Global and Stochastic Analysis
Vol. 11 No. 4 (September, 2024)

Received: 05th June 2024 Revised: 20th June 2024 Accepted: 04th August 2024

INVESTIGATION SOLVABILITY OF THE STOCHASTIC MODEL OF NONLINEAR DIFFUSION WITH RANDOM INITIAL VALUE

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Abstract. The article considers the study of stochastic model of a nonlinear diffusion, which describes the process of changing the concentration potential of a viscoelastic fluid filtered in a porous medium. The article presents conditions for the existence of solutions of the model under study with the Showalter–Sidorov initial condition.

Introduction

Consider a complete probability space $\Omega \equiv (\Omega, \mathcal{A}, \mathbf{P})$ and the set of real numbers $\mathbb{R}$, endowed with a σ-algebra. Measurable mapping $\xi : \Omega \to \mathbb{R}$ is called a *random variable*. The set of Gaussian random variables with $\mathbf{E}\xi = 0$ and $\mathbf{D}\xi < +\infty$ forms Hilbert space $\mathbf{L_2}$ with the inner product $(\xi_1, \xi_2) = \mathbf{E}\xi_1\xi_2$, where $\mathbf{E}$, $\mathbf{D}$ are the expectation and variance of the random variable, respectively. Consider a set $\mathcal{I} \subset \mathbb{R}$. Mapping $\eta : \mathbb{R} \times \Omega \to \mathbb{R}$ of the form $\eta = \eta(t, \omega) = g(f(t), \omega)$ is called a *stochastic process* [1], where $f : \mathcal{I} \to \mathbf{L_2}$ and $g : \mathbf{L_2} \times \Omega \to \mathbb{R}$. Let $\mathfrak{D} \subset \mathbb{R}^n$ be a bounded domain with a boundary C^∞. Consider a *$\mathcal{H}$-valued differentiable stochastic K-process* η, satisfying the stochastic nonlinear diffusion model:

$$(\lambda - \Delta) \overset{o}{\eta} - \operatorname{div}(|\nabla\eta|^{p-2}\nabla\eta) = 0, \; p \geq 2, \; (s,t) \in \mathfrak{D} \times (0,T), \tag{0.1}$$

$$\eta(s,t) = 0, \; (s,t) \in \partial\mathfrak{D} \times [0,T], \tag{0.2}$$

and initial Showalter–Sidorov condition

$$(\lambda - \Delta)(\eta(s,0) - \eta_0(s)) = 0, s \in \mathfrak{D}. \tag{0.3}$$

Here $\overset{o}{\eta}$ is Nelson–Gliklikh derivative of a stochastic process, which coincides with the classical function derivative in the deterministic case [2, 3]. Model (0.1), (0.2) with condition (0.3) describes the process of changing the concentration potential of a viscoelastic fluid filtered in a porous medium [4, 5], with the assumption of a randomly specified initial value η_0 of the fluid concentration potential. The parameter $\lambda \in \mathbb{R}$ characterizes viscosity of the fluid, and it was experimentally confirmed

2020 *Mathematics Subject Classification.* 35Q99, 60G65.

Key words and phrases. Sobolev type equations; stochastic model of nonlinear diffusion; Showalter–Sidorov problem; the Nelson–Glicklich derivative.

*This work was funded by Russian Science Foundation Grant No. 24-11-20037, https://rscf.ru/project/24-11-20037/.

that the negative value of the parameter λ does not contradict the physical meaning of the model [6], which leads to the study of a degenerate equation. Model of nonlinear diffusion in the deterministic case:

$$(\lambda - \Delta)x_t - \operatorname{div}(|\nabla x|^{p-2}\nabla x) = 0,\ p \geq 2,\ (s,t) \in \mathfrak{D} \times (0,T), \tag{0.4}$$

$$x(s,t) = 0,\ (s,t) \in \partial\mathfrak{D} \times [0,T], \tag{0.5}$$

has been studied previously. Unique local in time solvability of problem (0.4), (0.5) with the classical Cauchy initial condition ($x(s,0) = x_0(s)$) was estimated in the article [7], and in this case the initial value was taken from a specially constructed set called the phase space of the equation [8]. In non-degenerate case ($\lambda \in \mathbb{R}_+$) Cauchy problem was studied by Liu Changchung. He obtained conditions for the existence of a weak solution [9] and revealed the asymptotic properties of the solution [10]. Based on the projection method, the solvability of the degenerate model in the weak generalized sense was obtained for any given time interval [11].

