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INDEPENDENT TRANSVERSAL GEODETIC ENERGY OF A GRAPH

R.VASANTHI AND M.PERUMALSAMY

Abstract. Through out this research, we consider a simple connected graph with order n. A geodetic set $S \subseteq V(G)$ that intersects every maximum independent set (β_0-set) of G is termed as an independent transversal geodetic set [10]. The minimum cardinality of such a set is known as the independent transversal geodetic number, denoted by $g_{it}(G)$ [10]. An independent transversal geodetic set of minimum cardinality is referred as a g_{it}-set [10]. For each g_{it}-set S in G, we define an $n \times n$ matrix, called the independent transversal geo-adjacency matrix $M_{S_{g_{it}}}(G)$. The entries of this matrix are defined as follows: an entry is equal to 1 if at least one of the vertices v_j or v_k belongs to S and v_j is adjacent to v_k; otherwise, the entry is 0. In this research article, we introduce the new concepts independent transversal geodetic energy with respect to the g_{it}-set S and the independent transversal geodetic energy of the graph G. We compute various results to these parameters for several graphs and provide a detailed analysis of their properties.

1. Preliminaries

Throughout this research article,we work with a graph $G = (V, E)$ that is simple, undirected, finite, and connected. The number of edges incident with a vertex v in G is called the degree of v in G and is denoted by $deg_G(v)$ [1, 2, 3].

A set of vertices $I \subseteq V(G)$ is an independent set if no two vertices in I are adjacent [1, 2, 3]. I is said to be maximum if $|I| > |I'|$ for any other independent set I' [1, 2, 3]. A set $S \subseteq V(G)$ is called a *geodetic set* if every vertex of G lies on a shortest u-v path for some $u, v \in S$ [4, 5]. The minimum cardinality among all geodetic sets is called *geodetic number* and is denoted by $g(G)$ [4, 5]. A geodetic set $S \subseteq V(G)$ that intersects every maximum independent set (β_0-set) of G is called an independent transversal geodetic set [10]. The minimum cardinality of such a set is called the independent transversal geodetic number of G and is denoted by $g_{it}(G)$ [10]. An independent transversal geodetic set of minimum cardinality is denoted by g_{it}-set [10].

Independent transversal domination number and independent transversal geodetic number are studied in [6, 10]. Vertex covering transversal domination number of a graph and also of regular graphs are analyzed in [7, 8]. Vertex covering transversal Geodetic number and vertex covering transversal geodomatic number

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are introduced and studied in [9, 11] respectively.
For basic graph theoretic terminology, we refer to Harary [3].

In this paper, we define *independent transversal geo-adjacency matrix with respect to a g_{it}-set in G* and *independent transversal geodetic energy of G.* We provide a detailed analysis of the properties of independent transversal geo-adjacency matrix with respect to a g_{it}-set in G. We too analyze the independent transversal geodetic energy of graphs in detail.
The theorems and results listed below are referred where appropriate.

Theorem 1.1. [10] For a path graph P_n with $n \geq 3$ vertices, $g_{it}(P_n) = 2$

Theorem 1.2. [10] If C_{2n+1} is an odd cycle with $n \geq 1$, then

$$g_{it}(C_{2n+1}) = \begin{cases} 3 & if \quad n \quad is \quad odd \\ 4 & if \quad n \quad is \quad even \end{cases}$$

Theorem 1.3. [10] If C_{2n} is an even cycle with $n \geq 2$, then

$$g_{it}(C_{2n}) = \begin{cases} 2 & if \quad n \quad is \quad odd \\ 3 & if \quad n \quad is \quad even \end{cases}$$

Theorem 1.4. [10] If Q_n is a hypercube on n vertices with $n \geq 3$, then

$$g_{it}(Q_n) = \begin{cases} 2 & if \quad n \quad is \quad odd \\ 3 & if \quad n \quad is \quad even \end{cases}$$

2. Independent Transversal geo-adjacency matrix

Definition 2.1. Let G be a simple connected graph of order $n \geq 3$. The independent transversal geo-adjacency matrix of G corresponding to a g_{it}-set of a graph G of order $n \geq 3$, denoted by $M_{S_{g_{it}}}(G)$, is defined as an $n \times n$ matrix whose entries are given as follows

$$= \begin{cases} 1 & \text{if at least one of} \quad v_j, v_k \in S \quad \text{and} \quad v_j \quad \text{is adjacent to} \quad v_k \\ 0 & \text{otherwise} \end{cases}$$

Note 2.2. The independent transversal geo-adjacency matrix $M_{S_{g_{it}}}(G)$ is defined relative to a specific g_{it}-set S. Consequently, the set of all such matrices associated with G is indexed by collection of its g_{it}-sets.

Note 2.3. By the definition 2.1, it follows that the principal diagonal entries of the independent transversal geo-adjacency matrix $M_{S_{g_{it}}}(G)$ are zero, a direct consequence of G being a simple graph.

