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A NOVEL GRAPH, $K_N - DOME$ AND ITS DEGREE BASED TOPOLOGICAL INDEX

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Abstract. Currently topological indices have been recognized as the important areas of the graph theory and offering various applications. They are numerical quantities or measures interconnected with a graph layout or structure. These indices grant insights into multiple aspects of the graphs's connectivity, distance and other structural properties. Properties of the chemical compounds and topological indices are correlated.Here in this paper we depict a new graphical structure $K_n - dome$, and assess their topological indices namely Zagreb indices in different forms, randic and somber index. Furthermore we compute the correlation coefficients between them.

1. Introduction

Lately, graph theory is strengthening its connection with chemistry, informatoin sciences and mathematics in the form of Topological indices. These are real valued numbers related to a graph, that must be a structural invarient and perform an important role in mathematical chemistry, particularly QSPR/QSAR investigations[2][3][4]. All along this paper, we consider a finite connected simple graph G. The vertex set is symbolized by $V(G)$ and the edge set is symbolized by $E(G)$. The degree of the vertex v is the number of edges connected to this vertex denoted by $\delta(v)$. In practical applications, Zagreb indices are among the best applications to identify the physical properties, and chemical reactions. First zagreb index $M_1(G)$ and second zagreb index $M_2(G)$ were initially assessed by I.Gutman and N.Trinajstic in 1972[10]. They are defined as:

$$M_1(G) = \sum_{uv\in E(G)} \delta_G(u) + \delta_G(v)$$
$$M_2(G) = \sum_{uv\in E(G)} \delta_G(u)\delta_G(v)$$

These indices were inferred within the study to analyze and predict various properties of organic compounds.

In 2015, Furtula and gutman unveiled forgotten index also known as F-index[6] which defined as:

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$$F(G) = \sum_{uv\in E(G)} [\delta_G^2(u) + \delta_G^2(v)]$$

Ranjini et al in 2017 introduced thr re-defined zagreb indices[11], i.e., the redefined first, second and third zagreb indices for a graph G defined as:

$$R_eZG_1(G) = \sum_{uv\in E(G)} \frac{\delta(u)+\delta(v)}{(\delta(u))(\delta(v))},$$
$$R_eZG_2(G) = \sum_{uv\in E(G)} \frac{\delta(u)\delta(v)}{\delta(u)+\delta(v)},$$
$$R_eZG_3(G) = \sum_{uv\in E(G)} (\delta_G(u)\delta_G(v))(\delta_G(u)+\delta_G(v))$$

In 2013, Shirdel et al[14] introduced first Hyper-Zagreb index as:

$$HM_1(G) = \sum_{uv\in E(G)} (\delta_G(u)+\delta_G(v))^2.$$

Gao et al[15] in 2016 defined a new distance-based zagreb index named as second Hyper-Zagreb index as:

$$HM_2(G) = \sum_{uv\in E(G)} [\delta_G(u)\delta_G(v)]^2.$$

S. Ghobadi and M. Ghorbaninejad[7] in 2018 defined a new zagreb index named Hyper F-index as:

$$HF(G) = \sum_{uv\in E(G)} [\delta_G^2(u)+\delta_G^2(v)]^2.$$

The concept of sombor index was recently introduced by Gutman[8] in the chemical graph theory. It is a vertex-degree based topological index and denoted by Sombor index SO and defined as:

$$SO(G) = \sum_{uv\in E(G)} \sqrt{\delta^2(u)+\delta^2(v)}.$$

Milan Randic[13] in 1975 introduced the Randic index as:

$$R(G) = \sum_{uv\in E(G)} \frac{1}{\sqrt{\delta(u)\delta(v)}}$$

Apart from the above defined topological indices, there are several other degree based indices, for more such relative work one can go through [16] [9] [12].

Inspired from Ayache et al[5] in 2020 and the physical structures of dome shaped structure and their application to the field of physio- chemical studies we draw a new graphical structure named $K_n - Dome$, also depicted as $G(m,n)$ where the base of this structure will be any complete graph K_n which is later expanded with m number of C_n cycles.This portray like a *dome* with $|V(G)| = nm$ vertices and $|E(G)| = \frac{n^2+4mn-5n}{2}$ edges, that consists of mn-cycles $C_n^1, C_n^2, C_n^3, ..., C_n^m$ as shown in the Figure (1) which is named as $K_n - Dome$, where $n = 3, 4, 5, 6, ..., n$ be number of vertecies in each level as well as positive real numbers and $m = no.\ of\ levels$. Also we compute the same topological indices such as zagreb indices, Hyper zagreb indices, Redefined zagreb indices along with Sombor index and Randic index for the given graph.

