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A NOVEL GENERALIZED FUZZY DIVERGENCE MEASURE AND ITS APPLICATION IN DECISION MAKING

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ABSTRACT. Ambiguity plays a significant role in the contradictory observations about the outside environment. In order to evaluate degree of unpredictability in a probability distribution, Shannon (1948) proposed the notion of entropy, which is necessary for accessing uncertain information. The field of decision-making has made extensive use of fuzzy information measures. In this communication, we suggest using a function called divergence measure to assess the deviation between two fuzzy sets. Additionally, the study of their specific properties is examined to determine their validity. This article introduces a novel approach to decision-making criteria using a proposed measure. This study introduces a directed divergence measure for fuzzy sets and explores its properties. Multicriteria decision making is a practical and widely used approach in real-world situations, allowing for the selection of optimal choices based on given criteria. In recent years, researchers have extensively applied fuzzy directed divergence to multicriteria decision making, while others have investigated the use of parameterized Hesitant Fuzzy Soft Set theory in decision-making processes. Our research examines multiple criteria decision making within a fuzzy context. To illustrate the practical application of our suggested measure, we present a numerical example in the context of a decision-making scenario. Specifically, we examine a fuzzy multicriteria problem that focuses on a student's preferences when seeking admission to a medical program, demonstrating how our new approach can be utilized in such situations.

1. Introduction

Mathematical investigations into the issues surrounding the transmission, storage, and communication of signals gave rise to information theory. Shannon's [22] seminal work "The Mathematical Theory of Communication" served as its foundation. Shannon demonstrated a number of highly popular results with more profound implications and created mathematical frameworks for quantitatively describing the concepts of facts. Renyi [20] introduced a generalized version of Shannon entropy. Arimoto [1] examined the entropy measures for estimation problems. Sharma and Taneja [23] defined the entropy of order α, β. De Luca and Termini [4] proposed a set of properties to examine the correctness of entropy measures. The theoretical and fundamental properties of entropy measures were discussed by Kaufmann [11]. Peerzada et al. [19] also proposed some generalizations of Shannon entropy.

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The fundamental characteristics of human curiosity and many real-world goals are ambiguity and uncertainty. Fuzzy thinking is evident in the way we process information, make decisions, and speak. Eliminating ambiguity and uncertainty is the primary purpose of information. The degree of confusion and impreciseness of uncertainties is measured in practice as the degree of fuzziness. A measure of information is another name for the amount of uncertainty removed.

In order to represent ambiguous and indistinct occurrences, Zadeh [25] developed the hypothesis of fuzzy sets (FSs), which is an expansion of conventional set theory. This concept is a useful tool for understanding how humanistic systems behave, when human judgment, perceptions, and emotions are crucial. The exponential entropy concept of Pal and Pal [18], Anshu Ohlan [17], and was extended to the fuzzy circumstances by Bhandari and Pal [3]. Kapur [10] talked on the unpredictability of fuzzy measures because of the un-sureness of the information.

The fuzzy divergence measure measures the fuzzy deviation between two fuzzy sets by quantifying their differences. The similarity measure is an essential tool for managing uncertainty in decision-making problems when using IFS theory. Different researchers have offered different distance metrics. It has been noted that when determining the degree of separation between two IFSs, different distance measures yield varied results. Additionally, for a pair of IFSs, current distance metrics may not always provide a suitable and practical result. Because of this, improved measures are always required for better judgment.

