Global and Stochastic Analysis
Vol. 13 No. 1 (2026)

Received: 12th September 2025 *Revised: 20th December 2025* *Accepted: 09th January 2026*

ON SOME SUBALGEBRA BUNDLES OF A LIE ALGEBRA BUNDLE OF FINITE TYPE

K. KAMALAKSHI, HUSNIYAH ALZUBAIDI, HAKEEM A. OTHMAN, R. RAJENDRA, P. SIVA KOTA REDDY*, AND SAMI H. ALTOUM

Abstract. In this paper, we establish the existence of a finite dimensional ample Lie subalgebra in the sections of a Lie algebra bundle of finite type. We prove Engel's and Lie's theorem for bundles where the base space need not be compact. Also, we discuss about the existence of a Cartan subalgebra bundle, Engel and toral subalgebra bundle of a Lie algebra bundle of finite type.

1. Introduction

We begin by recalling the following notions of bundles from [1–4,8–17,19,20,22]. When a vector bundle $\xi = (\xi, p, X)$ is such that a morphism $\odot : \xi \oplus \xi \to \xi$ gives a Lie algebra structure on each fiber ξ_x, we call it as a Lie algebra bundle. If each fiber is a semisimple Lie algebra we call it a semisimple Lie algebra bundle.

A vector bundle ξ where each fiber is having a Lie algebra structure and for any x in X there is a neighborhood U of x in X, a Lie algebra L and a homeomorphism $\Phi : U \times L \to p^{-1}(U)$ such that $\Phi_x : \{x\} \times L \to p^{-1}(x)$ is an isomorphism of Lie algebras $\forall x \in U$, is called a locally trivial Lie algebra bundle.

Leonid N. Vaserstein [22] defined finite type vector bundles. Motivated by this concept Ranjitha Kumar et al. [15] introduced the concept of Lie algebra bundle of finite type. A Lie algebra bundle ξ over an arbitrary space X is of finite type if there exists a finite partition S of unity on X (that is, S is a finite set consisting of non-negative continuous functions on X whose sum equals 1) such that ξ restricted to the set $\{x \in X | g(x) \neq 0\}$ is a trivial Lie algebra bundle for each $g \in S$. As any compact Hausdorff space attains partition of unity, we observe that every Lie algebra bundle over a compact Hausdorff space is of finite type.

If $s : X \to \xi$ is a map such that $p \circ s = id_X$, then it is called as a section of ξ. We usually use $\Gamma(\xi)$ to denote the set of all sections of ξ. A map $f : \xi \to \eta$, where ξ and η are the vector bundles over a topological space X, is such that $f_x : \xi_x \to \eta_x$ is a homomorphism between the Lie algebras, ξ_x and η_x for all $x \in X$. We call

2000 *Mathematics Subject Classification.* 14XX, 14F05, 17Bxx, 17B62, 47A05.

Key words and phrases. Ample Lie subalgebra, Bundles of finite type, Cartan subalgebra, Lie algebra bundles, Semisimple algebra bundles.

*Corresponding author.

f as a morphism between ξ and η. If f is bijective and f^{-1} is continuous, f is an isomorphism. Existence of a finite partition of unity in the base space of any finite type Lie algebra bundles paves the way to prove some results by weakening the condition of compactness, which we discuss in this paper. We establish the existence of a finite dimensional ample Lie subalgebra in the sections of a Lie algebra bundle of finite type. We prove Engel's and Lie's theorem for bundles where the base space need not be compact. Also, we discuss about the existence of a Cartan subalgebra bundle, Engel and toral subalgebra bundle of a Lie algebra bundle of finite type. Some recent developments in the theory of algebra bundles can be found in [2, 3, 9, 10, 14–18, 20]. Throughout this paper, it is assumed that all underlying vector spaces are real and finite dimensional.

2. Engel's and Lie's theorem for Lie algebra bundles of finite type

In this section, we shall study the Engel's and Lie's theorem for finite type Lie algebra bundle ξ, by observing the existence of a finite dimensional ample Lie subalgebra in $\Gamma(\xi)$.