Note 2.4. We observe that for any g_{it}-set S in G, the resulting independent transversal geo-adjacency matrix $M_{S_{g_{it}}}(G)$ is symmetric.

Example 2.5. We refer to the graph G presented in Figure 1.

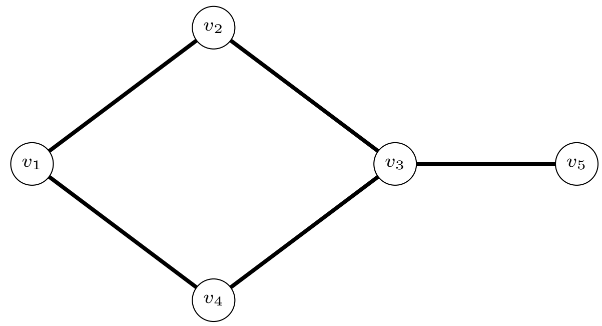

FIGURE 1. Graph G

Referring [10], $S = \{v_1, v_5\}$ is the unique g_{it}-set in G. Then the independent transversal geo-adjacency matrix with respect to S is given by

$$M_{S_{g_{it}}}(G) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

.

Example 2.6. We next illustrate the concept using Figure 2.

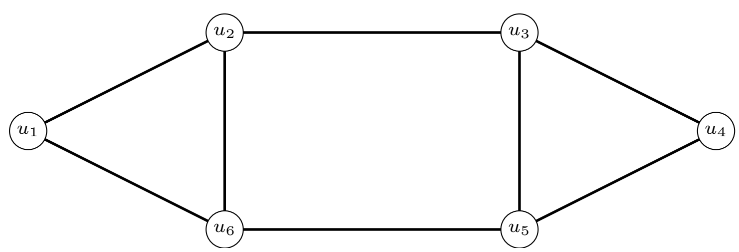

FIGURE 2. Graph G

Referring [10], $S_1 = \{u_1, u_2, u_4, u_6\}$ is an independent transversal geodetic set of minimum cardinality (i.e) a g_{it}-set.
Then the independent transversal geo-adjacency matrix with respect to S_1 is given by

$$M_{S_{1_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&0&1\\0&1&0&1&0&0\\0&0&1&0&1&0\\0&0&0&1&0&1\\1&1&0&0&1&0\end{bmatrix}$$

Also, $S_2=\{u_1,\,u_3,\,u_4,\,u_5\}$ is another g_{it}-set.

Then the independent transversal geo-adjacency matrix with respect to S_2 is given by

$$M_{S_{2_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&0&0\\0&1&0&1&1&0\\0&0&1&0&1&0\\0&0&1&1&0&1\\1&0&0&0&1&0\end{bmatrix}$$

It is possible to find two other g_{it}-sets in G, $S_3=\{u_1,\,u_2,\,u_3,\,u_4\}$ and $S_4=\{u_1,\,u_4,\,u_5,\,u_6\}$.
Then their corresponding independent transversal geo-adjacency matrices are

$$M_{S_{3_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&0&1\\0&1&0&1&1&0\\0&0&1&0&1&0\\0&0&1&1&0&0\\1&1&0&0&0&0\end{bmatrix}\quad\text{and}\quad M_{S_{4_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&0&0&1\\1&0&0&0&0&1\\0&0&0&1&1&0\\0&0&1&0&1&0\\0&0&1&1&0&1\\1&1&0&0&1&0\end{bmatrix}$$

Example 2.7. Take the graph G as depicted in Figure 3 [10].

Referring [10], the set $I=\{v_2,\,v_4,\,v_5,\,v_7\}$ is the unique β_0-set in G and $S_1=\{v_1,\,v_2,\,v_6\}$ is a g_{it}-set in G.
Obviously, $S_2=\{v_1,\,v_4,\,v_6\}$, $S_3=\{v_1,\,v_5,\,v_6\}$ and $S_4=\{v_1,\,v_6,\,v_7\}$ are also g_{it}-sets in G. Then their corresponding independent transversal geo-adjacency matrices are given by

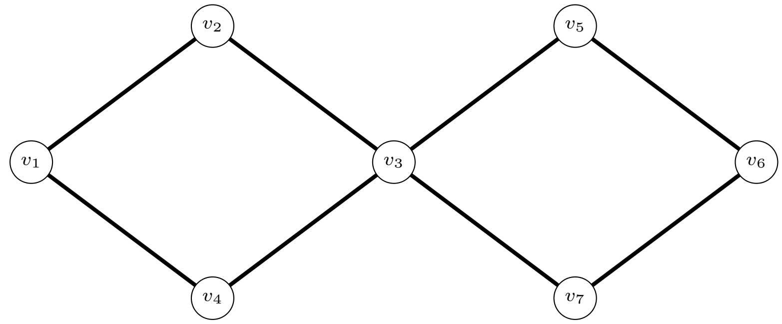

FIGURE 3. Graph G

$$M_{S_{1_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&1&0&0&0\\1&0&1&0&0&0&0\\0&1&0&0&0&0&0\\1&0&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&1&0&1\\0&0&0&0&0&1&0\end{bmatrix},\quad M_{S_{2_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&1&0&0&0\\1&0&0&0&0&0&0\\0&0&0&1&0&0&0\\1&0&1&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&1&0&1\\0&0&0&0&0&1&0\end{bmatrix}$$

