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AN ASSESSMENT OF THE ORTHOGONAL AND BIORTHOGONAL DISCRETE WAVELET TRANSFORM BASED DENOISING OF BIOLOGICAL SIGNALS – A STUDY BASED ON ECG

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Abstract. Wavelets transforms are effective mathematical tools useful in compression and denoising of n-D signals. Subjecting a signal to discrete wavelet transforms (DWT) generates approximation and detailed coefficients that are similar to the coefficients generated by passing the signal through lowpass and high pass filters respectively. Multilevel decomposition (filtering) can be carried out which can successively filter the signal at each bandpass. In this work, a normal sinus rhythm ECG signal noised with a sinewave was denoised by DWT using by members of mother wavelets like coiflet, symlet and biorthogonal families. The denoising performance of the wavelets were evaluated by comparing the signal characteristics of the original and the denoised signals.

1. Introduction

The Wavelet transform is a mathematical tool used in signal processing, Image compression, denoising a signal, feature extraction etc. Wavelet transforms can be broadly classified into two main types: the continuous wavelet transform (CWT) and the discrete wavelet transform (DWT). The continuous wavelet transform is a time-frequency transform, which is used for analyzing continuous signals or in other words, a CWT is used for analyzing a signal whose frequency-domain representation changes over time. A continuous wavelet transform provides a more detailed analysis, but it is computationally expensive.
The Continuous wavelet transform is defined as

$$W(a,b) = \frac{1}{|a|}\int_{-\infty}^{\infty} x(t)\psi\left(\frac{t-b}{a}\right)dt$$

Here, $W(a,b)$ is called the wavelet coefficient, 'a' and 'b' are the are the scaling and translational parameters respectively. $\psi(t)$ is called the mother wavelet. The mother wavelet serves as a basis for generating a family of wavelets by different dilations and translations. Morelet wavelet, Haar wavelet, Daubechies wavelet etc.

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are examples of some common mother wavelets.
With the discrete wavelet transform, scales are discretized more coarsely than with continuous wavelet transform. A discrete wavelet transform is more easier to compute and useful for denoising signals and compressing the images by preserving its important features. It can be used to perform multi resolution analysis and split signals into physically meaningful and interpretable components.

The discrete wavelet transform [1] is given by $\psi_{m,n}(t) = a_0^{\frac{-m}{2}\psi\left(\frac{t-nb_0}{a_0^m}\right)}$ where m and n are integer values related to scale and shift parameters 'a' and 'b' respectively, and discretized as $a = a_0^m$ and $b = nb_0$. For more properties of wavelet transforms one can refer [2],[3].
In discrete wavelet transform, the signal is decomposed into two levels such as coarse approximation and detail information. In DWT, the signal passes through a scaling filter band a wavelet filter. The output of these filters is down sampled by a factor of 2 by discarding every other sample [4].
There are different kinds of wavelets. The choice of wavelet function depends on the application. A Haar wavelet is the simplest type of wavelet. In our study, we have used the daughter wavelets from coiflet, symlet and biorthogonal wavelet families to assess denoising of ECG signals.
Coiflets are discrete wavelets designed by Ingrid Daubechies, at the request of Ronald Coifman, which possess scaling functions with vanishing moments. These wavelets are compactly supported with a higher number of vanishing moments for both scaling function and wavelet function. It is an orthogonal wavelet. This wavelet is not symmetric but near symmetric. A symlet is a reasonably short wavelet in time domain with a good degree of smoothness. Symlet is a modification of Daubechies wavelet which has almost symmetrical characteristics. A biorthogonal wavelet is a wavelet where the associated wavelet transform is invertible but not necessarily orthogonal.
Wavelets have been extensively used for compressing as well as denoising signals. 1-D signals like radiofrequency or sound waves, and N-D signals like images, video files etc can be effectively denoised with wavelets. However, the choice of the wavelet for each application is empirical. Wavelets have also been used to denoise biological signals like electrocardiogram (ECG), electroencephalogram etc. There are numerous reports of wavelet based denoising of ECG signals in literature. For a review on the wavelet families used for denoising ECG waves, one can refer to [5].
Orthogonal wavelet families [such as- haar, daubechies, coiflet (coif), symmlet (sym) etc.], and the biorthogonal wavelet (bior), at multiple levels of decomposition, have been extensively studied and employed for ECG denoising [6]. It has been revealed that both the members of both the wavelet type families are suitable for this purpose. In the current study, we used two orthogonal wavelets (coif5 and sym4) and one biorthogonal wavelet (bior6.8) up to four levels of decomposition to assess their capabilities in denoising a normal sinus rhythm ECG signal. In addition, we have employed computational (peak signal to noise ratio) and clinical assessment (by a cardiologist) to evaluate each wavelet's performance in denoising the signal.