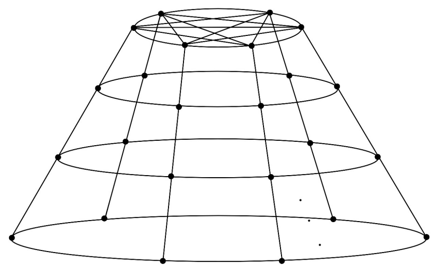

Fig. 1: $K_6 - Dome$

2. Main results

Though the computational properties of toplogical indices apperar simple to look at, it becomes challenging for one to identify the pairity and varsity of the vertex sets, edges sets according to vertex degree and cardinality. In this part we have attempted to make an clear picture of these above properties and computed the important toplogical indices.

Theorem 2.1. *Let $G(m, n)$ be a $K_n - dome$ graph then,*

$$\begin{aligned} M_1(G) &= n(n^2 - 3n + 16m - 11) \\ M_2(G) &= n\left(\frac{1}{2}n^3 - \frac{3}{2}n^2 + 32m - 27\right) \\ F(G) &= n(n^3 - 3n^2 + 64m - 53) \end{aligned}$$

Proof. The vertex set of $G(m, n)$ can be paritioned based on their respective degree as:

$V(G) = \{v_1^1, v_2^1, v_3^1, ..., v_n^1, v_1^2, v_2^2, v_3^2, ..., v_n^2, ..., v_1^m, v_2^m, v_3^m, ..., v_n^m, \}$

Degree of each vertex $= \begin{cases} n & : v = v_1^1 = v_2^1 = ... = v_n^1 \\ 3 & : v = v_i^m \\ 4 & : v = Otherwise \end{cases}$

The edge set of $G(m, n)$ can be written as, $E(G) = \bigcup_{k=0}^{3} = E_k : \bigcap E_k = \phi$,
Where,

$E_0 = \{v_1^1v_3^1, v_1^1v_4^1, v_1^1v_5^1, ..., v_1^1v_{n-1}^1, v_2^1v_4^1, v_2^1v_5^1, v_2^1v_6^1, ..., v_2^1v_n^1, v_3^1v_1^1, v_3^1v_5^1, v_3^1v_6^1, ..., v_3^1v_n^1, ..., v_n^1v_2^1, v_n^1v_3^1, v_n^1v_4^1, ..., v_n^1v_{n-2}^1\}$ and $|E_0| = \frac{n(n-3)}{2}$.

$E_1 = \{v_1^{m-1}v_1^m, v_2^{m-1}v_2^m, ..., v_n^{m-1}v_n^m\}$ and $|E_1| = n$.

$E_2 = \{v_1^mv_2^m, v_2^mv_3^m, ..., v_{n-1}^mv_n^m, v_n^mv_1^m\}$ and $|E_2| = n$

and $E_3 = E_+ \bigcup E_*$ such that $\forall\ j = 1, 2, 3, ..., m-1$

$E_+ = \{v_1^j v_2^j, v_2^j v_3^j, v_3^j v_4^j, ..., v_{n-1}^j v_n^j, v_n^j v_1^j\}$ and $|E_+| = n(m-1)$
$E_* = \{v_1^{j-1} v_1^m, v_2^{j-1} v_2^m, v_3^{j-1} v_3^m, ..., v_n^{j-1} v_n^m\}$ and $|E_+| = n(m-2)$

Thus, $|E_3| = |E_+| + |E_*| = n(m-1) + n(m-2) = n(2m-3)$.
For $i = 1, 2, 3, ..., n$, we have, $E(G) = \bigcup_{k=0}^{3} E_k$, and
- If $uv \in E_0$, then $\delta(u) = n$ and $\delta(v) = n$.
- If $uv \in E_1$, then $\delta(u) = 4$ and $\delta(v) = 3$.
- If $uv \in E_2$, then $\delta(u) = 3$ and $\delta(v) = 3$.
- If $uv \in E_3$, then $\delta(u) = 4$ and $\delta(v) = 4$.