,

$$M_{S_{3_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&1&0&0&0\\1&0&0&0&0&0&0\\0&0&0&0&1&0&0\\1&0&0&0&0&0&0\\0&0&1&0&0&1&0\\0&0&0&0&1&0&1\\0&0&0&0&0&1&0\end{bmatrix}\ \&\ M_{S_{4_{g_{it}}}}(G)=\begin{bmatrix}0&1&0&1&0&0&0\\1&0&0&0&0&0&0\\0&0&0&0&0&0&1\\1&0&0&0&0&0&0\\0&0&0&0&0&1&0\\0&0&0&0&1&0&1\\0&0&1&0&0&1&0\end{bmatrix}$$

Remark 2.8. In Example 2.5, the independent transversal geo-adjacency matrix is unique as there is a unique g_{it}-set in G, whereas in Examples 2.6 and 2.7, there are 4 independent transversal geo-adjacency matrices as there are four g_{it}-sets in each graph.

The following theorem follows immediately from definition 2.1.

Theorem 2.9. Let $V(G)=\{v_1, v_2, ..., v_n\}$ and let S be a g_{it}-set of G. Let $M_{S_{g_{it}}}(G)$ be the independent transversal geo-adjacency matrix with respect to the g_{it}-set S. Then the entries of the k-th row(column) in $M_{S_{g_{it}}}(G)$ is all zero, provided $v_k \notin S$ and is not adjacent to any of the vertices in S.

Theorem 2.10. Let $V(G)=\{v_1, v_2, ..., v_n\}$ and let S be a g_{it}-set of G. Let $M_{S_{g_{it}}}(G)$ denote the independent transversal geo-adjacency matrix with respect to S. If $v_k \in S$, then the sum of the entries in the k-th row (column) of $M_{S_{g_{it}}}(G)$ is equal to $deg_G(v_k)$.

Proof: Let S be a g_{it}-set of graph G consisting of m vertices from $V(G)$ where $m \leq n$.
It is clear that all the entries of $M_{S_{g_{it}}}(G)$ are either 0 or 1.
Moreover, entry in the (j, k)th position is equal to 1 if and only if at least one of the vertices v_j or v_k belongs to S and are adjacent in G.
Suppose that $v_k \in S$ for some k.
Then even if $v_j \notin S$ but v_j is adjacent to v_k, then the (j, k)th entry of $M_{S_{g_{it}}}(G)$ is 1.
Consequently, the total number of 1's in the k-th row of $M_{S_{g_{it}}}(G)$ corresponds to the number of vertices that are adjacent to v_k in G.
Hence, the sum of the entries in the k-th row of $M_{S_{g_{it}}}(G)$ equals the number of edges incident to v_k, which is precisely $deg_G(v_k)$.

□

Remark 2.11. The assumption $v_k \in S$ in Theorem 2.9 is sufficient, but it is not a necessary condition. In fact, even $v_k \notin S$, the sum of entries in the k-th row(column) of the matrix $M_{S_{g_{it}}}(G)$ is equal to $deg_G(v_k)$".
To illustrate this, consider the path graph P_3.
In this graph, the set $S = \{v_1, v_3\}$ is the unique g_{it}-set of P_3.
Consequently, the independent transversal geo-adjacency matrix corresponding to S is the 3×3 matrix given by

$$M_{S_{g_{it}}}(P_3) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Clearly $v_2 \notin S$ and $deg(v_2) = 2$.
Also the sum of the elements in the 2nd row of $M_{S_{g_{it}}}(P_3)$ is also equal to 2.

Remark 2.12. There are graphs in which the property does not hold if the condition fails. That is, "The sum of the elements in the k-th row(column) of $M_{S_{g_{it}}}(G)$ is less than $deg_G(v_k)$ for $v_k \notin S$".
For instance, let us consider the graph G shown in Example 2.5.
$S = \{v_1, v_5\}$ is the unique g_{it}-set in G.
Then the independent transversal geo-adjacency matrix with respect to S is given by

$$M_{S_{g_{it}}}(G) = \begin{bmatrix} 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \end{bmatrix}$$

.

Here, $v_2, v_3, v_4 \notin S$ and $deg_G(v_2) = 2$, $deg_G(v_3) = 3$ & $deg_G(v_4) = 2$.
The sum of the entries of the second row of $M_{S_{g_{it}}}(G)$ equals 1, which is strictly less than $deg_G(v_2) = 2$. In the same way, the sums of the entries of the third and fourth rows of $M_{S_{g_{it}}}(G)$ are each equal to 1, and both are smaller than the corresponding degrees of v_3 and v_4 in G.

