_{\mathfrak{A}}}}$.*

Proof.

Theorem 3.2. *Let* $\overline{\mathfrak{V}_{\mathfrak{A}}} = [\overline{\tau_{\mathfrak{A}}}, \overline{\sigma_{\mathfrak{A}}}]$ *be a I-V vague ideal of* N *such that if* $\overline{\tau_{\mathfrak{A}}}(i) + \overline{\sigma_{\mathfrak{A}}}(i) \leq \overline{\tau_{\mathfrak{A}}}(0) + \overline{\sigma_{\mathfrak{A}}}(0)\ \forall\ i \in N$ *where* $\overline{\mathfrak{V}_{\mathfrak{A}+}} = [\overline{\tau_{\mathfrak{A}+}}, \overline{\sigma_{\mathfrak{A}+}}]$ *defined as* $\overline{\tau_{\mathfrak{A}+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1$ *and* $\overline{\sigma_{\mathfrak{A}+}}(i) = \overline{\sigma_{\mathfrak{A}}}(i) - \overline{\sigma_{\mathfrak{A}}}(0)$ *then* $\overline{\mathfrak{V}_{\mathfrak{A}+}}$ *is normal I-V vague ideal of* N*.*

Proof. Firstly we can shown easily that $\overline{\tau_{\mathfrak{A}+}}, \overline{\sigma_{\mathfrak{A}+}}$ is I-V vague ideal, now $\overline{\tau_{\mathfrak{A}+}}(i) + \overline{\sigma_{\mathfrak{A}+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1 + \overline{\sigma_{\mathfrak{A}}}(i) - \overline{\sigma_{\mathfrak{A}}}(0) \leq 1$, thus $\overline{\tau_{\mathfrak{A}+}}, \overline{\sigma_{\mathfrak{A}+}}$ is I-V vague ideal and $\overline{\tau_{\mathfrak{A}+}}(0) = 1$, $\overline{\sigma_{\mathfrak{A}+}}(0) = 0$ then $\overline{\mathfrak{V}_{\mathfrak{A}+}}$ is normal I-V vague ideal of N.

Corollary 3.3. *if* $\overline{\mathfrak{V}_{\mathfrak{A}}}$ *is a I-V vague ideal of* N *and if* $\exists\ \overline{\mathfrak{V}_{\mathfrak{A}+}}(i) = 0$ *for some* $i \in M$ *then* $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = 0\ \forall\ i \in M$*.*

Theorem 3.4. *A I-V vague ideal of* $\overline{\mathfrak{V}_{\mathfrak{A}}}(i)$ *is normal if and only if* $\overline{\mathfrak{V}_{\mathfrak{A}}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}+}}(i)$*.*

Proof. Suppose that $\overline{\mathfrak{V}_{\mathfrak{A}}}(i)$ is normal I-V vague ideal of N let $i \in N$ then $\overline{\tau_{\mathfrak{A}+}}(i) = \overline{\tau_{\mathfrak{A}}}(i) + \overline{\tau_{\mathfrak{A}}}(0) - 1 = \overline{\tau_{\mathfrak{A}}}(i)$ and $\overline{\sigma_{\mathfrak{A}+}}(i) = \overline{\sigma_{\mathfrak{A}}}(i) + \overline{\sigma_{\mathfrak{A}}}(0) = \overline{\sigma_{\mathfrak{A}}}(i)$ and converse is obvious.

4. Direct product of interval valued vague ideal of nearrings

Definition 4.1. Let $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ be an I-V vague ideal of N_l for l = 1, 2...n, then direct product of $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ is a function $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$: $N_1 \times N_2 \times ... \times N_n \to$ D[0,1] defined by $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2..., i_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2)...\overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)\}$.