Definition 2.1. A Lie subalgebra $V \subseteq \Gamma(\xi)$ is said to be an ample Lie subalgebra if a Lie morphism, $\phi : X \times V \to \xi$ defined by $\phi(x, v) = v(x)$ is surjective.

Lemma 2.2. *Let $p : \xi \to X$ be lie algebra bundle of finite type. Then $\Gamma(\xi)$ contains a finite dimensional ample Lie subalgebra.*

Proof. Since ξ is a Lie algebra bundle of finite type, we get a finite partition of unity $\{f_i\}_{i=1}^n$ such that ξ when restricted to each U_i is trivial, where $U_i = \{x \in X : f_i(x) \neq 0\}$. We observe that $\{U_i\}_{i=1}^n$ covers X. Since each $\xi|_{U_i}$ is a trivial we can find a finite dimensional ample Lie subalgebra $V_i \subset \Gamma(\xi|_{U_i})$. Now define, $p_i : V_i \to \Gamma(\xi)$ for each i such that if $s \in V_i$, $p_i(s)$ is defined as

$$p_i(s)(x) = \begin{cases} f_i(x)s(x), & \text{for } x \in U_i; \\ 0, & x \notin U_i. \end{cases}$$

Then p_i is a Lie algebra homomorphism. Now consider $\{U_i\}$ in the increasing order of indices. Define $\phi : \prod_{i=1}^{n} V_i \to \Gamma(\xi)$, as $\phi((s_1(x), s_2(x), \ldots, s_n(x))) = p_i(s_i)(x)$, where i is the least index for which $x \in U_i$. Then ϕ is a homomorphism. Consider $V = \phi(\prod_{i=1}^{n} V_i)$ which is a subalgebra of $\Gamma(\xi)$. We observe that V is finite dimensional. Now we shall prove that the natural map from $X \times V \to \xi$ is a surjective Lie bundle morphism. Let $u \in \xi$. Then there exists $x \in X$ such that $u \in \xi_x$. Since $\{f_i\}$ is a partition of unity, there exists a U_i such that $x \in U_i$. Then $u \in \xi|_{U_i}$. Now since V_i is an ample subalgebra of $\xi|_{U_i}$, we have $s \in V_i$ such that $s(x) = u$. Now let $v \in V$ such that $v = \phi(s_1, s_2, \ldots, s_n)$, where $s_i = s$. Now $v(x) = \phi(s_1, s_2, \ldots, s_n)(x) = s(x) = u$. Hence V is a finite dimensional ample Lie subalgebra of $\Gamma(\xi)$. □

Remark 2.3. Suppose $p : \xi \to X$ is a Lie algebra of finite type. Then $End(\xi) = \bigcup_{x \in X} End(\xi_x)$ is also a Lie algebra bundle of finite type. For, let $\{f_1, f_2, \ldots f_n\}$ be finite partition of unity and $U_i = \{x \in X; f_i(x) \neq 0\}$ for all i, such that $\xi|_{U_i}$ is trivial. Let $\phi_i : U_i \times L_i \to \bigcup_{x\in U_i} \xi_x$ be the isomorphism. Then $\psi_i : U_i \times End(L_i) \to \bigcup_{x\in U_i} End(\xi_x)$ is an isomorphism. Thus by above Lemma, $\Gamma(End(\xi))$ has a finite dimensional ample Lie subalgebra.

Definition 2.4. Let $\mathcal{G}$ be Lie subalgebra of $HOM(\xi)$. Then $T \in \mathcal{G}$ is said to be nilpotent on ξ, if $T_x = T|_{\xi_x} : \xi_x \to \xi_x$ is nilpotent for every $x \in X$. For $T \in \mathcal{G}$ we can define ad $T : \mathcal{G} \to \mathcal{G}$ as

$$\text{ad } T(S) = [T, S] : \xi \to \xi,$$

where ad $T(S)(u) = [T_x, S_x](u) = T_x(S_x(u)) - S_x(T_x(u))$, for $u \in \xi_x$.