Theorem 2.13. Let $V(G)$ =$\{v_1,\ v_2,\ ...,\ v_n\}$ and let $S_1,\ S_2,\ \cdots,\ S_l$ denote the g_{it}-sets. Let $M_{S_{j_{g_{it}}}}(G)$ be the independent transversal geo-adjacency matrix with respect to the g_{it}-set S_j for $j = 1, 2, \cdots l$. Then the elements in the mth row(column) of $M_{S_{j_{g_{it}}}}(G)$ and $M_{S_{k_{g_{it}}}}(G)$ remain the same for all $v_m \in S_j \cap S_k$.

Proof: Since $v_m \in S_j \cap S_k$, it follows that $v_m \in S_j$ as well as $v_m \in S_k$.
Then by Theorem 2.10, the sum of the entries in the m-th row(or column) of $M_{S_{j_{g_{it}}}}(G)$ is equal to $deg_G(v_m)$.
Similarly, the sum of the entries in the m-th row(or column) of $M_{S_{k_{g_{it}}}}(G)$ is also equal to $deg_G(v_m)$.
This implies that the number of 1's occuring in the m-th row(column) of $M_{S_{j_{g_{it}}}}(G)$ as well as $M_{S_{k_{g_{it}}}}(G)$ is equal to $deg_G(v_m)$. So the (m, l)th entry of $M_{S_{j_{g_{it}}}}(G)$ as well as $M_{S_{k_{g_{it}}}}(G)$ is equal to 1 iff v_m is adjacent to v_l.
Therefore the entries in the m-th row(or column) of $M_{S_{j_{g_{it}}}}(G)$ and $M_{S_{k_{g_{it}}}}(G)$ are

$$= \begin{cases} 1 \quad \text{if} \quad v_m \quad \text{is adjacent to} \quad v_l \\ 0 \qquad\qquad\quad \text{if not} \end{cases}$$

Consequently, for every vertex $v_m \in S_j \cap S_k$, the m-th row(or column) of $M_{S_{j_{g_{it}}}}(G)$ coincides with that of $M_{S_{k_{g_{it}}}}(G)$. $\square$

Corollary 2.14. Let $V(G)$ =$\{v_1,\ v_2,\ ...,\ v_n\}$ and $S_1,\ S_2,\ \cdots,\ S_l$ be the g_{it}-sets of G. Let $M_{S_{j_{g_{it}}}}(G)$ be the independent transversal geo-adjacency matrix with respect to the g_{it}-set S_j for $j = 1, 2, \cdots l$. Then the elements in the mth row(column) of $M_{S_{j_{g_{it}}}}(G)$, $M_{S_{k_{g_{it}}}}(G)$, $\cdots$, $M_{S_{p_{g_{it}}}}(G)$ remain the same for all $v_m \in S_j \cap S_k \cap \cdots \cap S_p$.

3. Independent transversal geodetic energy

Definition 3.1. Let G be a simple connected graph of order $n \geq 3$, and S be a g_{it}-set of G. Let $M_{S_{g_{it}}}(G)$ denote the independent transversal geo-adjacency matrix corresponding to S. Then the Independent Transversal Geodetic Energy with respect to the g_{it}-set S, denoted by $ITGE_S(G)$, is defined as the sum of the absolute values of all the eigenvalues of the matrix $M_{S_{g_{it}}}(G)$. Then the Independent Transversal Geodetic Energy of the Graph G, denoted by $ITGE(G)$, is defined as

$$ITGE(G) = \max.\{ITGE_S(G)\text{: S is a } g_{it}\text{-set of G}\}.$$

Note 3.2. From the above Definition 3.1, it is evident that the Independent Transversal Geodetic Energy of a graph G is always a positive real number.

Example 3.3. For the graph G in Figure 4,
the set $S = \{v_1,\ v_4,\ v_5\}$ is a g_{it}-set, referring [10].
Then the independent transversal geo-adjacency matrix with respect to S is given

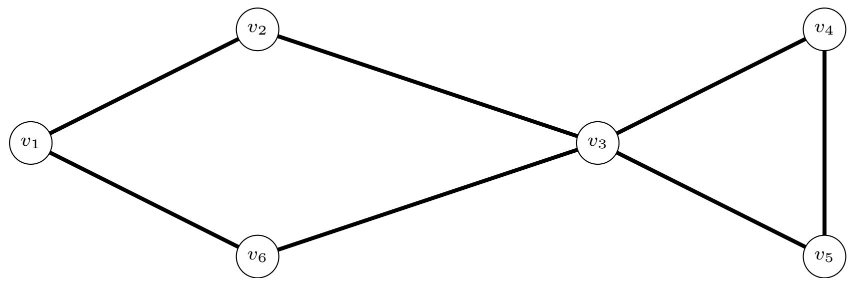

FIGURE 4. Graph G

by

$$M_{S_{g_{it}}}(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The eigenvalues of $M_{S_{g_{it}}}(G)$ are $0, -1, -1, 2, \sqrt{2}$ and $-\sqrt{2}$.
Therefore the Independent Transversal Geodetic Energy with respect to the g_{it}-set S, $ITGE_S(G) = 4 + 2\sqrt{2} = 6.828$ (approximately).
Since S is unique, it follows that $ITGE(G) = 6.828$

Example 3.4. Let G denote the graph illustrated in Figure 5.