2. Methods

An ECG signal from MIT-BIH Normal Sinus Rhythm Database available on PhysioNet [7], was used to perform denoising of signals using DWT. The database includes ECG recording of 18 subjects from Boston's Beth Israel Hospital (BIH). All the recordings show normal sinus rhythm, with no significant arrhythmia. For this research work, we have chosen the first record (No. 16265) without any pre-processing. Record 16265 is a two-channel ECG recording containing 2 signals (ECG1 and ECG2), with a sample rate of 128/sec and 25 minutes long (11730944 samples). The last 3 minutes of the recording consists of only noise, without any ECG waves. For this study, we have used the initial 32000 samples from the ECG1 signal of the record.
Data analyses and visualization was performed using *Python* (3.12.1 stable release). The ECG record from MIT-BIH Normal Sinus Rhythm Database was accessed using the *Python* library- *Waveform Database Software Package* (WFDB version 4.1.0) [8]. The other Python libraries used for data processing include- *SciPy* (for scientific computing) and its subpackages -*io, signal and stats* [9], *numpy* (for numerical computing) [10], *matplotlib* (for visualization) [11], *Librosa* (for audio file processing) [12], *os, pandas* (for data analysis) [13] and *scikit-image* [image and signal processing] [14]. The orthogonal and biorthogonal wavelet analysis were performed using the *PyWavelets* [15] package for *Python*, an open-source wavelet transform software with 1D, 2D and nD multilevel DWT and IDWT capabilities. The processing and denoising of ECG signal using orthogonal and biorthogonal DWT was performed in the following manner.

A. Adding noise to the ECG signal

The initial 32001 data samples from the ECG signal (record 16265) were obtained and used for further analyses. The signal was analyzed using *Fast Fourier Transform* to check for any noise frequencies (data not shown). However, the signal was already preprocessed by the original investigators before it was made publicly available. Therefore, we added a sinewave of 40 Hz with the same sampling rate (180/s) and time duration (32000 samples) as that of the signal was added to it to obtain noised ECG signal.

B. Multilevel 1D Discrete Wavelet Transform of signals

The *pywt.wavedec()* function of *PyWavelets* library was used for multilevel decomposition of the signal that was obtained as a *NumPy* array. Along with the signal, the other arguments of the function *pywt.wavedec()* include– wavelet type and decomposition level (1-4). We used symmetric-padding, i.e., signal extension by mirroring the samples, to extrapolate the input data before computing the Discrete Wavelet Transform. We performed DWT of the noised signal using two orthogonal wavelets (coif5 and sym4) and a biorthogonal wavelet (bior6.8), each up to four levels. The computation generated sets of *approximation coefficient* (cA, the lowpass sub band) and *detailed coefficient* (cD, the high pass sub band) at each level of the decomposition. The *approximation coefficient* (cA) obtained at each level of the decomposition was used for further analysis.

C. Comparison of the original signal and the transformed signal at each level of wavelet decomposition

We determined the peak signal to noise ratio (PSNR) between the original clean signal and the cA (lowpass band) obtained at each level to determine the level of noise in the processed signal. However, while using the *pywt.wavedec()* function, at each level of decomposition, the data points obtained in cA approximately reduced to half its previous value. That is, at level-1 the number of data points in cA ($\sim$ 16000) was half of the initial signal data points (32000 samples). Similarly, at Level-2 decomposition, the number of cA data points obtained ($\sim$ 8000) was half as that of the data points of Level-1 cA, and so on. Therefore, the original clean signal had to be down sampled (halved) at each level for calculating the PSNR between the clean signal and the processed signal. We used *scipy.signal.resample()* function to downsample the original clean signal. The 1-D signals were normalized and the PSNR was calculated using the $skimage.metrics.peak\_signal\_noise\_ratio()$ function.
The transformed signal was also subjected to clinical evaluation by a consultant cardiologist blinded to these processes to avoid any bias. The obtained cA (up to four levels) for each of the three wavelets (coif5, sym4 and bior6.8) were evaluated visually and compared with the original signal to check similarities and precision of ECG components and features. The transformed signal that resembles the original signal the most was identified by the observer.