Theorem 2.5. *Consider a Lie algebra bundle of finite type ξ. Let $\mathfrak{X}$ be a finite dimensional ample Lie subalgebra of $\Gamma(End(\xi))$ such that $\mathfrak{X}$ consists of nilpotent sections. Then there is a non-zero section s_ξ in $\Gamma(\xi)$ satisfying the condition $\sigma_{\mathfrak{X}}(x)(s_\xi(x)) = 0$, $x \in X$, $\sigma_{\mathfrak{X}} \in \mathfrak{X}$.*

Proof. Let ω be an isomorphism between $\Gamma(End(\xi))$ and $HOM(\xi)$. Then for any $\sigma_{\mathfrak{X}} \in \mathfrak{X}$, we have $F = \omega(\sigma_{\mathfrak{X}})$ is a morphism in $Hom(\xi)$. Hence it suffices to show that there is a section $s_\xi \neq 0$ such that $F(s_\xi) = 0$ for any $F \in \omega(\mathfrak{X})$.
Let $\psi : HOM(\xi) \to gl(\Gamma(\xi))$ be defined by $\psi(F) : \Gamma(\xi) \to \Gamma(\xi)$, $\psi(F)(s) = F \circ s$. Then clearly ψ is a Lie algebra homomorphism. Since $\omega(\mathfrak{X})$ is a Lie subalgebra of $HOM(\xi)$, $\psi(\omega(\mathfrak{X}))$ is a Lie subalgebra of $gl(\Gamma(\xi))$ containing nilpotent operators on $\Gamma(\xi)$. We can prove the result by induction on $dim\mathfrak{X}$, following the methods in Theorem 3.1, in [19] □

Corollary 2.6 (Engel's Structure Theorem for Lie Bundles of Finite Type)**.** *Suppose $\xi = (\xi, p, X)$ be a Lie algebra bundle of finite type and $\mathfrak{X}$ be a finite dimensional ample Lie subalgebra of $\Gamma(End(\xi))$ such that every section of $\sigma_{\mathfrak{X}}$ of $\mathfrak{X}$ is nilpotent on ξ. Let V be a finite dimensional ample Lie subalgebra of $\Gamma(\xi)$. Then there is a basis $\{s_i\}_{i=1}^n$ of V such that for any element $\sigma_{\mathfrak{X}} \in \mathfrak{X}$, we have*

$$\sigma_{\mathfrak{X}}(x)\,(s_j(x)) = \sum_{i<j} \alpha_i(x) s_i(x), \qquad \forall x \in X, \quad 1 \le j \le n.$$

Proof. If we consider any section σ of $End(\xi)$, we can get a unique morphism $F \in HOM(\xi)$ which gives raise to an equivalent conclusion: Then there is a basis $\{s_i\}_{i=1}^n$ of V such that for every element F of $\omega(\mathfrak{X})$, we have

$$F(s_j(x)) = \sum_{i<j} \alpha_i(x) s_i(x), \qquad \forall x \in X, \quad 1 \le j \le n.$$

Now the result can be concluded by induction on $dimV$ as written in Corollary 1 in [19]. □

Theorem 2.7 (Engel's Theorem for Lie Bundles of Finite Type)**.** *For any Lie algebra bundle of finite type ξ and a finite dimensional ample Lie subalgebra V of $\Gamma(\xi)$, ξ is nilpotent if and only if* ad $s : V \to V$ *is nilpotent for all s in V.*

Proof. We can observe that, the given necessary and sufficient condition between the nilpotency of ξ and the nilpotency of ad s can be easily established by using the definitions and the fact that if each ad s is nilpotent, ad V will be a nilpotent

Lie algebra which gives raise to a surjective homomorphism between ad V and ξ_x (which follows from Theorem 3.2 in [19]). □

Definition 2.8. Let ξ be a Lie algebra bundle. The derived series of ξ can be defined as follows:

$$\xi^{(0)} = \xi,\ \xi^{(1)} = [\xi,\ \xi] = \bigcup_{x\in X} [\xi_x, \xi_x]\,,\ \xi^{(k)} = \bigcup_{x\in X} \left[\xi_x^{(k-1)}, \xi_x^{(k-1)}\right] \quad \text{for } k \geq 0.$$