Consider the spaces $\mathcal{H} = L_2(\mathfrak{D})$, $\mathfrak{H} = \overset{0}{W}{}^1_2(\mathfrak{D})$, $\mathcal{B} = \overset{0}{W}{}^1_p(\mathfrak{D})$, $\mathfrak{H}^* = W_2^{-1}(\mathfrak{D})$, $\mathcal{B}^* = W_q^{-1}(\mathfrak{D})$, $\frac{1}{p} + \frac{1}{q} = 1$. Let us identify $\mathcal{H}$ with its conjugate and define in $\mathcal{H}$ the scalar product

$$\langle x, y\rangle = \int\limits_{\mathfrak{D}} xyds\, \forall x, y \in \mathcal{H}.$$

Due to the choice of function spaces $\mathfrak{H}$ and $\mathcal{B}$ there exists a dense and continuous embedding

$$\mathcal{B} \hookrightarrow \mathfrak{H} \hookrightarrow \mathcal{H} \hookrightarrow \mathfrak{H}^* \hookrightarrow \mathcal{B}^*, \tag{0.6}$$

which corresponds to the evolutionary case of embeddings and space splitting for the problem under study [11]. Let's construct operators:

$$\langle Lx, y\rangle = \int\limits_{\mathfrak{D}} (\lambda xy + \nabla x \cdot \nabla y)ds, \quad x,\, y \in \mathfrak{H}; \tag{0.7}$$

$$\langle N(x), y\rangle = \int\limits_{\mathfrak{D}} |\nabla x|^{p-2}\nabla x \cdot \nabla y ds, \quad x,\, y \in \mathcal{B}. \tag{0.8}$$

The operator $L \in \mathcal{L}(\mathfrak{H}; \mathfrak{H}^*)$ is self-adjoint, non-negative definite in case $\lambda \geq -\lambda_1$ and Fredholm, the operator $N \in C^\infty(\mathcal{B}; \mathcal{B}^*)$ is s-monotonous and p-coercive [11]. Here $\{\lambda_k\}, \{\psi_k\}$ are the sequences of eigenfunctions and eigenvalues of the homogeneous Dirichlet problem for the Laplace operator $(-\Delta)$ in the domain $\mathfrak{D}$, numbered in non-decreasing order taking into account the multiplicity. Note that the orthonormal family of functions $\{\psi_k\}$ is total in space $\mathcal{H}$. Thus, in case of determined initial function $\eta_0(s)$, problem (0.3) – (0.5) is reduced to an abstract semilinear equation of Sobolev type:

$$L\dot{x} + N(x) = 0,\ \ker L \neq \{0\}, \tag{0.9}$$

with Showalter–Sidorov initial condition

$$L\left(x(s,0) - x_0(s)\right) = 0. \tag{0.10}$$

Note that considering condition (0.10) when studying degenerate equations allows us to avoid the difficulties of studying the Cauchy problem outlined in non-existence of solution under random initial conditions η_0, which is especially important in the stochastic case. G.A. Sviridyuk developed the phase space method [12], which allows one to study the phenomenon of degenerate equations. This method was successfully applied to the study of the abstract equation (0.9) and its specific interpretations in case of (L,p)-bounded, (L,p)-sectional and (L,p)-radial operator M [11], as well as in case of s-monotone, p-coercive operator N and Fredholm operator L [13]. The transition from studying a deterministic model to a stochastic one is caused by the fact that measurement noise may occur in experiments, which leads to the need to consider a stochastic model [14, 15]. We will study problem (0.1) – (0.3) on the basis of the developed research method for the abstract stochastic equation [16]

$$L \overset{o}{\eta} + N(\eta) = 0, \ker L \neq \{0\}, \tag{0.11}$$

in case of s-monotone, p-coercive operator N and Fredholm operator L.

Let's consider stochastic K-processes $\eta = \eta(t)$ and $\zeta = \zeta(t)$ them equal if almost certainly each trajectory of one of the processes coincides with the trajectory of another process. Construct spaces of differentiable K-"noises", i.e. space of $\mathcal{H}$-valued continuous stochastic K-processes almost surely differentiable by Nelson–Gliklikh, to study problem (0.1) – (0.3).