Theorem 4.2. *If* $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ *and* $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *be two I-V vague ideals in nearrings* M *and* N *respectively then* $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ *is a I-V vague ideal of* $M \times N$*.*

Corollary 4.3. *Direct product of I-V vague ideal of a nearrings* N *is also an I-V vague ideal of* N*.*

Proof. Let $\overline{\mathfrak{V}_{\mathfrak{A}_i}}$ be I-V vague ideal of nearrings N_i, for i = 1, 2...n. Now let $N = N_1 \times N_2 \times ... \times N_n$ be an direct product of nearrings. Let $i = (i_1, i_2, ... i_n)$, $j = (j_1, j_2, ... j_n)$, w = (w$_1$, w$_2$,... w$_n$) $\in N$ now $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}} = \overline{\mathfrak{V}_{\mathfrak{A}}}$ is a product of N, now (i) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}((i_1, i_2, ... i_n) - (j_1, j_2, ... j_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i_1 - j_1, i_2 - j_2, ... i_n - j_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1 - j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2 - j_2), ...\overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n - j_n)\} \geq \min\{\min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1)\}, \min\{\overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2)\}, ...\min\{\overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n), \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n)\} = \min\{(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2, ..., i_n), (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(j_1, j_2, ... j_n)\} = \min\{\overline{\mathfrak{V}_{\mathfrak{A}}}(i), \overline{\mathfrak{V}_{\mathfrak{A}}}(\mathrm{j})\}$. (ii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(j + i - j) = \overline{\mathfrak{V}_{\mathfrak{A}}}((j_1, j_2, ... j_n) + (i_1, i_2, ... i_n) - (j_1, j_2, ... j_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}\{j_1 + i_1 - j_1, j_2 + i_2 - j_2, ... j_n + i_n - j_n\} = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1 + i_1 - j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2 + i_2 - j_2) ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n + i_n - j_n)\} \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2) ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)\} = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2, ... i_n) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i)$. (iii) $\overline{\mathfrak{V}_{\mathfrak{A}}}(i(j + k) - ik) = \overline{\mathfrak{V}_{\mathfrak{A}}}((i_1, i_2, ... i_n)(j_1 + k_2, j_2 + k_2, ... j_n + k_n) - (i_1, i_2, ... i_n)(k_1, k_2, ... k_n)) = \overline{\mathfrak{V}_{\mathfrak{A}}}(i_1(j_1 + k_1) - i_1k_1, i_2(j_2 + j_2) - i_2j_2, ... i_n(j_n + k_n) - i_nk_n) = \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1(\mathrm{j}_1 + k_1) - i_1k_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2(j_2 + k_2) - i_2k_2), ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n(j_n + k_n) - i_nk_n)\} \geq \min\{\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j_1), \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j_2), ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}(j_n)\} = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(j_1, j_2 ..., j_n) = \overline{\mathfrak{V}_{\mathfrak{A}}}(j)$. Hence $\overline{\mathfrak{V}_{\mathfrak{A}}}$ is I-V vague ideal of N. □

Lemma 4.4. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ and $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ be a two I-V vague subcollection of nearrings M and N respectively, if $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of $M \times N$, then*
(i) $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0,\, 0^{'}) \geq (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i,\, j)$.
(ii) $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(0_1,\, 0_2 ... 0_n) \geq (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1,\, i_2 ... i_n)$.

Theorem 4.5. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ and $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ be a two vague subset of near ring M and N respectively. If $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of nearrings $M \times N$, then atleast one of the following must be true*
(i) $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \geq \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j)\ \forall\ j \in N$
(ii) $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)\ \forall\ i \in M$

Proof. Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is the I-V vague ideal of $M \times N$. By contradiction suppose that $\exists\ i \in M$ and $j \in N$ such that $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0') < \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$ and $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) < \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j)$.
Now $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i,\, j) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(j) > \overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0') = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0,\, 0')$. Thus $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideals of the $M \times N$ satisfying $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(i,\, j) > (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})(0,\, 0')$. This is a contradict to Lemma(25). □

Theorem 4.6. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ and $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ be a two vague subset of nearrings M and N respectively, if $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of nearring $M \times N$, then either $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ is a I-V vague ideal M or $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal N.*

