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STOCHASTIC ANALYSIS IN MODELLING AND EFFICIENCY ESTIMATION OF MODERN NETWORKS

VLADIMIR A. GORLOV & ALLA V. MAKAROVA

Abstract. Optimization and control are the central problem of the investigation. There are lots of methods to analyse systems, one of them is to use the stochastic analysis. This approach are more accurate and give us the best solution of optimization [1, 2]. In current article we use Kolmogorov equation to find out the optimal parameters of system in various conditions.It also helps us to predict the productivity of the networks and to build the self-defined networks with the ability to rebuilt or restore their operation automatically. To solve this problem, it is necessary to describe all possible conditions for system with automatic rebuilding or restoring and without these characteristics. The important part of the research is to implement the graph theory and on its base to build the mathematical model of the whole system. Then the methods of stochastic analysis using Kolmogorov equation solve the problem of the system elements productivity. At the end of this approach we obtain the accurate probabilities for each condition. The significance of the method in such problems lies in the fact that we can create on it base the automatic algorithm for complex networks with the vast amount of conditions.

1. Introduction

Nowadays the various amount of researches is investigating in widespread areas. For instance, we can see how the mathematical and stochastic methods apply in different fields, such as theoretical mathematics, theoretical physics, medicine, engineering and networking systems [3, 4]. So, from this point of view it's necessary to make some researches in controlling networks stability and to find out the results how we can to increase the stability modern and perspective networks. The SDN networks have the implementation to the wired and wireless systems. They help to increase the efficiency and here we have several questions: what kind of methods do we need to use to find out about the efficiency level and how do we need to use them [5]. That's why in this article we try to describe the full-length method for investigating the SDN networks.

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2. SDN network mathematical modelling

Let consider the SDN network based on principles and architecture used for building those kind of networks. Also let the network consists of one server and two switchers. As well-known, in SDN the switchers have limited option list and all control functions, routing and other operations lies on SDN controller. SDN controller is a technically complex device for the network, so it's important to keep it in online mode and we need to minimize the errors and the faults. Reservation is one of the method that can help us to solve the problem of the faults and unstable processing SDN network.

In current study we have some assumptions:

1. If one route is offline, the system stays stable.
2. The SDN network is unworkable if SDN controller or all switchers are offline.
3. At the same time only one server is fault.
4. The restore time of controller is more than the restore time of switchers.

In context with the assumptions defined above we can describe the list of conditions for the network:

1. System is online;
2. One switcher is fault;
3. Two switchers are fault;
4. Restoring switcher if the server is online;
5. Restoring two switchers if the server is online;
6. Server is offline;
7. Server and switcher are offline;
8. Server and two switchers are offline;
9. Restoring controller;
10. Controller from set of servers is unworkable.

So, in this terms system with restoring is unworkable in conditions 9 and 10, system without restoring is unworkable in conditions 6, 7, 8 and 9.

3. Transfers between conditions in SDN

As we understand it's important how the system transfers from one condition to another and what it means. To answer this question, we built the spreadsheet with conditions and description (look at Figure 1).

Furthermore, if we combine all these condition in whole one, we obtain the state graph for system without reservation (Figure 2).

If we notice that we have the system with reservation, there is one new state where the system transfers to condition 10 - SDN controller from the set of servers is unworkable. So, now we can build a new state graph for the system with reservation (Figure 3).

Further using the state graph for system without reservation we can write the equations that will help us to define the probabilities for each state in considered Markov process [6, 7, 8].