The construction of such functional spaces made it possible to apply already developed research methods for a determined equation (0.9), based on methods of functional analysis, to study equations (0.11) [16]. This approach has been widely used in recent years to solve stochastic equations of Sobolev type in works [17, 18, 19, 20]. Note the other approaches for study stochastic equations. I.V. Melnikova has studied stochastic equations in Schwarz spaces, using the traditional approach to the concept of white noise as a generalized derivative of the Wiener process [21]. M. Kovács and S. Larsson have studied stochastic equations in Schwarz spaces non-degenerate models of mathematical physics using the Ito–Stratonovich–Skorokhod approach [22].

1. Spaces of differentiable K-"noises"

Following the idea first presented in the article [1], and widely applied in the study of linear stochastic equations of Sobolev type [18, 19, 20], let's construct spaces of differentiable K-"noises" necessary to study problem (0.1) – (0.3). The set of continuous one-dimensional random processes forms a Banach space denoted $\mathbf{C}(\mathcal{I}, \mathbf{L}_2)$. Fix $\eta \in \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$ and $t \in \mathcal{I}$ and denote $\mathcal{N}_t^\eta$ σ-algebra generated by the random variable $\eta(t)$. Denote $\mathbf{E}_t^\eta = \mathbf{E}(\cdot|\mathcal{N}_t^\eta)$.

Definition 1.1. Suppose that $\eta \in \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$. The derivative

$$\overset{o}{\eta} = \lim_{\triangle t \to 0+} \mathbf{E}_t^\eta \left(\frac{\eta\,(t + \triangle t, \cdot) - \eta(t, \cdot)}{\triangle t} \right) + \lim_{\triangle t \to 0+} \mathbf{E}_t^\eta \left(\frac{\eta\,(t, \cdot) - \eta\,(t - \triangle t, \cdot)}{\triangle t} \right)$$

is called the symmetric mean derivative or Nelson–Gliklikh derivative *of a random process η at the point $t \in \mathcal{I}$,* if the limit exists in the sense of a uniform metric on $\mathbb{R}$.

Denote the $l \in \mathbb{N}$ is order of the Nelson–Gliklikh derivative of the stochastic process η. Note that the Nelson–Gliklikh derivative coincides with the classical derivative, if $\eta(t)$ is a determined function. Consider space of "noises" $\mathbf{C}^l(\mathcal{I}, \mathbf{L}_2)$, $l \in \mathbb{N}$, i.e. the space of random processes from $\mathbf{C}(\mathcal{I}, \mathbf{L}_2)$, which trajectories are almost surely differentiable by Nelson–Gliklikh up to the order l inclusively.

Choose a monotonely decreasing numerical sequence $K = \{\mu_k\}$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 < +\infty$. Consider a sequence of random variables $\{\xi_k\} \subset \mathbf{L}_2$ such that $\sum\limits_{k=1}^{\infty} \mu_k^2 \mathbf{D}\xi_k < +\infty$. Choose an orthonormal basis $\{\varphi_k\}$ in the space $\mathcal{H}$ and require that the following condition is met

$$\{\varphi_k\} \subset \mathcal{B}. \tag{1.1}$$

Denote by $\mathcal{H}_K\mathbf{L}_2$ Hilbert space of *random K-variables* of the form

$$\xi = \sum_{k=1}^{\infty} \mu_k \xi_k \varphi_k. \tag{1.2}$$

Note that the space $\mathcal{H}_K\mathbf{L}_2$ is a Hilbert space with the scalar product $(\xi^1, \xi^2) = \sum\limits_{k=1}^{\infty} \mu_k^2 \mathbf{E}\xi_k^1 \xi_k^2$.

Consider a sequence of random processes $\{\eta_k\} \subset \mathbf{C}(\mathcal{I}, \mathbf{L}_2)$ and define *$\mathcal{H}$-valued continuous stochastic K-process*

$$\eta(t) = \sum_{k=1}^{\infty} \mu_k \eta_k(t) \varphi_k, \tag{1.3}$$

if series (1.3) converges uniformly in the norm $\mathcal{H}_K\mathbf{L}_2$ on any compact set in $\mathcal{I}$.