To describe the distinctions between fuzzy sets, the divergence measure was developed and is thought of as a parallel model of the correspondence measure. Many scholars have used distance measurements to define fuzzy entropy, such as Kaufmann [11], Arora, Hari Darshan, and Anjali Dhiman [2], Kosko [12], and Yager [24].A variety of earlier methods for fuzzy information produced by divergence measure were expanded upon by Fan et al. [7]. The detachment between two fuzzy points were described by Dubois and Prade [5]. Rosenfeld [21] described the minimum distance of fuzzy sets on non-negative real numbers. Bhandari and Pal [3], Anshu Ohlan [17] and Arora et al. [2] stated the next measure of fuzzy directed divergence, which is connected to Kullback and Leibler [13] probabilistic measure of divergence. An axiomatic form for measuring the deviation of fuzzy sets was proposed by Montes et al. [15], Perveen P. A. Fathima et al. [8]. Gupta and Tiwari [9] suggested a cosine measure for intuitionistic fuzzy sets and Datta and Goala [6], Yaya Liu, Keyun Qin [14] suggested cosine measure for interval-valued intuitionistic fuzzy sets. Application of Pythagorean fuzzy sets in fault diagnosis by using similarity measures suggested by Hoang Nguyen [16]. In proposed work we introduce a new fuzzy divergence measure of cosine function. Some terms associated to fuzzy set and information theory are given in Section 2. The validity of the proposed measure has been examined in Section 3 and finally, the overall work has been summarized in Section 4.

2. Preliminaries

Definition 2.1. Assume that in an experiment, we use the probability distribution $P = (p_1, p_2, \ldots, p_n)$, where X is a discrete random variable. The well-known

Shannon entropy [22], $H(P)$, is given by:

$$H(P) = -\sum_{i=1}^{n} p_i \log p_i \tag{2.1}$$

This quantity represents the information content of the experiment.

Definition 2.2. Recall that a membership function $\tilde{\mu}_{\mathbb{A}} : \Omega \to [0,1]$ characterizes a fuzzy subset $\mathbb{A}$ in Ω (a collection of speech), and expresses the degree of membership of $\omega \in \Omega$ in $\mathbb{A}$ as:

$$\tilde{\mu}_{\mathbb{A}}(\tau) = \begin{cases} 0 & \text{if } \omega \notin \mathbb{A} \text{ (uncertainty is 0)} \\ 1 & \text{if } \tau \in \mathbb{A} \text{ (uncertainty is 0)} \\ 0.5 & \text{if the uncertainty is maximum} \end{cases}$$

Definition 2.3. De Luca and Termini [4] suggested the following measure of fuzzy entropy:

$$H(A) = -\left[\sum_{i=1}^{n} \tilde{\mu}_A(\tau_i) \log \tilde{\mu}_A(\tau_i) + \sum_{i=1}^{n} \big(1 - \tilde{\mu}_A(\tau_i)\big) \log \big(1 - \tilde{\mu}_A(\tau_i)\big)\right] \tag{2.2}$$

A collection of properties presented by De Luca and Termini [4] is commonly used as a standard for establishing new fuzzy entropies. These minimum requirements are listed below:

(1) $H(A) = 0$ when $\tilde{\mu}_A(\tau_i) = 0$ or 1.
(2) $H(A)$ increases as $\tilde{\mu}_A(\tau_i)$ lies between 0 and 0.5.
(3) $H(A)$ decreases as $\tilde{\mu}_A(\tau_i)$ lies between 0.5 and 1.
(4) $H(A) = H(\bar{A})$, i.e., $\tilde{\mu}_{\bar{A}}(\tau_i) = 1 - \tilde{\mu}_A(\tau_i)$.
(5) $H(A)$ represents a concave function of $\tilde{\mu}_A(\tau_i)$.

Definition 2.4. The divergence measure of the probability distribution $\mathbb{P} = (p_1, p_2, \ldots, p_n)$ from the probability distribution $\mathbb{Q} = (q_1, q_2, \ldots, q_n)$ was defined by Kullback and Leibler [13] as:

$$\mathbb{D}(\mathbb{P} \parallel \mathbb{Q}) = \sum_{i=1}^{n} p_i \log \frac{p_i}{q_i} \tag{2.3}$$