3. Results

The raw ECG signals from record 16265 obtained from the MIT-BIH normal sinus rhythm database is shown in **Figure 1**. The individual (P, Q, R, S, T) wave components can be observed in the clean signal. Addition of 40 Hz sinewave created noised wave which was then subjected to denoising using DWT. We have performed the wavelet transforms only till level-4, since beyond that the resultant transformed signals started losing pertinent features of ECG components.

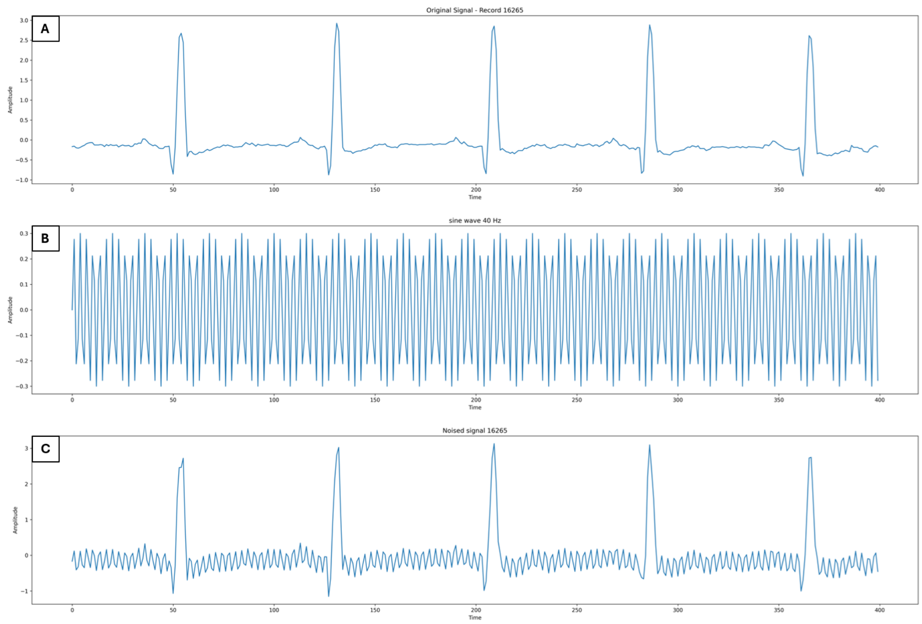

FIGURE 1. (A) The ECG signal (record no. 16265) accessed from MIT-BIH Normal Sinus Rhythm Database. (Sampling rate: 180/s, first 400 data points are shown in the plot); (B) sinewave of 40 Hz (noise, sampling rate: 180/s, sampling points: 400); (C) Noised signal.

1. Multilevel decomposition using coif5 wavelet **Figure 2** shows the four levels of decomposition using coif5 wavelet. The PSNR values at each level are given in Table 1. The highest PSNR is observed at level-1, where the cA plot exhibited all the individual ECG wave components and matched with original signal, according to the observations by the clinician.