Specify the Nelson–Gliklikh derivative of the stochastic K-process

$$\overset{o}{\eta}{}^{(l)}(t) = \sum_{k=1}^{\infty} \mu_k \overset{o}{\eta}{}_k^{(l)}(t) \varphi_k$$

under condition that all series converge uniformly in the norm $\mathcal{H}_K\mathbf{L}_2$ on any compact from $\mathcal{I}$. The space $\mathbf{C}^l(\mathcal{I}; \mathcal{H}_K\mathbf{L}_2)$, $l \in \mathbb{N}$, of continuous $\mathcal{H}$-valued stochastic K-process, which trajectories are almost surely continuously differentiable by Nelson–Gliklikh, which is called *the space of differentiable K-"noises"*.

Similar to the construction of space $\mathbf{C}^l(\mathcal{I}; \mathcal{H}_K\mathbf{L}_2)$ let's construct spaces of differentiable K-"noises" $\mathbf{C}^l(\mathcal{I}; \mathcal{B}_K\mathbf{L}_2)$ and $\mathbf{C}^l(\mathcal{I}; \mathfrak{H}_K\mathbf{L}_2)$, where spaces of *random K-values* of form (1.2) denote by $\mathcal{B}_K\mathbf{L}_2$ and $\mathfrak{H}_K\mathbf{L}_2$. Note that due to the density and continuity of embeddings (0.6) and condition (1.1) it follows that the orthonormal basis $\{\varphi_k\}$ in $\mathcal{H}$ is also the basis of spaces $\mathcal{B}$ and $\mathfrak{H}$.

Lemma 1. (i) *For all $\lambda \geq -\lambda_1$ the operator $L \in \mathcal{L}(\mathfrak{H}\mathbf{L}_2; \mathfrak{H}^*\mathbf{L}_2)$ is self-adjoint, Fredholm and non-negative definite.*

(ii) *The operator $N \in C^\infty(\mathcal{B}\mathbf{L}_2; \mathcal{B}^*\mathbf{L}_2)$ is s-monotone and p-coercive.*

Proof. In case $\lambda \geq -\lambda_1$

$$\ker L = \begin{cases} \{0\}, \text{ if } \lambda > -\lambda_1; \\ \operatorname{span}\{\psi_1\}, \text{ if } \lambda = -\lambda_1. \end{cases}$$

Then

$$\operatorname{im} L = \begin{cases} \mathfrak{H}^*\mathbf{L}_2, \text{ if } \lambda > -\lambda_1; \\ \{\eta \in \mathfrak{H}^*\mathbf{L}_2 : \langle \eta, \psi_1 \rangle = 0\}, \text{ if } \lambda = -\lambda_1, \end{cases}$$

$$\operatorname{coim} L = \begin{cases} \mathfrak{H}\mathbf{L}_2, \text{ if } \lambda > -\lambda_1; \\ \{\eta \in \mathfrak{H}\mathbf{L}_2 : \langle \eta, \psi_1 \rangle = 0\}, \text{ if } \lambda = -\lambda_1. \end{cases}$$

Due to the construction of spaces, the proof of this lemma is based on the idea of proving for deterministic case in [11]. □

Due to the properties of the operator L eigenfunction system $\{\psi_k\}$ is total in space $\mathcal{H}$. Thus, further as a basis of $\{\varphi_k\}$ you can take $\{\psi_k\}$.

2. Solvability Research

Let us present the conditions for the existence of a trajectory solution to problem (0.1) – (0.3). Let $\mathcal{I} = (0, T)$. By solving a problem we mean a $\mathcal{H}$-valued K-process satisfying the following definition:

Definition 1.2. A random K-process $\eta \in \mathbf{C}^k(\mathcal{I}; \mathcal{B}_K\mathbf{L}_2)$ is called *a solution to equation* (0.11), if almost surely all trajectories of η satisfy equation (0.11) for all $t \in \mathcal{I}$. A solution $\eta = \eta(t)$ to equation (0.11) is called *a solution to Showalter–Sidorov problem* (0.10), (0.11), if solution satisfies condition (0.10) for some random K-variable $\eta_0 \in \mathcal{B}_K\mathbf{L}_2$.