Proof. Suppose $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal $M \times N$ then by using the property $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$ If $\overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$ then (i) for $i,\, j \in M$, $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i$ - $j) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i$ - $j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i$ - $j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}$ - $0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((i,\, 0^{'})$ - $(j,\, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge (\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)$. (ii) $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j + i$ - $j) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j + i$ - $j) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((j,\, 0^{'}) + (i,\, 0^{'})$ - $(j,\, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$.
(iii) For $i,\, j,\, k \in M$ so $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(j(i + k)$ - $jk) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(k(i + k)$ - $jk) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}})((j,\, 0^{'})((i,\, 0^{'}) + (k,\, 0^{'}))$ - $(j,\, 0^{'})(k,\, 0^{'})) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(0^{'}) = \overline{\mathfrak{V}_{\mathfrak{A}_1}}(i)$.
Hence in this case $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ is a I-V vague ideal M. Simillary we can show $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal N. □

Theorem 4.7. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}$ and $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ be vague subsets of nearrings M and N respectively, such that $\overline{\mathfrak{V}_{\mathfrak{A}_1}}(0) \geq \overline{\mathfrak{V}_{\mathfrak{A}_1}}(j)$, $\forall\ j \in N$ and $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of $M \times N$ then $\overline{\mathfrak{V}_{\mathfrak{A}_2}}$ is a I-V vague ideal of N .*

Corollary 4.8. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}, \overline{\mathfrak{V}_{\mathfrak{A}_2}}, ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ be a vague subset of nearrings $N_1,\, N_2 ... N_n$ respectively if $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is a I-V vague ideal of nearring $N_1 \times N_2 \times ... \times N_n$, then atleast for one l, $\overline{\mathfrak{V}_{\mathfrak{A}_l}}(0_l) \geq \overline{\mathfrak{V}_{\mathfrak{A}_m}}(i_m)\ \forall\ i \in N_m$, for m = 1, 2,...n where 0_l is a identity of N_l must be true.*

Proof. We know that if $N_1, N_2 ... N_n$ is nearrings then their product $N_1 \times N_2 \times ... \times N_n$ is also a nearring. Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is a I-V vague ideals of $N_1 \times N_2 \times ... \times N_n$. by contradiction, suppose that none of condition is true for any l is hold. then we can find $i_m \in N_m$ such that $\overline{\mathfrak{V}_{\mathfrak{A}_l}}(\mathrm{i}_l) > \overline{\mathfrak{V}_{\mathfrak{A}_m}}(0_m)$, Now $[(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1,\, i_2 ... i_n)] = [\overline{\mathfrak{V}_{\mathfrak{A}_1}}(i_1) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_2}}(i_2) \wedge ...\ \overline{\mathfrak{V}_{\mathfrak{A}_n}}(i_n)] > \overline{\mathfrak{V}_{\mathfrak{A}_n}}(0_n) \wedge \overline{\mathfrak{V}_{\mathfrak{A}_{n-1}}}(0_{n-1}) \wedge ...\ \overline{\mathfrak{V}_{\mathfrak{A}_1}}(0_1) = (\overline{\mathfrak{V}_{\mathfrak{A}_n}} \times \overline{\mathfrak{V}_{\mathfrak{A}_{n-1}}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_1}})(0_n,\, 0_{n-1} ......,\, 0_1)$.
Thus $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is vague with identity $(0_1,\, 0_2 ...,\, 0_2)$ of $N_1 \times N_2 \times ... \times$

N_n satisfying $(\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(i_1, i_2, ... i_n) > (\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}})(0_n, 0_{n-1} ... 0_1)$ this is contradiction.

Corollary 4.9. *Let $\overline{\mathfrak{V}_{\mathfrak{A}_1}}, \overline{\mathfrak{V}_{\mathfrak{A}_2}}, ... \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ be a I-V vague ideal of nearrings N_1, $N_2, ... N_n$ respectively if $\overline{\mathfrak{V}_{\mathfrak{A}_1}} \times \overline{\mathfrak{V}_{\mathfrak{A}_2}} \times ... \times \overline{\mathfrak{V}_{\mathfrak{A}_n}}$ is a I-V vague ideal of $N_1 \times N_2 \times ... \times N_n$, then atleast for one of l, $\overline{\mathfrak{V}_{\mathfrak{A}_l}}$ is a I-V vague ideal of N_l.*

Conclusion

In this paper we give talked about interval value vague ideals in nearrings. After that we give definition, properties, theorems, examples of fuzzy vague ideals in nearrings. We also study normal interval valued vague ideals and direct product of interval value vague ideals in nearrings.