The graph G has three maximum independent sets(that is, β_0-sets), namely,

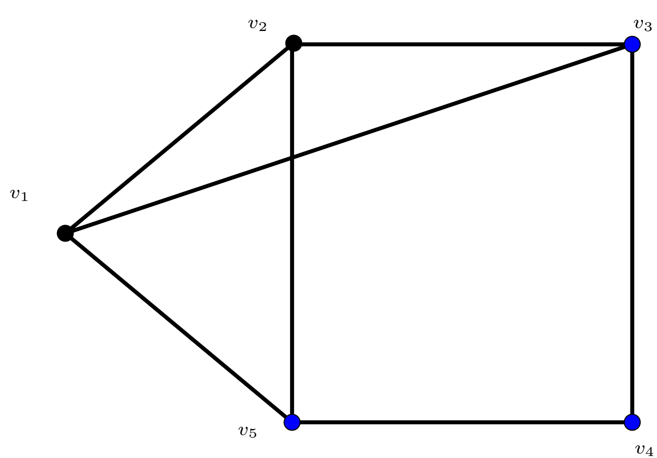

FIGURE 5. Graph G

I_1 ={v_1, v_4}, I_2 ={v_2, v_4}, and I_3 ={v_3, v_5}.
Consider the set S={v_3, v_4, v_5}. The set S is a geodetic set and has a non-empty

intersection with each β_0-set of G.
Hence, S qualifies as a g_{it}-set of G.
Then the independent transversal geo-adjacency matrix with respect to S is given by

$$M_{S_{g_{it}}}(G) = \begin{bmatrix} 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 1 & 0 & 1 & 0 \end{bmatrix}$$

The eigenvalues of $M_{S_{g_{it}}}(G)$ are $0, 0, 0, -\sqrt{6}$ and $\sqrt{6}$
Therefore the independent transversal geodetic energy with respect to the g_{it}-set S, $ITGE_S(G) = 2\sqrt{6} = 4.899$
Since S is unique, it is evident that $ITGE(G) = 4.899$

Example 3.5. Refer to the graph G depicted in Example 2.5.
$S = \{u_1, u_5\}$ is the unique g_{it}-set in G and $M_{S_{g_{it}}}(G)$ is the independent transversal geo-adjacency matrix with respect to S given in Example 2.5.
Its eigenvalues are 0, 1, -1, $\sqrt{2}$ and $-\sqrt{2}$
Therefore the Independent Transversal Geodetic Energy with respect to the g_{it}-set S, $ITGE_S(G) = 2 + 2\sqrt{2} = 4.828$
Since S is unique, it follows that $ITGE(G) = 4.828$

Example 3.6. Consider G shown in Example 2.6.
$S_1 = \{v_1, v_2, v_4, v_6\}$, $S_2 = \{v_1, v_3, v_4, v_5\}$, $S_3 = \{v_1, v_2, v_3, v_4\}$ and $S_4 = \{v_1, v_4, v_5, v_6\}$ are g_{it}-sets in G.
Their corresponding independent transversal geo-adjacency matrices $M_{S_{1_{g_{it}}}}(G)$, $M_{S_{2_{g_{it}}}}(G)$, $M_{S_{3_{g_{it}}}}(G)$ and $M_{S_{4_{g_{it}}}}(G)$ are as given in Example 2.6.
$M_{S_{1_{g_{it}}}}(G)$ and $M_{S_{2_{g_{it}}}}(G)$ have the same eigenvalues $-1.618, 0.618, -0.820, 2.438, -1.75$ and 1.139.
Therefore the independent transversal geodetic energy with respect to the g_{it}-sets S_1 and S_2, $ITGE_{S_1}(G) = ITGE_{S_2}(G) = 8.39$
Also $M_{S_{3_{g_{it}}}}(G)$ and $M_{S_{4_{g_{it}}}}(G)$ have the same eigenvalues $-1, -1, -\sqrt{3}, \sqrt{3}, -\sqrt{2}+1, \sqrt{2}+1$.
So the independent transversal geodetic energy with respect to both the g_{it}-sets S_3 and S_4, $ITGE_{S_3}(G) = ITGE_{S_4}(G) = 8.292$
Further, there are no other g_{it}-sets in G.
Therefore the independent transversal geodetic energy of G is
$ITGE(G) = \max.\{ITGE_S(G)$: S is a g_{it}-set of G$\} = 8.39$