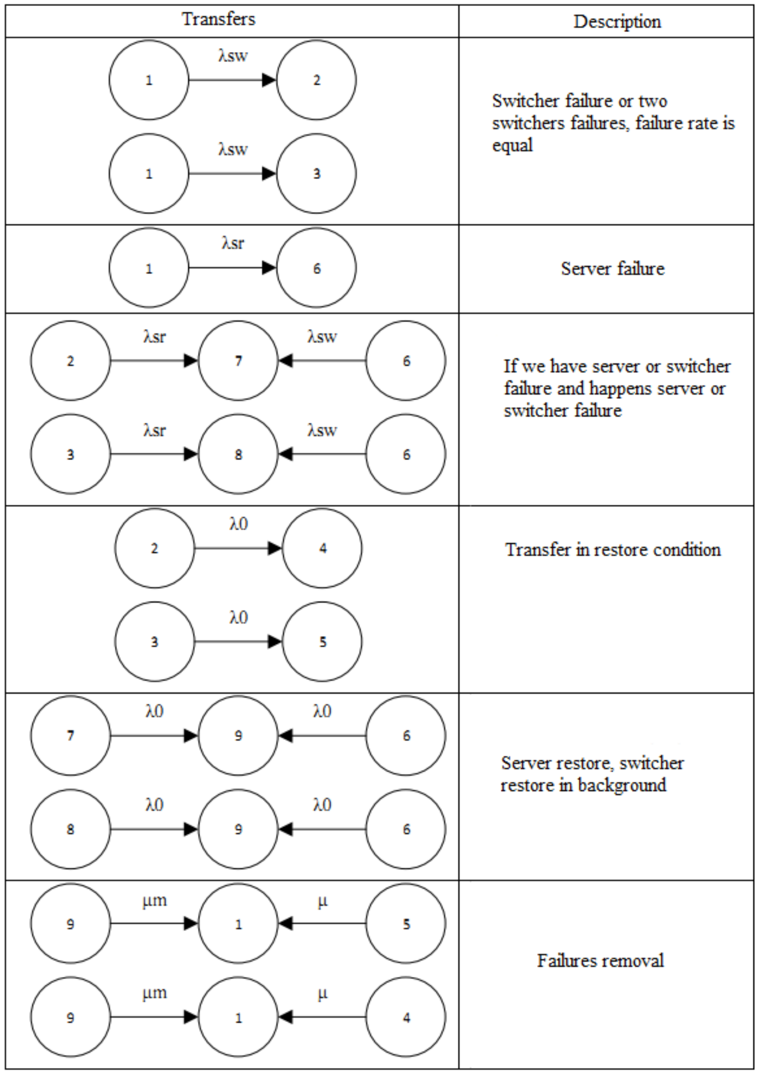

FIGURE 1. Transfers between conditions in SDN with description

Then we obtain the following system of Kolmogorov differential equations:

$$\begin{cases} \frac{dP_1(t)}{dt} = -(2\lambda_{sr})P_1 + \mu P_3 + \mu P_4 + \mu P_8, \\ \frac{dP_2(t)}{dt} = \lambda_{sw}P_1 - (\lambda_0 + \lambda_{sr})P_2, \\ \frac{dP_3(t)}{dt} = \lambda_{sw}P_1 + (\lambda_0 + \lambda_{sr})P_3, \\ \frac{dP_4(t)}{dt} = \lambda_0 P_2 - \mu P_4, \\ \frac{dP_5(t)}{dt} = \lambda_0 P_3 - \mu P_5, \\ \frac{dP_6(t)}{dt} = \lambda_{sr}P_1 - (2\lambda_{sw} + \lambda_0)P_6, \\ \frac{dP_7(t)}{dt} = \lambda_{sr}P_2 + \lambda_{sw}P_6 - \lambda_0 P_7, \\ \frac{dP_8(t)}{dt} = \lambda_{sr}P_3 + \lambda_{sw}P_6 - \lambda_0 P_8, \\ \frac{dP_9(t)}{dt} = \lambda_0 P_6 + \lambda_0 P_7 - \lambda_0 P_8 - \mu P_9. \end{cases} \tag{3.1}$$