Definition 2.9. A Lie algebra bundle $\xi = (\xi, p, X)$ is a solvable Lie algebra bundle if $\xi^{(m)} = 0$ for some positive integer m.
Consider the dual bundle $(\text{End}\xi)^*$ of $(\text{End}\xi)$. Let $\beta \in \Gamma((\text{End}\xi)^*)$. By denoting $\beta(x) = \beta_x$, we have for each β in $\Gamma((\text{End}\xi)^*)$ the joint eigenspace corresponding to η, a Lie subalgebra of $\Gamma(\text{End}\xi)$ defined as

$$(\Gamma(\xi))_\beta = \{s \in \Gamma(\xi)/\ \sigma(x)(s(x)) = \beta_x(\sigma(x)) \cdot s(x),\ \forall\ x \in X,\ \sigma \in \eta\}$$

The non zero element in $(\Gamma(\xi))_\beta$ is called joint eigenvector in $\Gamma(\xi)$ corresponding to η with respect to $\beta \in \Gamma((\text{End}\xi)^*)$.

Theorem 2.10. *If (ξ, p, X) be a Lie algebra bundle of finite type and W is a finite dimensional solvable ample Lie subalgebra of $\Gamma(\text{End}(\xi))$, then there is a non zero joint eigenvector in $\Gamma(\xi)$ corresponding to W.*

Proof. If $\omega : \Gamma(End(\xi)) \to HOM(\xi)$ is a Lie algebra isomorphism, then $\omega(W)$ is a solvable Lie subalgebra of $HOM(\xi)$. Now we can get desired non-zero eigenvector by using the induction on $\omega(W)$ (which follows from Theorem 4.1 in [19]). □

Theorem 2.11 (Lie's theorem for bundles of finite type)**.** *Let $\xi = (\xi, p, X)$ be a Lie algebra bundle of finite type. Let V be a finite dimensional ample Lie subalgebra of $\Gamma(\xi)$. If W is a finite dimensional solvable ample Lie subalgebra of $\Gamma(\text{End}\xi)$, then we get a basis $\{s_i\}_{i=1}^n$ of V such that $\sigma \in W$ satisfies*

$$\sigma(x)(s_j(x)) = \sum_{i\leq j} \alpha_i s_i(x), \qquad x \in X.$$

Proof. Theorem 2.10, signifies the existence of a nonzero element in V. Now the proof follows by induction on the $dimV$. □

3. Root System of a Lie Agebra Bundle of Finte Type

In this section, we discuss about the existence of a Cartan subalgebra bundle of a Lie algebra bundle of finite type which in turn gives the root space decomposition of the bundle with respect to the Cartan subalgebra bundle.

Definition 3.1. Any subalgebra $\mathfrak{h}$ of a finite dimensional Lie algebra $\mathfrak{g}$ is called a Cartan subalgebra if it is nilpotent and equal to its normalizer.

Definition 3.2. If $\mathfrak{h}$ is a Cartan subalgebra of a complex semisimple Lie algebra $\mathfrak{g}$, Then $\mathfrak{g}$ can be written as a direct sum $\mathfrak{g} = \mathfrak{h} \oplus \left(\bigoplus_{\alpha\in\Delta} \mathfrak{g}_\alpha\right)$, called as the root space decomposition of $\mathfrak{g}$ relative to $\mathfrak{h}$, where $\alpha \in \mathfrak{h}^*$ and $\mathfrak{g}_\alpha = \{g \in \mathfrak{g} : [h, g] = \alpha(h)(g), \forall h \in \mathfrak{h}\}$. ([7], page 35)

Definition 3.3. For any Cartan subalgebra $\mathfrak{h}$ of a semisimple Lie algebra $\mathfrak{g}$, linear functional $\alpha \in \mathfrak{h}^*$, $\alpha \neq 0$ such that $\mathfrak{g}_\alpha \neq 0$ is called a root of $\mathfrak{g}$ relative to $\mathfrak{h}$.

Definition 3.4. Let $\pi : \xi \to M$ be a semisimple Lie algebra fiber bundle with standard fiber L. A Lie algebra subbundle η of ξ with standard fibre H is said to be a Cartan subalgebra bundle of ξ if H is a Cartan subalgebra of L and further for each $m \in M$, the fibre η_m of η over m is a Cartan subalgebra of ξ_m the fibre of ξ over m.