Example 3.7. Consider G shown in Example 2.7.
$S_1 = \{v_1, v_2, v_6\}$, $S_2 = \{v_1, v_4, v_6\}$, $S_3 = \{v_1, v_5, v_6\}$ and $S_4 = \{v_1, v_6, v_7\}$ are g_{it}-sets in G. Then their corresponding independent transversal geo-adjacency matrices $M_{S_{1_{g_{it}}}}(G)$, $M_{S_{2_{g_{it}}}}(G)$, $M_{S_{3_{g_{it}}}}(G)$ and $M_{S_{4_{g_{it}}}}(G)$ are as given in Example 2.7.
All these matrices produce the same eigenvalues $0, \sqrt{2}, -\sqrt{2}, \frac{\sqrt{5}-1}{2}, \frac{\sqrt{5}+1}{2}, \frac{-\sqrt{5}-1}{2}$ and $\frac{-\sqrt{5}+1}{2}$.

So the independent transversal geodetic energy with respect each of the g_{it}-sets S_1, S_2, S_3 and S_4 is equal to $2\sqrt{2}+2\sqrt{5}=7.3$(approximately).
Therefore the independent transversal geodetic energy of G is $ITGE(G)=7.3$.

4. Independent transversal geodetic energy of standard graphs

Result 4.1. For the complete graph K_n on n vertices, $S=\{v_1,\ v_2\ ,...,\ v_n\}$ is the unique g_{it}-set [10].
Then the independent transversal geo-adjacency matrix with respect to S is an $n\times n$ matrix with entries 0 along the main diagonal and 1 elsewhere.

It is given by

$$M_{S_{g_{it}}}(K_n)=\begin{bmatrix}0&1&1&\cdots&1\\1&0&1&\cdots&1\\1&1&0&\cdots&1\\\vdots&\vdots&\vdots&\ddots&\vdots\\1&1&1&\cdots&0\end{bmatrix}$$

The spectrum of the matrix $M_{S_{g_{it}}}(K_n)$ consists of the eigenvalues $n-1$ and -1 where -1 with multiplicity $n-1$. Therefore the independent transversal geodetic energy of the complete graph K_n is $2n-2$.

Result 4.2. Consider the star graph consisting of $n+1$ vertices with u as the central vertex and $v_1,\ v_2,\ ...,\ v_n$ on its leaves. The set S $=\{v_1,\ v_2,\ ...,\ v_n\}$ is the only g_{it}-set of G.
With respect to the set S, the associated independent transversal geo-adjacency matrix of G is an $(n+1)\times(n+1)$ matrix, which is defined as

$$M_{S_{g_{it}}}(G)=\begin{bmatrix}0&1&1&\cdots&1\\1&0&0&\cdots&0\\1&0&0&\cdots&0\\\vdots&\vdots&\vdots&\ddots&\vdots\\1&0&0&\cdots&0\end{bmatrix}$$

Its eigenvalues are $\sqrt{n}$, $-\sqrt{n}$, $0,0,\cdots$ and 0 (that is, 0 appears $(n-1)$ times). Therefore the independent transversal geodetic energy of G is $2\sqrt{n}$.

Theorem 4.3. If P_n is a path on $n\geq 3$ vertices, then

$$ITGE(P_n)=\begin{cases}2\sqrt{2} & if \quad n=3\\ 4 & if \quad n>3\end{cases}$$

Proof: Let $V(P_n)=\{v_1,\ v_2,\ ...,\ v_n\}$.
Referring Theorem 1.1[10], $S=\{v_1,\ v_n\}$ is the unique minimum independent transversal geodetic set (i.e)g_{it}-set in P_n.
<u>Case 1:</u> $n=3$.
Then $S=\{v_1,\ v_3\}$ is the unique g_{it}-set in P_3.

Therefore the independent transversal geo-adjacency matrix with respect to S in this case is the 3×3 matrix given by

$$M_{S_{g_{it}}}(P_3) = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$$

Its eigenvalues are 0, $\sqrt{2}$ and $-\sqrt{2}$.
$\therefore ITGE(P_n) = 2\sqrt{2}$ if $n = 3$.
Case 2: $n > 3$.
Here the independent transversal geo-adjacency matrix with respect to S is an $n \times n$ matrix given by

$$M_{S_{g_{it}}}(P_n) = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 1 & 0 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 1 & 0 \end{bmatrix}$$

The eigenvalues of $M_{S_{g_{it}}}(P_n)$ are $1, -1, 1, -1, 0, 0, \cdots$ and 0 (that is, 0 appears $(n-4)$ times).
Therefore the independent transversal geodetic energy of P_n in this case is 4.
$\therefore ITGE(P_n) = 4$ if $n > 3$. $\square$