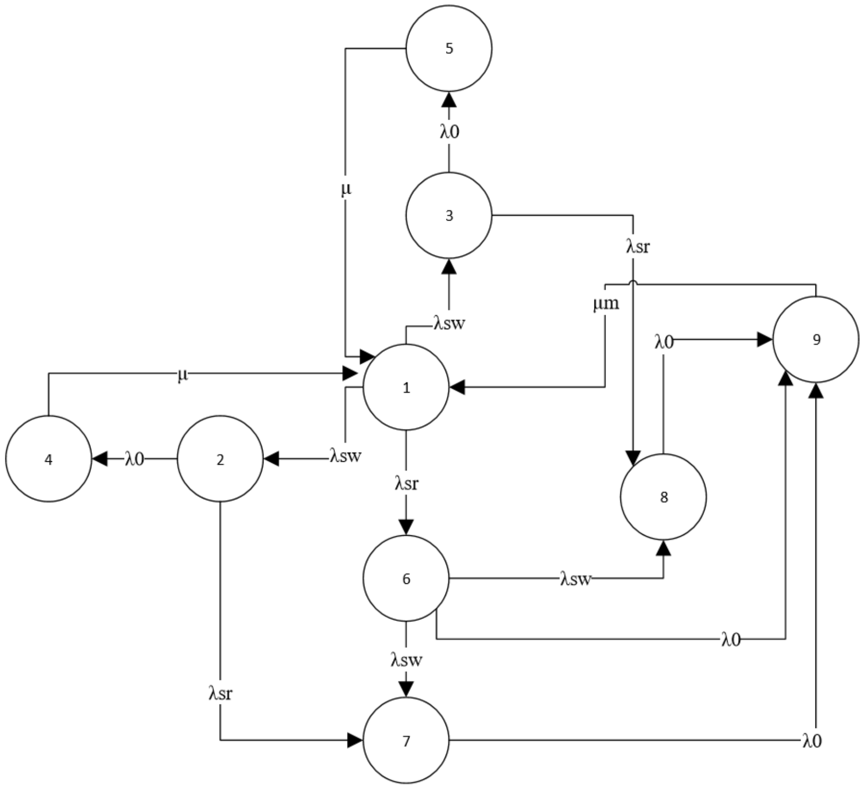

FIGURE 2. State graph for SDN without reservation

The solution of this system could be obtained by well-known algorithm for differential equations systems. However, we can simplify the calculation if notice: in this network there is a stationary Markov process, so then $dP_i = 0$, i.e. probabilities aren't changing through the time.

Now we overwrite the left-side of the equations to zero and put in system normalization condition:

$$\sum_{i=0}^{n} P_i = 1.$$

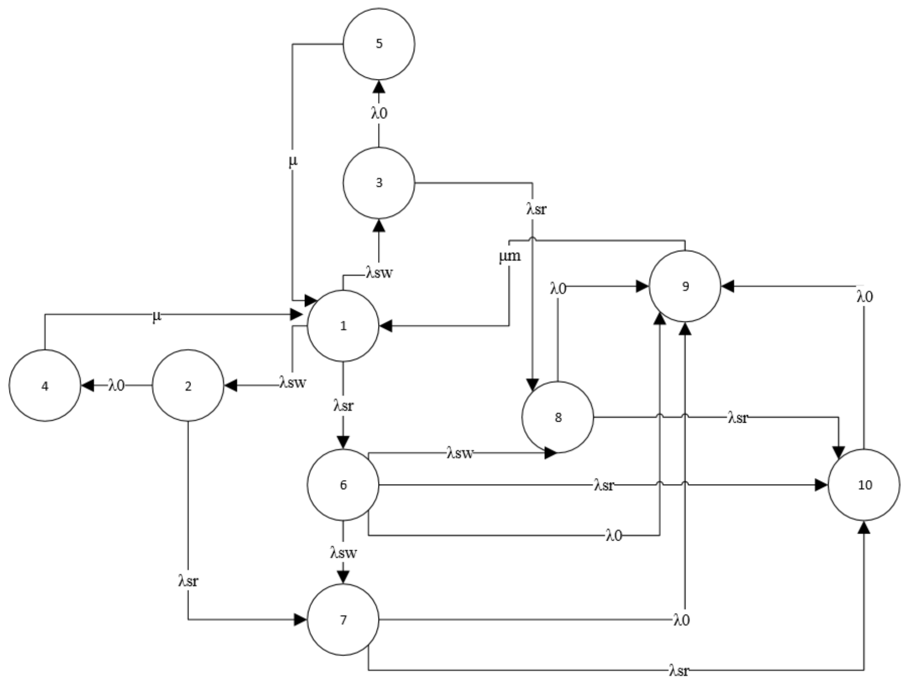

Figure 3. State graph for SDN with reservation

Considering all these remarks we obtain:

$$\begin{cases} -(2\lambda_{sr})P_1 + \mu P_3 + \mu P_4 + \mu P_8 = 0, \\ \lambda_{sw}P_1 - (\lambda_0 + \lambda_{sr})P_2 = 0, \\ \lambda_{sw}P_1 + (\lambda_0 + \lambda_{sr})P_3 = 0, \\ \lambda_0 P_2 - \mu P_4 = 0, \\ \lambda_0 P_3 - \mu P_5 = 0, \\ \lambda_{sr}P_1 - (2\lambda_{sw} + \lambda_0)P_6 = 0, \\ \lambda_{sr}P_2 + \lambda_{sw}P_6 - \lambda_0 P_7 = 0, \\ \lambda_{sr}P_3 + \lambda_{sw}P_6 - \lambda_0 P_8 = 0, \\ \lambda_0 P_6 + \lambda_0 P_7 - \lambda_0 P_8 - \mu P_9 = 0. \end{cases} \tag{3.2}$$

As we see from system of differential equations written above, it can be simplified, because some of probabilities are eqaul.