Definition 2.5. Bhandari and Pal [3] proposed the fuzzy divergence measure as:

$$\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = \sum_{i=1}^{n} \tilde{\mu}_{\mathbb{A}}(\tau_i) \log \frac{\tilde{\mu}_{\mathbb{A}}(\tau_i)}{\tilde{\mu}_{\mathbb{B}}(\tau_i)} + \sum_{i=1}^{n} (1 - \tilde{\mu}_{\mathbb{A}}(\tau_i)) \log \frac{1 - \tilde{\mu}_{\mathbb{A}}(\tau_i)}{1 - \tilde{\mu}_{\mathbb{B}}(\tau_i)} \tag{2.4}$$

This measure holds the following conditions:

(1) $\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) \geq 0$ and is a convex function in $(0, 1)$.
(2) $\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = 0$ if and only if $\mathbb{A} = \mathbb{B}$.
(3) $\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = \mathbb{D}(\mathbb{B} \parallel \mathbb{A})$.
(4) $\mathbb{D}(\mathbb{A} \parallel \mathbb{C}) \leq \mathbb{D}(\mathbb{A} \parallel \mathbb{B}) + \mathbb{D}(\mathbb{B} \parallel \mathbb{C})$.
(5) $\mathbb{D}(\mathbb{A} \parallel \mathbb{B})$ is a convex function.

Now, we propose a new cosine measure of fuzzy directed divergence in Section 3.

3. Results and Discussion

3.1. New Directed Divergence Measure. Let $\Omega = \{\tau_1, \tau_2, \tau_3, \ldots, \tau_n\}$ be the collection of points. Let $\mathbb{A} = \{(\tau_i, \tilde{\mu}_{\mathbb{A}}(\tau_i)) : \tau_i \in \Omega\}$ and $\mathbb{B} = \{(\tau_i, \tilde{\mu}_{\mathbb{B}}(\tau_i)) : \tau_i \in \Omega\}$ be two fuzzy sets. Then we propose a new divergence measure as:

$$\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = \frac{2}{n} \sum_{i=1}^{n} \frac{\cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)} \tag{3.1}$$

Theorem 3.1. *The proposed fuzzy divergence measure is an authentic measure of fuzzy divergence.*

Proof. We now demonstrate each of these requirements individually:
i) Let

$$\mathbb{A} = \{(\tau_i, \tilde{\mu}_{\mathbb{A}}(\tau_i)) : \tau_i \in \Omega\}, \quad \mathbb{B} = \{(\tau_i, \tilde{\mu}_{\mathbb{B}}(\tau_i)) : \tau_i \in \Omega\}$$

be fuzzy sets. Then,

$$0 \leq \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right| \leq 1 \quad \Rightarrow \quad 0 \leq \frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right| \leq \frac{\pi}{2}$$

$$\Rightarrow 0 \leq \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right) \leq 1$$

$$\Rightarrow 0 \leq 1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right) \leq 2$$

From the above, we have

$$0 \leq \frac{2\cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)} \leq 1$$

$$\Rightarrow 0 \leq \frac{2}{n} \sum_{i=1}^{n} \frac{\cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)} \leq 1$$

$$\Rightarrow 0 \leq \mathbb{D}(\mathbb{A} \parallel \mathbb{B}) \leq 1$$

Also, $\mathbb{D}(\mathbb{A} \parallel \mathbb{B})$ is a convex function in $(0, 1)$.

ii) We have $\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = 0$ if and only if

$$\frac{2}{n} \sum_{i=1}^{n} \frac{\cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)} = 0$$

$$\Rightarrow \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right) = 0$$

$$\Rightarrow \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right| = 0 \quad \Rightarrow \quad \tilde{\mu}_{\mathbb{A}}(\tau_i) = \tilde{\mu}_{\mathbb{B}}(\tau_i)$$

$$\Rightarrow \mathbb{A} = \mathbb{B}$$

Therefore, $\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = 0$ if and only if $\tilde{\mu}_{\mathbb{A}}(\tau_i) = \tilde{\mu}_{\mathbb{B}}(\tau_i)$ for all $\tau_i \in \Omega$.
iii) As

$$\mathbb{D}(\mathbb{A} \parallel \mathbb{B}) = \frac{2}{n} \sum_{i=1}^{n} \frac{\cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1 + \cos\left(\frac{\pi}{2} \left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}$$

$$+\frac{2}{n}\sum_{i=1}^{n}\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{A}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{A}}(\tau_i)\right|\right)}$$
$$=\mathbb{D}(\mathbb{B}\parallel\mathbb{A})$$