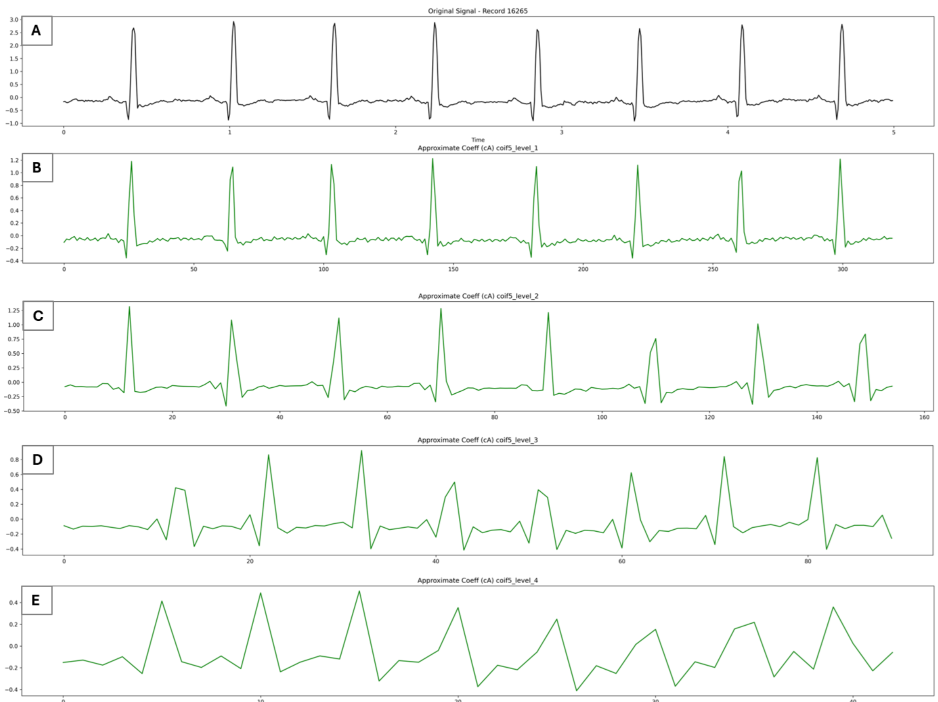

Figure 2. Plot of the original signal and the approximation coefficients (cA) obtained from multilevel decomposition of the noises signal using the orthogonal wavelet- coif5 (A) original signal; (B) cA at level-1; (C) cA at level-2; (D) cA at level-3; (E) cA at level-4.

2. Multilevel decomposition using syn4 wavelet

The four levels of noised wave decomposition using sym4 wavelet is shown in **figure 3** and their PSNR values at each level in **Table 1**. According to the computation the highest PSNR is observed at level-1, whereas clinical comparison showed that the level-2 retains the most features of ECG (P, Q, R, S, T) wave components.

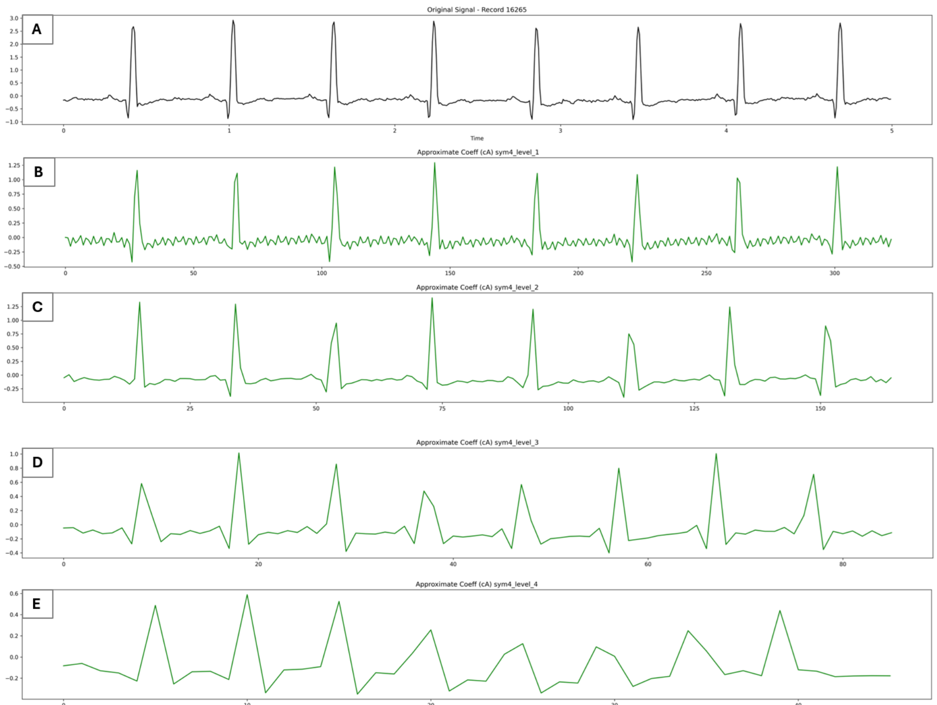

FIGURE 3. Plot of the original signal and the approximation coefficients (cA) obtained from multilevel decomposition of the noises signal using the orthogonal wavelet- sym4 (A) original signal; (B) cA at level-1; (C) cA at level-2; (D) cA at level-3; (E) cA at level-4.

3. Multilevel decomposition using bior6.8 wavelet

The plot of the cA at each level of noised wave decomposition by bior6.8 is shown in **figure 4**. Here the level-1 shows the highest PSNR value (**Table 1**). However, the clinical examination showed that level-2 coefficients were mst similar to the original ECG wave.