Theorem 4.4. If C_{2n+1} is an odd cycle with $n \geq 2$, then

$$ITGE(C_{2n+1}) = \begin{cases} 2 + 2\sqrt{5} & if \quad n = 2 \\ 2\sqrt{2} + 2\sqrt{5} & if \quad n \quad is \quad odd \\ 4\sqrt{5} & if \quad n \quad is \quad even \end{cases}$$

Proof: Let $V = \{v_1, v_2, ..., v_{2n+1}\}$ be the vertex set of the odd cycle C_{2n+1}.
Case 1: $n = 2$
Clearly, $S_i = V - \{v_i\}$, $i = 1, 2, .., 5$ is a g_{it}-set in C_5. That is, each S_i contains any of the 4 vertices from $V(C_5)$.
Then the independent transversal geo-adjacency matrix with respect to S_i is a 5×5 matrix given by

$$M_{S_{i_{g_{it}}}}(G) = \begin{bmatrix} 0 & 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

for each $i = 1, 2, .., 5$
Each of these have the eigenvalues 2, $\frac{-\sqrt{5}-1}{2}$, $\frac{\sqrt{5}-1}{2}$, $\frac{-\sqrt{5}-1}{2}$ and $\frac{\sqrt{5}-1}{2}$.
Therefore the independent transversal geodetic energy with respect to each of the g_{it}-sets S_i, $ITGE_{S_i}(G) = 2 + 2\sqrt{5}$.
Thus the independent transversal geodetic energy of C_5 is $2 + 2\sqrt{5}$.
Case 2: n is odd.

Referring Theorem 1.2[10], $S_i = \{v_i,\ v_{i+1},\ v_{i+n+1}\}$ is a g_{it}-set for every $i = 1, 2, ..., n$.
Then the eigenvalues of the independent transversal geo-adjacency matrix with respect to S_i are $\sqrt{2}$, $-\sqrt{2}$, $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{5}+1}{2}$, $\frac{-\sqrt{5}-1}{2}$, $\frac{-\sqrt{5}+1}{2}$ and 0, 0, ..., 0($2n-5$ times).
Therefore the independent transversal geodetic energy with respect to the g_{it}-set S_i, $ITGE_{S_i}(G) = 2\sqrt{2} + 2\sqrt{5}$.
If we consider any other g_{it}-set of the form $S_j = \{v_j,\ v_{j+n},\ v_{(j+n+1)(mod(2n+1))}\}$, $j = 1, 2, ..., n+1$, then also we obtain the same eigenvalues.
Thus the independent transversal geodetic energy of C_{2n+1} is
$ITGE(C_{2n+1}) = 2\sqrt{2} + 2\sqrt{5}$ if n is odd.
Case 3: n is even.
Referring Theorem 1.2[10], $S_i = \{v_i,\ v_{i+1},\ v_{i+n+1},\ v_{(i+n+2)mod(2n+1)}\}$ is a g_{it}-set for every $i = 1, 2, ..., n$.
Then the eigenvalues of the independent transversal geo-adjacency matrix with respect to S_i are $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{5}+1}{2}$, $\frac{-\sqrt{5}-1}{2}$, $\frac{-\sqrt{5}+1}{2}$, $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{5}+1}{2}$, $\frac{-\sqrt{5}-1}{2}$, $\frac{-\sqrt{5}+1}{2}$ and 0, 0, ..., 0($2n-7$ times).
Therefore the independent transversal geodetic energy with respect to the g_{it}-set S_i, $ITGE_{S_i}(G) = 4\sqrt{5}$.
$\therefore$ $ITGE(C_{2n+1}) = 4\sqrt{5}$ in this case. □

Theorem 4.5. Let C_{2n} be an even cycle with $n \geq 2$, then the independent transversal geodetic energy of C_{2n} is given by

$$ITGE(C_{2n}) = \begin{cases} 4 & if \quad n = 2 \\ 4\sqrt{2} & if \quad n \quad is \quad odd \\ 2\sqrt{2} + 2\sqrt{5} & if \quad n \quad is \quad even \quad and \quad n \geq 4 \end{cases}$$

Proof: Let $V = \{v_1,\ v_2,\ ...,\ v_{2n}\}$ denote the vertex set of C_{2n}.
Case 1: $n = 2$
Obviously, $S_i = V - \{v_i\}$, $i = 1, 2, 3, 4$ is a g_{it}-set in C_4. That is, each S_i contains any of the 3 vertices from $V(C_4)$. Then the independent transversal geo-adjacency matrix with respect to S_i is a 4×4 matrix given by

$$M_{S_{i_{g_{it}}}}(G) = \begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$$

The eigenvalues of $M_{S_{i_{g_{it}}}}(G)$ are 0, 0, 2 and -2.
Therefore $ITGE_{S_i}(C_4) = 4$ for each $i = 1, 2, 3, 4$.
Thus $ITGE(C_4) = 4$.
Case 2: n is odd.
Referring Theorem 1.3[10], $S_i = \{v_i,\ v_{i+n}\}$ is a g_{it}-set for every $i = 1, 2, ..., n$.
Then the eigenvalues of the independent transversal geo-adjacency matrix with respect to S_i are $\sqrt{2}$, $-\sqrt{2}$, $\sqrt{2}$, $-\sqrt{2}$ and 0, 0, ..., 0($2n-4$ times).
Therefore $ITGE_{S_i}(C_{2n}) = 4\sqrt{2}$.
It is not possible to find any g_{it}-set other than S_i in C_{2n} when n is odd.