References

[1] Satyanarayana B. and Prasad K. S.: *Nearrings, Fuzzy Ideals and Graph Theory*, Chapman and Hall/CRC, 2019.

[2] Marcel G. and Vetrivelan N.: QOS-Aware Transmission For Multimedia Applications in Manet Using ACO With Fuzzy Logic, *Int. J. Inno. in Sci. Eng. Res.* **9** (2015) 199–213.

[3] Prasad K. S., Srinivas K. B., Harikrishnan P. K. and Satyanarayana B.: *Nearrings, Nearfields and Related Topics* World Scientific Publishing Company, 2017.

[4] Zadeh L. A.: Fuzzy sets, *Inf. Control.* **8** (1965) 338-–353.

[5] Bhaskar L.: Vague ideals of a nearrings, *Int. J. Pure and Appl. Math.* **117** (2017) 219–227.

[6] Bhaskar L., Nagaiah T.: P. Narasimha Swamy and T. Srinivas, Sum of vague ideals of Gamma-nearring, *Int. J. Eng. Technol.* **7** (2018) 4–7.

[7] Akram M. and Shum K. P.: Intuitionistic (T, S)-fuzzy ideals of nearrings, *J. Algebra Comb. Discrete Struct. Appl.* **6** (2008) 37–52.

[8] Swamy P. N.: A fundamental theorem of homomorphism for semirings, *Int. j. math. sci. appl.* **4** (2010) 423–435.

[9] Sharma P. K.: Intuitionistics fuzzy ideal of nearrings, *Int. Math. Forum* **7** (2012) 769–776.

[10] Allen P. J.: A fundamental theorem of homomorphism for semirings, in: *Proc. Amer. Math. Soc.* **21** (1969) 412–416.

[11] Swamy P. N.: A note on fuzzy ideal of a nearring, *Int. J. Math. Math. Sci.* **4** (2010) 423–435.

[12] Biswas R.: Vague groups, *Int. j. Cogn. Computing in Eng.*, **4** (2000) 4–7.

[13] Kim S. D. and Kim H. S.: On fuzzy ideal of nearrings, *Int. J. Pure Appl. Math.* **8** (1998) 133–139.

[14] Ragamayi S. and Bhargavi Y.: A study Of vague Γ-nearrings, *Int. J. Sci. Technol. Res.* **9** (2020) 3960–3963.

[15] Gue W. L. and Buehrer D. J.: Vague sets, *Osaka J. Math.*, **23** (1993) 610–614.

[16] Cho Y.U. and Jun Y.B.: On intuitionistic fuzzy R-subgroups of near-rings, *J. Appl. Math. and Computing*, **18** (2005) 665–677.

*Neha Gahlot: Department of Mathematics and Statistics, Manipal University Jaipur, Jaipur, Rajasthan-303 007, India.
Email address: gahlotneha1995@gmail.com

Nagaraju Dasari: Department of Mathematics and Statistics, Manipal University Jaipur, Jaipur, Rajasthan-303 007, India.
Email address: dasari.nagaraju@gmail.com

Mohiddin Shaw Shaik: Department of Mathematics, Narasaraopeta Engineering College, Narasaraopet-522 601, AP, India.
Email address: mohiddin_shaw26@yahoo.co.in

Babu Prasad Munagala: Department of Freshmen Engineering, NRI Institute of Technology (Autonomous), Pothavarappadu (V), Agiripalli (M), Vijayawada Rural, Krishna (DT), Andhra Pradesh, PIN -521 212, India.
Email address: bpsirvijayawada@gmail.com