TABLE 1. Relationship between Zagreb indices and its degrees of $K_n - dome$ graphs.

N0.	$uv \in E(G)$	$uv \in E_0$	$uv \in E_1$	$uv \in E_2$	$uv \in E_3$
1	$\delta(u) + \delta(v)$	$2n$	7	6	8
2	$\delta(u)\delta(v)$	n^2	12	9	16
3	$\delta^2(u)\delta^2(v)$	$2n^2$	25	18	32

Now, by Table 1, definition of first, second and forgotten topological index, we have:

$$M_1(G) = \sum_{uv \in E_0} (n+n) + \sum_{uv \in E_1} (4+3) + \sum_{uv \in E_2} (3+3) + \sum_{uv \in E_3} (4+4)$$
$$= \sum_{uv \in E_0} (2n) + \sum_{uv \in E_1} (7) + \sum_{uv \in E_2} (6) + \sum_{uv \in E_3} (8)$$
$$= 2n\left(\frac{n(n-3)}{2}\right) + 7n + 6n + 8(n(2m-3))$$
$$M_1(G) = n(n^2 - 3n + 6m - 11)$$

$$M_2(G) = \sum_{uv \in E_0} (n^2) + \sum_{uv \in E_1} (12) + \sum_{uv \in E_2} (9) + \sum_{uv \in E_3} (16)$$
$$= n^2\left(\frac{n(n-3)}{2}\right) + 12n + 9n + 16(n(2m-3))$$
$$M_2(G) = n\left(\frac{n^3}{2} - \frac{3n^2}{2} + 32m - 27\right)$$

$$F(G) = \sum_{uv\in E_0}(n^2+n^2) + \sum_{uv\in E_1}(16+9) + \sum_{uv\in E_2}(9+9) + \sum_{uv\in E_3}(16+16)$$
$$= (2n^2)\sum_{uv\in E_0}(1) + (25)\sum_{uv\in E_1}(1) + (18)\sum_{uv\in E_2}(1) + (32)\sum_{uv\in E_3}(1)$$
$$= 2n^2\left(\frac{n(n-3)}{2}\right) + 25n + 18n + 32(n(2m-3))$$
$$F(G) = n(n^3 - 3n^2 + 64m - 53)$$

□

Example 1. Suppose G is a $K_n - dome$ with $m = 5$ levels and $n = 6$ vertices in each level, then, by *theorem 2.1* we have
$M_1(G) = 522$, $M_2(G) = 1122$, $F(G) = 2250$.

Theorem 2.2. *Let $G(m, n)$ be a $K_n - dome$ graph then,*

$$HM_1(G) = n(2n^3 - 6n^2 + 128m - 107)$$
$$HM_2(G) = n(\frac{n^5}{2} - \frac{3n^4}{2} + 512m - 543)$$
$$HF(G) = n(2n^5 - 6n^4 + 2048m - 2123)$$

Proof. According to Theorem 1. considering the same edge set partition based on their degree with Table 2. given below.

Table 2. Relationship between hyper Zagreb indices and its degrees of $K_n - dome$ graphs.

N0.	$uv \in E(G)$	$uv \in E_0$	$uv \in E_1$	$uv \in E_2$	$uv \in E_3$
1.	$(\delta(u)+\delta(v))^2$	$4n^2$	49	36	64
2.	$(\delta(u)\delta(v))^2$	n^4	144	81	256
3.	$(\delta^2(u)\delta^2(v))^2$	$4n^4$	625	324	1024

$$HM_1(G) = \sum_{uv\in E_0}(n+n)^2 + \sum_{uv\in E_1}(4+3)^2 + \sum_{uv\in E_2}(3+3)^2 + \sum_{uv\in E_3}(4+4)^2$$
$$= (4n^2)\sum_{uv\in E_0}(1) + (49)\sum_{uv\in E_1}(1) + (36)\sum_{uv\in E_2}(1) + (64)\sum_{uv\in E_3}(1)$$
$$= 4n^2\left(\frac{n(n-3)}{2}\right) + 49n + 36n + 64(n(2m-3))$$
$$HM_1(G) = n(2n^3 - 6n^2 + 128m - 107)$$