Remark 1. Due to the degeneracy of equation (0.11) all its solutions $\eta = \eta(t)$ for all $t \in \mathcal{I}$ belongs to set

$$\mathfrak{M} = \begin{cases} \{\eta \in \mathcal{B}_K\mathbf{L}_2 : (\mathbf{I} - Q)N(\eta) = 0\}, \quad \text{if } \ker L \neq \{0\}; \\ \mathcal{B}_K\mathbf{L}_2, \quad \text{if} \quad \ker L = \{0\}, \end{cases} \tag{2.1}$$

which is called the phase manifold of equation (0.11). Here

$$Q = \begin{cases} \mathbb{I}, \text{ if } \lambda \neq -\lambda_k; \\ \mathbb{I} - \sum\limits_{k=1}^{\infty} \langle \cdot, \varphi_k \rangle, \text{ if } \lambda = -\lambda_k, \end{cases}$$

is an orthonormal space projector $\mathcal{B}_K^*\mathbf{L}_2$. In the case of studying model (0.1), (0.2), the phase manifold $\mathfrak{M}$ takes the following form

$$\mathfrak{M} = \begin{cases} \mathcal{B}\mathbf{L}_2, \text{ if } \lambda \neq -\lambda_k; \\ \{\eta \in \mathcal{B}\mathbf{L}_2 : \mathbf{E} \int\limits_{\mathfrak{D}} (|\nabla\eta|^{p-2}\nabla\eta \cdot \nabla)\varphi_k \, ds = 0, \text{ if } \lambda = -\lambda_k. \end{cases}$$

Define $\eta_0 \in \mathcal{B}\mathbf{L}_2$ in form

$$\eta_0 = \sum_{k=1}^{\infty} \mu_k \eta_{0k} \varphi_k,$$

where $\{\eta_{0k}\} \subset \mathbf{L}_2$ is a sequence of random variables. Then the following theorem is true.

Theorem 1. *Let $\lambda \geq -\lambda_1$, then for any sequence of random variables $\{\eta_{0k}\} \subset \mathbf{L}_2$, for any $T \in \mathbb{R}_+$ there exists a solution $\eta \in \mathbf{C}^k(\mathcal{I}; \mathcal{B}_K\mathbf{L}_2)$ to problem* (0.1) – (0.3).

Proof. Taking into account that the operator L is self-adjoint and Fredholm, we identify $\mathfrak{H} \supset \ker L \equiv \operatorname{coker} L \subset \mathfrak{H}^*$. We use the subspace $\ker L$ in order to construct the subspace $[\ker L]_K \mathbf{L}_2 \subset \mathcal{H}_K \mathbf{L}_2$ and, similarly, the subspace $[\operatorname{coker} L]_K \mathbf{L}_2 \subset \mathcal{H}^*_K \mathbf{L}_2$. Taking into account that embeddings (0.6) are continuous and dense, we construct the spaces $\mathfrak{H}_K \mathbf{L}_2 = [\ker L]_K \mathbf{L}_2 \oplus [\operatorname{coim} L]_K \mathbf{L}_2$ and $\mathfrak{H}^*_K \mathbf{L}_2 = [\operatorname{coker} L]_K \mathbf{L}_2 \oplus [\operatorname{im} L]_K \mathbf{L}_2$. Similarly, denote by $\mathcal{B}_K \mathbf{L}_2 = [\ker L]_K \mathbf{L}_2 \oplus [\operatorname{coim} L \cap \mathcal{B}]_K \mathbf{L}_2$ and $\mathcal{B}^*_K \mathbf{L}_2 = [\operatorname{coker} L]_K \mathbf{L}_2 \oplus [\overline{\operatorname{im} L}]_K \mathbf{L}_2$, where $\overline{\operatorname{im} L}$ is closure $\operatorname{im} L$ in topology $\mathcal{B}^*$.

Fix $\omega \in \Omega$. Since the stochastic component in problem (0.1) – (0.3) is found only in the initial condition (0.3), then when ω is fixed, the derivative $\overset{o}{\eta}$ coincides with classical derivative η' from (0.4). Thus, problem (0.1) – (0.3) is reduced to the deterministic case (0.3) – (0.5). By virtue of the theorem on the existence of a unique solution [11], the existence of a trajectory solution to problem (0.1) – (0.3) is proved. □

Remark 2. In the deterministic case, there is a unique solution to problem (0.3) – (0.5) [11]. Therefore, each trajectory for a fixed $\omega \in \Omega$ is unique.

Acknowledgments. This work was funded by Russian Science Foundation 24-11-20037, https://rscf.ru/project/24-11-20037/.

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