Hence, $ITGE(C_{2n}) = 4\sqrt{2}$ in this case.
Case 3: n is even.
Referring Theorem 1.3[10], $S_i = \{v_i,\ v_{i+1},\ v_{i+n}\}$ is a g_{it}-set for every $i = 1, 2, ..., n$. Then the eigenvalues of the independent transversal geo-adjacency matrix with respect to S_i are $\sqrt{2}$, $-\sqrt{2}$, $\frac{\sqrt{5}-1}{2}$, $\frac{\sqrt{5}+1}{2}$, $\frac{-\sqrt{5}-1}{2}$, $\frac{-\sqrt{5}+1}{2}$ and 0, 0, ..., 0($2n-6$ times).
Therefore $ITGE_{S_i}(C_{2n}) = 2\sqrt{2} + 2\sqrt{5}$.
If we consider any other g_{it}-set of the form $S_j = \{v_j,\ v_{j+n},\ v_{(j+n+1)(mod(2n))}\}$, where $j = 1, 2, ..., n$, then also we obtain the same eigenvalues.
Thus the independent transversal geodetic energy of C_{2n} is
$ITGE(C_{2n}) = 2\sqrt{2} + 2\sqrt{5}$.

□

Theorem 4.6. If $G = C_n \odot K_1$ is the corona graph on 2n vertices, then $ITGE(G) = 2n$.

Proof: Consider the n-sunlet graph on 2n vertices, also known as corona graph. This graph is formed by adjoining exactly one vertex to each vertex of the cycle C_n. Formally, the structure of $G = C_n \odot K_1$ is illustrated in Figure 6 and it has $2n$ vertices as $V(G) = \{v_1,\ v_2,\ ...,\ v_n,\ u_1,\ u_2,\ ...,\ u_n\}$.

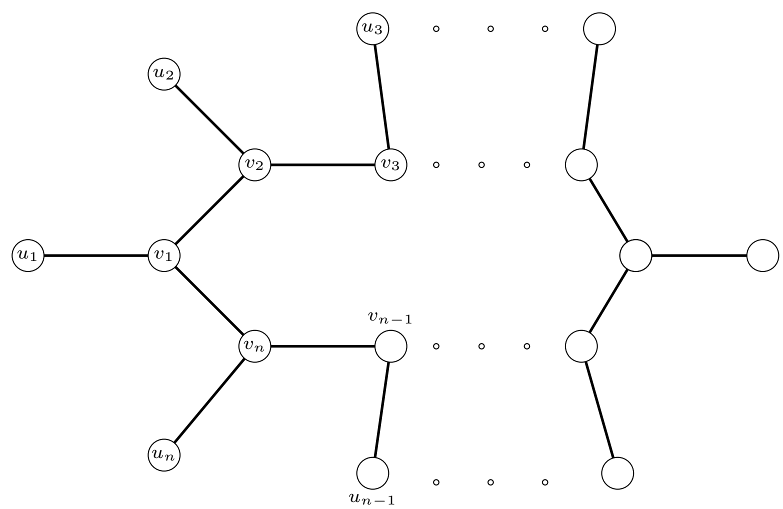

FIGURE 6. Graph G = $C_n \odot K_1$

Consider the set S =$\{u_1,\ u_2,\ ...,\ u_n\}$ consisting of all end vertices of G. Then $|S| = n$. Obviously S is the unique g_{it}-set in G.
Then the independent transversal geo-adjacency matrix with respect to S is a $2n \times 2n$ matrix given by

$$M_{S_{git}}(G) = \begin{bmatrix} 0 & 0 & 0 & \cdots & 0 & 1 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 1 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 1 \\ 1 & 0 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \cdots & 1 & 0 & 0 & 0 & \cdots & 0 \end{bmatrix}.$$

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The eigenvalues of $M_{S_{git}}(G)$ are 1 and -1 each with multiplicity n. Therefore $ITGE_S(G) = 2n$.

Hence the independent transversal geodetic energy of G is $ITGE(G) = 2n$.

□

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R.Vasanthi:Department of Mathematics, Alagappa Chettiar Government College of Engineering and Technology, Karaikudi, India

Email address: vasanthi2014accet@gmail.com

M.Perumalsamy:Department of Mathematics, Government College of Technology, Coimbatore, India

Email address: pervas2014@gmail.com