Proposition 3.5. *Every complex semisimple Lie algebra bundle of finite type admits a Cartan subalgebra bundle.*

Proof. Let $\xi = (\xi, \pi, M)$ be a semisimple lie algebra bundle of finite type. Then there exists a partition of unity $\{f_i\}_{i=1}^n$ on M such that $\xi|_{U_i} = \xi_i$ is trivial where $U_i = \{m \in M : f_i(m) \neq 0\}$ and $\{U_i\}_{i=1}^n$ cover M. Let L be the standard fibre of U_i and H be a Cartan subalgebra of L. Now $\eta_i = U_i \times H$ is a Cartan subalgebra bundle of ξ_i. Let $\xi_i = \eta_i \oplus \left(\bigoplus_{\hat{\alpha}_i \in \hat{\Delta}_i} \xi_{i_{\hat{\alpha}_i}} \right)$ be the root space decomposition of ξ_i with respect to η_i. $\Delta_i \subset \Gamma(\eta_i^*)$, the root system of ξ_i with respect to η_i, $\hat{\Delta}_i = \{\hat{\alpha}_i : \alpha_i \in \Delta_i\}$ (Follows from the proof of Proposition 6.9 in [1]). Construct a Lie algebra bundle η, using the technique of clutching construction, by defining quotient topology on the topological union of η_i's. Let η^* be the dual bundle of η over M. For $\alpha \in \Delta$, define $\hat{\alpha} : M \to \eta^*$ by $\hat{\alpha} = \sum_{i=1}^{n} g_i \hat{\alpha}_i$. Then $\hat{\Delta} = \{\hat{\alpha} : \alpha \in \Delta\}$ and $\xi_{\hat{\alpha}} = \bigcup_{m \in M} \xi_{\hat{\alpha}_m}$ where $\xi_{\hat{\alpha}_m} = \{l \in \xi_m : \theta_m(h, l) = \hat{\alpha}(m)(h)l, \forall h \in \eta_m\}$ where the bundle structure on $\xi_{\hat{\alpha}}$ is given by clutching construction using $(\xi_i)_{\hat{\alpha}_i}$ [20]. Then we have $\xi = \eta \oplus \left(\bigoplus_{\hat{\alpha} \in \hat{\Delta}} \xi_{\hat{\alpha}} \right)$, which is the root space decomposition of ξ with respect to η and $\hat{\Delta}$ is the root system of the Lie algebra bundle ξ with respect to the Cartan subalgebra bundle η. $\square$

Engel and Toral Subalgebra Bundles. Now we shall define Engel and toral subalgebra bundle and we prove their existence where the underlying field is of characteristic zero. [7]. If $\mathfrak{g}$ is a Lie algebra, then $\mathfrak{g} = \mathfrak{g}_0(\text{ad } s) \oplus \mathfrak{g}_*(\text{ad } s)$, where $\mathfrak{g}_*(\text{ad } s)$ is the sum $\mathfrak{g}_f(\text{ad } s) = ker(\text{ad } s - f)^k$, for $f \neq 0$ (k is the multiplicity of f) [from [7], 15.1 Decomposition of $\mathfrak{g}$ relative to ad s]. Now we consider the following lemma:

Lemma 3.6 (Lemma 15.1, in [7])**.** *If $u, v \in \mathbb{F}$, $[\mathfrak{g}_u(\text{ad } s), \mathfrak{g}_v(\text{ad } s)] \subset \mathfrak{g}_{u+v}(\text{ad } s)$. In particular, $\mathfrak{g}_0(\text{ad } s)$ is a subalgebra of $\mathfrak{g}$. If $char\mathbb{F} = 0$, then every element of $\mathfrak{g}_u(\text{ad } s)$ is* ad- *nilpotent for $u \neq 0$.*

By the above lemma, $\mathfrak{g}_0(\text{ad } s)$ is a subalgebra of $\mathfrak{g}$, for $s \in \mathfrak{g}$, called an *Engel subalgebra* of $\mathfrak{g}$ with respect to s (It is a subalgebra of elements annihilated by some power of ad s.).