$$
\begin{aligned}
HM_2(G) &= \sum_{uv\in E_0}((n)(n))^2 + \sum_{uv\in E_1}((4)(3))^2 + \sum_{uv\in E_2}((3)(3))^2 + \sum_{uv\in E_3}((4)(4))^2 \\
&= (n^4)\sum_{uv\in E_0}(1) + (144)\sum_{uv\in E_1}(1) + (81)\sum_{uv\in E_2}(1) + (256)\sum_{uv\in E_3}(1) \\
&= n^4\left(\frac{n(n-3)}{2}) + 144n + 81n + 256(n(2m-3)\right) \\
HM_2(G) &= n\left(\frac{n^5}{2} - \frac{3n^4}{2} + 512m - 543\right)
\end{aligned}
$$

$$
\begin{aligned}
HF(G) &= \sum_{uv\in E_0}(n^2+n^2)^2 + \sum_{uv\in E_1}(16+9)^2 + \sum_{uv\in E_2}(9+9)^2 + \sum_{uv\in E_3}(16+16)^2 \\
&= (2n^2)^2\sum_{uv\in E_0}(1) + (25)^2\sum_{uv\in E_1}(1) + (18)^2\sum_{uv\in E_2}(1) + (32)^2\sum_{uv\in E_3}(1) \\
&= 4n^4\sum_{uv\in E_0}(1) + (625)\sum_{uv\in E_1}(1) + (324)\sum_{uv\in E_2}(1) + (1024)\sum_{uv\in E_3}(1) \\
&= 4n^4\left(\frac{n(n-3)}{2}) + 625n + 324n + 1024(n(2m-3)\right) \\
HF(G) &= n(2n^5 - 6n^4 + 2048m - 2123)
\end{aligned}
$$

□

Example 2. Suppose G is a $K_n - dome$ with $m = 4$ levels and $n = 5$ vertices in each level, then, by *theorem 3.1* we have
$HM_1(G) = 2525$, $HM_2(G) = 10650$, $HF(G) = 42845$.

Theorem 2.3. *Let $G(m, n)$ be a $K_n - dome$ graph with $|V(G)| = nm$, $|E(G)| = \frac{n^2+4mn-5n}{2}$, then,*

$R_eZG_1(G) = mn + \frac{3n}{4} - 3$,
$R_eZG_2(G) = n\left(\frac{n^2}{4} - \frac{3n}{4} + 4m - \frac{39}{14}\right)$,
$R_eZG_3(G) = n(n^4 - 3n^3 + 256m - 246)$.

Proof. Again, according to the edge set partition considered in theorem 1. and with reference to table 3.

TABLE 3. Relationship between redefined Zagreb indices and its degrees of $K_n - dome$ graphs.

N0.	$uv \in E(G)$	$uv \in E_0$	$uv \in E_1$	$uv \in E_2$	$uv \in E_3$
1.	$\frac{\delta(u)+\delta(v)}{\delta(u)\delta(v)}$	$\frac{2}{n}$	$\frac{7}{12}$	$\frac{2}{3}$	$\frac{1}{2}$
2.	$\frac{\delta(u)\delta(v)}{\delta(u)+\delta(v)}$	$\frac{n}{2}$	$\frac{12}{7}$	$\frac{3}{2}$	2
3.	$\delta(u)\delta(v)(\delta(u)+\delta(v))$	$2n^3$	84	54	128

$$\begin{aligned}
R_eZG_1(G) &= \sum_{uv\in E_0} \frac{n+n}{(n)(n)} + \sum_{uv\in E_1} \frac{4+3}{(4)(3)} + \sum_{uv\in E_2} \frac{3+3}{(3)(3)} + \sum_{uv\in E_3} \frac{4+4}{(4)(4)} \\
&= \sum_{uv\in E_0} \frac{2}{n} + \sum_{uv\in E_1} \frac{7}{12} + \sum_{uv\in E_2} \frac{2}{3} + \sum_{uv\in E_3} \frac{1}{2} \\
&= \frac{2}{n}\left(\frac{n(n-3)}{2}) + \frac{7}{12}n + \frac{2}{3}n + \frac{1}{2}(n(2m-3)\right) \\
R_eZG_1(G) &= mn + \frac{3n}{4} - 3
\end{aligned}$$