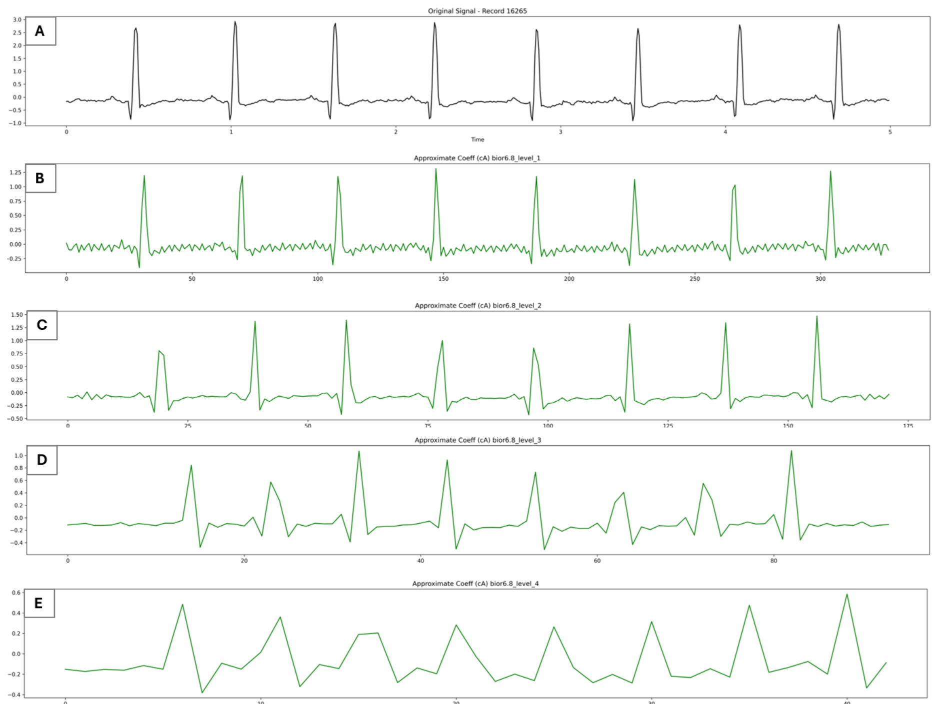

FIGURE 4. Plot of the original signal and the approximation coefficients (cA) obtained from multilevel decomposition of the noises signal using the biorthogonal wavelet- bior6.8 (A) original signal; (B) cA at level-1; (C) cA at level-2; (D) cA at level-3; (E) cA at level-4.

Wavelet	PSNR(db) values at			
	Level-1	Level-2	Level-3	Level-4
coif5	26.87*	14.19	12.32	10.25
sym4	15.21	14.20*	12.64	10.83
bior6.8	14.80	14.06*	12.32	10.42

Table1.PSNR values of the approximation coefficient at four levels of decomposition. * Indicates the level at which the signal exhibits closest resemblance to the original wave in terms of precision when examined by the clinician.

4. Discussion and conclusion

Discrete Wavelet Transform is a set of mathematical functions employed to compress and denoise N-dimensional signals. In this research work, we performed multilevel decomposition on a noised ECG signal using two orthogonal wavelets- coif5 and sym4, and a biorthogonal wavelet- bior6.8 using the PyWavelet library

for *Python*. Four levels of decomposition were performed and the approximation coefficients at each level were plotted and compared with the original clean signal computationally by calculating peak signal to noise ratio (PSNR), and visually by a cardiologist who was blinded to the computational aspects of the work.
The PSNR values of each wavelet kept decreasing with each increasing decomposition levels, indicating that, with each level of decomposition the loss pass sub band was becoming noisier. With each increasing level, the signal was deviating more from the original wave. However, visual inspection and comparison of the cAs with the original wave by a clinician resulted in a different outcome. The comparison of the clinician was based on the presence or absence of of ECG sub-wave features in the cAs and the level of precision with the original ECG wave. In two out of three wavelets (Table 1), the clinician chose signals that has lower PSNR values to be more precise than their higher PSNR counterparts. There included sym4 (orthogonal wavelet) and bior6.8 (biorthogonal wavelet).
Among the three signals that exhibited the maximum resemblance with the original signal, the approximation coefficient of coif5 had most precision comparison to sym4 and bior6.8. Therefore, it can be concluded that coif5 is more suitable for denoising ECG signals than sym4 ans bior6.8. In addition, for denoising of biological signals, the computational tools and PSNR should not be the only criteria for determining the suitability of a wavelets, the clinical interpretation and opinion should also be an important factor in their selection.

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