Definition 3.7. Suppose T is a subalgebra of a Lie algebra $\mathfrak{g}$ such that every element of T is semisimple, that is it has no nonzero nilpotent element. Then T is called toral subalgebra of $\mathfrak{g}$.

Definition 3.8. Let $\pi : \xi \to M$ be a semisimple Lie algebra bundle with standard fiber L. A Lie algebra subbundle η of ξ with standard fibre E is said to be an Engel (toral) subalgebra bundle of ξ if E is an Engel (toral) subalgebra of L and further for each $m \in M$, the fibre η_m of η over m is an Engel (toral) subalgebra of ξ_m the fibre of ξ over m.

Remark 3.9. We have, a subalgebra A of Lie algebra $\mathfrak{g}$ is a Cartan subalgebra if and only if it is a minimal Engel subalgebra. Also A is Cartan if and only if it is a maximal toral subalgebra. (From Theorem 15.3 and Corollary 15.3 in [7]).

Theorem 3.10. *Every complex semisimple Lie algebra bundle of finite type admits an Engel subalgebra bundle and a toral subalgebra bundle.*

Proof. We have proved the existence of a Cartan subalgebra bundle in Proposition 3.5. But every Cartan subalgebra is a minimal Engel subalgebra and a maximal toral subalgebra. Hence the result. □

Lemma 3.11. *Suppose $\theta : \xi \to \xi'$ is a surjective morphism between two complex semisimple Lie algebra bundles ξ and ξ' of finite type. Then*
(a) θ carries Cartan subalgebra bundles to Cartan subalgebra bundles.
(b) If η' is a Cartan subalgebra bundle of ξ', then any Cartan subalgebra bundle of $\theta^{-1}(\eta')$ is a Cartan subalgebra bundle of ξ.

Proof. Proof of (a): Let $\mathfrak{g}$ and $\mathfrak{g}'$ be standard fibers of ξ and ξ' respectively. Since θ is onto, if H is a standard fiber of a Cartan subalgebra bundle η of ξ, $\theta_H(H)$ is a Cartan subalgebra of $\mathfrak{g}'$ (By Lemma 15.4 in [7]). Also $\theta_m(\eta_m)$ is a Cartan subalgebra in ξ'_m. Thus $\theta(\eta)$ is Cartan subalgebra bundle of ξ'.
Similarly we can prove (b). □

Acknowledgments

The authors extend their appreciation to Umm Al-Qura University, Saudi Arabia for funding this research work through grant number: 26UQU4350370GSSR03. Also, the authors would like to thank the anonymous reviewers for their comments and suggestions.

Funding Statement

This research work was funded by Umm Al-Qura University, Saudi Arabia under grant number: 26UQU4350370GSSR03.

References

[1] K. Ajaykumar, *Some Topics in Lie Algebra and Lie Group Bundles*, Doctor of Philosophy thesis, University of Mysore, 2019.
[2] H. A. AlFran, K. Kamalakshi, R. Rajendra, and P. Siva Kota Reddy, *On Smooth Lie Algebra Bundles of Finite Type*, Glob. Stoch. Anal., **11**(1) (2024), 75–79.
[3] H. A. AlFran, K. Kamalakshi, R. Rajendra, and P. Siva Kota Reddy, *On Malcev Algebra Bundles*, J. Appl. Math. Inform., **42**(1) (2024), 207–212.
[4] M. F. Atiyah, *K- Theory*, W.A. Benjamin, Inc., New York, 1967.
[5] D. Coppersmith, *A familly of Lie algebras not extendible to a family of Lie groups*, Proc. Amer. Math. Soc., **66**(2) (1977), 365–366.

[6] A. Douady and M. Lazard, *Espace fibrès algèbres de Lie et en groupes*, Invent. Math., **1** (1966), 133–151.

[7] J. E. Humphreys, *Introduction to Lie Algebras and Representation Theory*, Springer-Verlag New York, Heidelberg Berlin, 1980.

[8] D. Husemoller, *Fibre Bundles*, Springer-Verlag New York, Heidelberg Berlin, 1994.