$$\begin{aligned}
R_eZG_2(G) &= \sum_{uv\in E_0} \frac{n^2}{n+n} + \sum_{uv\in E_1} \frac{12}{7} + \sum_{uv\in E_2} \frac{3}{2} + \sum_{uv\in E_3} 2 \\
&= \frac{n}{2}\left(\frac{n(n-3)}{2}) + \frac{12n}{7} + \frac{3n}{2} + 2(n(2m-3)\right) \\
R_eZG_2(G) &= n\left(\frac{n^2}{4} - \frac{3n}{4} + 4m - \frac{39}{14}\right)
\end{aligned}$$

$$\begin{aligned}
R_eZG_3(G) &= \sum_{uv\in E_0} (n^2)(2n) + \sum_{uv\in E_1} (12)(7) + \sum_{uv\in E_2} (9)(6) + \sum_{uv\in E_3} (16)(8) \\
&= 2n^3\left(\frac{n(n-3)}{2}) + 84n + 54n + 128(n(2m-3)\right) \\
R_eZG_3(G) &= n(n^4 - 3n^3 + 256m - 246)
\end{aligned}$$

□

Example 3. Suppose G is a $K_n - dome$ with $m = 4$ levels and $n = 4$ vertices in each level, then, by *theorem 4.1* we have $R_eZG_1(G) = 16$, $R_eZG_2(G) = 56.86$, $R_eZG_3(G) = 3368$.

Theorem 2.4. *Let $G(m,n)$ be a $K_n - dome$ graph with then, $R(G) = n(\frac{2\sqrt{3}+1}{12}) + \frac{mn}{2} - \frac{3}{2}$.*

Proof. With reference to edge sets partition from Theorem 1. and table 4 given below.

TABLE 4. Relationship between Randic index and its degrees of $K_n - dome$ graphs.

N0.	$uv \in E(G)$	$uv \in E_0$	$uv \in E_1$	$uv \in E_2$	$uv \in E_3$
1.	$\frac{1}{\sqrt{\delta(u)\delta(v)}}$	$\frac{1}{n}$	$\frac{1}{2\sqrt{3}}$	$\frac{1}{3}$	$\frac{1}{4}$

$$\begin{aligned}
R(G) &= \sum_{uv\in E_0}\left(\frac{1}{\sqrt{(n)(n)}}\right)+\sum_{uv\in E_1}\left(\frac{1}{\sqrt{(4)(3)}}\right)+\sum_{uv\in E_2}\left(\frac{1}{\sqrt{(3)(3)}}\right)+\sum_{uv\in E_3}\left(\frac{1}{\sqrt{(4)(4)}}\right)\\
&= \sum_{uv\in E_0}\left(\frac{1}{\sqrt{n^2}}\right)+\sum_{uv\in E_1}\left(\frac{1}{\sqrt{12}}\right)+\sum_{uv\in E_2}\left(\frac{1}{\sqrt{9}}\right)+\sum_{uv\in E_3}\left(\frac{1}{\sqrt{16}}\right)\\
&= \frac{1}{n}\left(\frac{n(n-3)}{2}\right)+\frac{1}{2\sqrt{3}}(n)+\frac{1}{3}(n)+\frac{1}{4}(n(2m-3))\\
R(G) &= n\left(\frac{2\sqrt{3}+1}{12}\right)+\frac{mn}{2}-\frac{3}{2}
\end{aligned}$$

□

Theorem 2.5. *Let $G(m,n)$ be a $K_n - dome$ graph with then,*

$$SO(G) = n\left(\frac{\sqrt{2}n^2}{2}-\frac{3\sqrt{2}n}{2}+8\sqrt{2}m+(5-9\sqrt{2})\right)$$

.

Proof. With reference to edge sets partition from Theorem 1., and Table 5 given below.

TABLE 5. Relationship between Sombor index and its degrees of $K_n - dome$ graphs.