So, we have:

$$\begin{cases} \lambda_{sw} P_1 - (\lambda_0 + \lambda_{sr}) P_2 = 0, \\ \lambda_0 P_2 - \mu P_4 = 0, \\ \lambda_{sr} P_1 - (2\lambda_{sw} + \lambda_0) P_6 = 0, \\ \lambda_{sr} P_2 + \lambda_{sw} P_6 - \lambda_0 P_7 = 0, \\ \lambda_0 P_6 + \lambda_0 P_7 - \lambda_0 P_8 - \mu P_9 = 0, \\ P_1 + 2P_2 + 2P_4 + P_6 + 2P_7 + P_9 = 1. \end{cases} \tag{3.3}$$

To find out the solution, we involve Mathcad where define matrix X and vector Y. Furthermore, if we have the intensity points, we obtain the accurate probabilities for each state, illustrated on Figure 4.

λ_0 / P_i	0.5	1	2	4	5	10	15	20	24
P_1	0.9999787550 80811	0.999976525300732	0.999972065770406	0.99996314682908	0.99995868741808	0.999936390959674	0.999914095495552	0.999891801025647	0.999873966165514
P_2	9.6785908923 41742*10^-7	1.935713292823544 *10^-6	3.871407043200608 *10^-6	7.742735917677192 *10^-6	9.678371042307647 *10^-6	1.935625354999978 *10^-5	2.903364755625712 *10^-5	3.871055309425739 *10^-5	4.645172584980608 *10^-5
P_3	9.6785908923 41742*10^-7	1.935713292823544 *10^-6	3.871407043200608 *10^-6	7.742735917677192 *10^-6	9.678371042307647 *10^-6	1.935625354999978 *10^-5	2.903364755625712 *10^-5	3.871055309425739 *10^-5	4.645172584980608 *10^-5
P_4	7.7428727138 73395*10^-6	7.742853171294175 *10^-6	7.742814086401218 *10^-6	7.742735917677193 *10^-6	7.742696833846117 *10^-6	7.742501419999912 *10^-6	7.742306015001899 *10^-6	7.742110618851476 *10^-6	7.741954308301015 *10^-6
P_5	7.7428727138 73395*10^-6	7.742853171294175 *10^-6	7.742814086401218 *10^-6	7.742735917677193 *10^-6	7.742696833846117 *10^-6	7.742501419999912 *10^-6	7.742306015001899 *10^-6	7.742110618851476 *10^-6	7.741954308301015 *10^-6
P_6	2.9411082922 52383*10^-7	5.882192081687397 *10^-7	1.176428615302545 *10^-6	2.352818027201446 *10^-6	2.94099803233458* 10^-6	5.881751057197453 *10^-6	8.822259097588638 *10^-6	1.176252217650559 *10^-5	1.411455628283036 *10^-5
P_7	5.6932840872 69202*10^-13	2.277306017735746 *10^-12	9.109163135140848 *10^-12	3.643616505983808 *10^-11	5.693112706903878 *10^-11	2.27716891786334* 10^-10	5.123458703796127 *10^-10	9.108066403557052 *10^-10	1.311526471032283 *10^-9
P_8	5.6932840872 69202*10^-13	2.277306017735746 *10^-12	9.109163135140848 *10^-12	3.643616505983808 *10^-11	5.693112706903878 *10^-11	2.27716891786334* 10^-10	5.123458703796127 *10^-10	9.108066403557052 *10^-10	1.311526471032283 *10^-9
P_9	3.5293436145 8467*10^-6	3.529342576684651 *10^-6	3.529340500886445 *10^-6	3.529336349297348 *10^-6	3.529334273506462 *10^-6	3.529323894588615 *10^-6	3.529313515731758 *10^-6	3.52930313693589* 10^-6	3.529294833943107 *10^-6
ΣP_i	1	1	1	1	1	1	1	1	1

FIGURE 4. Current probabilities for the states

Thus, every network can be simulated by mathematical modelling and applying the stochastic analysis to investigation of efficiency and reliability networks. As a result, we obtain accurate characteristics.

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Vladimir A. Gorlov : N.E. Zhukovsky and Y.A. Gagarin Air Force Academy, Voronezh, Russian Federation
E-mail address: gorlov.v.a@gmail.com

Alla V. Makarova: N.E. Zhukovsky and Y.A. Gagarin Air Force Academy, Voronezh, Russian Federation
E-mail address: allagm@mail.ru