[9] K. Kamalakshi, H. A. Elsayir, R. Rajendra, P. Siva Kota Reddy and J. López-Bonilla, *Lie Algebra Bundles Defined by Jordan Algebra Bundles of Finite Type*, Glob. Stoch. Anal., **11**(3) (2024), 14–20.

[10] K. Kamalakshi, R. Rajendra and P. Siva Kota Reddy, *On Cohomology of Lie Algebra Bundles of finite type*, Bol. Soc. Parana. Mat. (3), 43 (2025), Article Id: 69209, 10 Pages.

[11] B. S. Kiranagi, *Lie algebra bundles*, Bull. Sci. Math., 2e series, **102** (1978), 57–62.

[12] B. S. Kiranagi, *Semi-simple Lie algebra bundles*, Bull. Math. Soc. Sci. Math. R. S. Roumanie (N.S.), **27**(75)(3) (1983), 253–257.

[13] B. S. Kiranagi, *Lie algebra bundles and Lie rings*, Proc. Nat. Acad. India, **54** (1984), 38–44.

[14] B. S. Kiranagi and R. Rajendra, *Revisiting Hochschild cohomology for algebra bundles*, J. Algebra Appl., **7**(6) (2008), 685–715.

[15] R. Kumar, G. Prema and B. S. Kiranagi, *Lie algebra bundles of finite type*, Acta Univ. Apulensis Math. Inform., **39** (2014), 151–160.

[16] R. Kumar, H. A. Elsayir and P. Siva Kota Reddy, *Note On Leger Cohomology of Lie Algebra Bundles*, Glob. Stoch. Anal., **11**(2) (2024), 29–33.

[17] R. Kumar, T. Kim, J. López-Bonilla and P. Siva Kota Reddy, *A note on Characteristic Ideal Bundles and Structure of Lie Algebra Bundles*, Montes Taurus Journal of Pure and Applied Mathematics, **6**(1) (2024), 36-39.

[18] R. Kumar, T. Kim and P. Siva Kota Reddy, *On Shukla Cohomology of Associative Algebra Bundles*, Adv. Stud. Contemp. Math. (Kyungshang), **34**(2) (2024), 117–121.

[19] B. Madhu, R. Rangarajan, B. S. Kiranagi and K. Ajaykumar, *Engel's and Lie's theorems for Lie algebra bundles*, Adv. Stud. Contemp. Math. (Kyungshang), **31**(1) (2021), 99–106.

[20] R. Rajendra, Ranjitha Kumar and B. S. Kiranagi, *On pull back Lie algebra bundles*, Adv. Stud. Contemp. Math. (Kyungshang), **22**(4) (2012), 521–524.

[21] G. B. Seligman, *Characteristic ideals and the structure of Lie algebras*, Proc. Amer. Math. Soc., **8**(1) (1957), 159–164.

[22] L. N. Vaserstein, *Vector bundles and projective modules*, *Trans. Amer. Math. Soc.*, **294**(2) (1986), 749–755.

K. Kamalakshi: PG Department of Mathematics, Poornaprajna College and Post Graduate Center, Udupi-576 101, India
Email address: kamalakshiaithal@yahoo.com; kamalakshik@ppc.ac.in

Department of Mathematics, AL-Qunfudhah University college, Umm Al-Qura University, Kingdom of Saudi Arabia
Email address: hazbaidi@uqu.edu.sa

Department of Mathematics, Albaydha University, Al Bayda, Yemen
Email address: hakim_albdoie@yahoo.com

R. Rajendra: Department of Mathematics, Field Marshal K. M. Cariappa College (A Constituent College of Mangalore University), Madikeri-571 201, India
Email address: rrajendrar@gmail.com

P. Siva Kota Reddy: Department of Mathematics, JSS Science and Technology University, Mysuru-570 006, India
and
Universidad Bernardo O'Higgins, Facultad de Ingeniería, Ciencia y Tecnología, Departamento de Formación y Desarrollo Científico en Ingeniería, Av. Viel 1497, Santiago, Chile
Email address: pskreddy@jssstuniv.in; pskreddy@sjce.ac.in

Sami H. Altoum: Manar Al Janoub College of Science and Technology, Jazan, Kingdom of Saudi Arabia
Email address: s.altom@mjc.edu.sa