N0.	$uv \in E(G)$	$uv \in E_0$	$uv \in E_1$	$uv \in E_2$	$uv \in E_3$
1.	$\sqrt{\delta^2(u)+\delta^2(v)}$	$\sqrt{2}n$	5	$3\sqrt{2}$	$4\sqrt{2}$

$$\begin{aligned}
SO(G) &= \sum_{uv\in E_0}(\sqrt{n^2+n^2})+\sum_{uv\in E_1}(\sqrt{4^2+3^2})+\sum_{uv\in E_2}(\sqrt{3^2+3^2})+\sum_{uv\in E_3}(\sqrt{4^2+4^2})\\
&= \sum_{uv\in E_0}(\sqrt{2n^2})+\sum_{uv\in E_1}(\sqrt{25})+\sum_{uv\in E_2}(\sqrt{18})+\sum_{uv\in E_3}(\sqrt{32})\\
&= \sqrt{2}n\left(\frac{n(n-3)}{2}\right)+5(n)+3\sqrt{2}(n)+4\sqrt{2}(n(2m-3))\\
SO(G) &= n\left(\frac{\sqrt{2}n^2}{2}-\frac{3\sqrt{2}n}{2}+8\sqrt{2}m+(5-9\sqrt{2})\right)
\end{aligned}$$

□

3. CORRELATION COEFFICIENT OF SOME SPECIAL $K_n - DOME$ GRAPHS

In this section, we calculate the Coefficient Correlation for Zagreb Indices, Hyper Zagreb Indices, Redefined Zagreb Indices, Randic Index and Sombor Index of $K_n - Dome$ graphs $G(m, n)$ with m level where $m = 3, 4, ..., 10, 11$ and $n = 4$ vertices, we have $|V(G)| = nm = 4m$, $|E(G)| = \frac{n^2+4mn-5n}{2} = 2(4m-1)$, where, $m = 3, 4, ..., 10, 11$.
Table 6. gives the clear picture of the numerical values of these indices for increasing value of m for the fixed value $n = 4$.The correlation coefficient of $K_n - Dome$ graphs $G(m, n)$ are found in Table 7.

Table 6. The zagreb indices, hyper zagreb indices, redefined zagreb indices, Randic index, and Sombor index of $K_n - dome$ graphs $G(m, n)$ with $m = 3, 4, 5, 6, ..., 11$ levels and n = 4 vertices.

$m =$	3	4	5	6	7	8	9	10	11
M_1	164	228	292	356	420	484	548	612	676
M_2	308	436	564	692	820	948	1076	1204	1332
F	620	876	1132	1388	1644	1900	2156	2412	2668
HM	1236	1748	2260	2772	3284	3796	4308	4820	5332
HM_2	4484	6532	8580	10628	12676	14724	16772	18820	20868
HF	18132	26324	34516	42708	50900	59092	67284	75476	83668
R_eZG_1	12	16	20	24	28	32	36	40	44
R_eZG_2	40.86	56.86	72.86	88.86	104.86	120.86	136.86	152.86	168.86
R_eZG_3	2344	3368	4392	5416	6440	7464	8488	9512	10536
$R(G)$	5.99	7.99	9.99	11.99	13.99	15.99	17.99	19.99	21.99
$SO(G)$	116.17	161.42	206.68	251.93	297.19	342.44	387.70	432.95	478.20

TABLE 7. The correlation coefficient of $K_n - dome$ graphs $G(m,n)$ between the Zagreb indices, Hyper zagreb indices, Redefined zagreb indices, Randic index and sombor index.

Correlation	M_1	M_2	F	HM	HM_2	HF	R_eZG_1	R_eZG_2	R_eZG_3	$R(G)$	$SO(G)$
M_1	1	1	1	1	1	1	1	1	1	1	1
M_2	1	1	1	1	1	1	1	1	1	1	1
F	1	1	1	1	1	1	1	1	1	1	1
HM	1	1	1	1	1	1	1	1	1	1	1
HM_2	1	1	1	1	1	1	1	1	1	1	1
HF	1	1	1	1	1	1	1	1	1	1	1
R_eZG_1	1	1	1	1	1	1	1	1	1	1	1
R_eZG_2	1	1	1	1	1	1	1	1	1	1	1
R_eZG_3	1	1	1	1	1	1	1	1	1	1	1
$R(G)$	1	1	1	1	1	1	1	1	1	1	1
$SO(G)$	1	1	1	1	1	1	1	1	1	1	1

In Table 7. We have shown the physicochemical properties of some special cases of $K_n - Dome$ graphs for which give perfect correlations, i.e., corelation point is 1.

Conclusion

This work is inspired from the structral properties of domes and other chemical apparatus that are intended to have a thick base. They are designed as such in order to increase the stability and tipping over. Hence these toplogical indices having the perfect corelation these structural studies can help solve the numerical problems associated to the QSPR properties of the apparatus used.

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