Therefore, $\mathbb{D}(\mathbb{A}\parallel\mathbb{B})=\mathbb{D}(\mathbb{B}\parallel\mathbb{A})$.

iv) To prove
$$\mathbb{D}(\mathbb{A}\parallel\mathbb{C})\leq\mathbb{D}(\mathbb{A}\parallel\mathbb{B})+\mathbb{D}(\mathbb{B}\parallel\mathbb{C}),$$
we first note that
$$\cos(\mathbb{A}+\mathbb{B})\leq\cos\mathbb{A}+\cos\mathbb{B}\quad\text{or}\quad\cos\mathbb{A}+\cos\mathbb{B}-\cos(\mathbb{A}+\mathbb{B})\geq 0$$
$$\Rightarrow\cos\mathbb{A}+\cos\mathbb{B}-\cos\mathbb{A}\cos\mathbb{B}+\sin\mathbb{A}\sin\mathbb{B}\geq 0$$
$$\Rightarrow\cos\mathbb{A}(1-\cos\mathbb{B})+\cos\mathbb{B}+\sin\mathbb{A}\sin\mathbb{B}\geq 0$$
$$\Rightarrow\cos\mathbb{A}\cdot 2\sin^2\left(\frac{\mathbb{B}}{2}\right)+\cos\mathbb{B}+\sin\mathbb{A}\sin\mathbb{B}\geq 0$$

Since $\mathbb{A}$ and $\mathbb{B}$ are acute angles, this identity holds good.
Consider $\mathbb{A}=\{(\tau_i,\tilde{\mu}_{\mathbb{A}}(\tau_i)):\tau_i\in\Omega\}$, $\mathbb{B}=\{(\tau_i,\tilde{\mu}_{\mathbb{B}}(\tau_i)):\tau_i\in\Omega\}$, and $\mathbb{C}=\{(\tau_i,\tilde{\mu}_{\mathbb{C}}(\tau_i)):\tau_i\in\Omega\}$ be three fuzzy sets.
As
$$|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)|\leq|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)|+|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)|$$
$$\Rightarrow\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)\leq\cos\left(\frac{\pi}{2}\left(\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|+\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)\right)$$
$$\Rightarrow\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)\leq\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)$$
Also,
$$1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)\leq 1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)$$
$$\Rightarrow\frac{1}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}\geq\frac{1}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)$$
$$\Rightarrow 1-\frac{1}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}\leq 1-\frac{1}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)$$
$$\Rightarrow\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}\leq\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}+\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}$$
$$\Rightarrow\frac{2}{n}\sum_{i=1}^{n}\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}\mathbb{C}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}\leq\frac{2}{n}\sum_{i=1}^{n}\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i)-\tilde{\mu}_{B}(\tau_i)\right|\right)}$$
$$+\frac{2}{n}\sum_{i=1}^{n}\frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{B}}(\tau_i)-\tilde{\mu}_{\mathbb{C}}(\tau_i)\right|\right)}$$

Therefore,

$$\mathbb{D}(\mathbb{A} \parallel \mathbb{C}) \leq \mathbb{D}(\mathbb{A} \parallel \mathbb{B}) + \mathbb{D}(\mathbb{B} \parallel \mathbb{C})$$

□

Hence, the measure in (3.1) satisfies all five essential properties, so it is an authentic measure of directed divergence.
The graph of the proposed divergence measure is given in Figure 1.

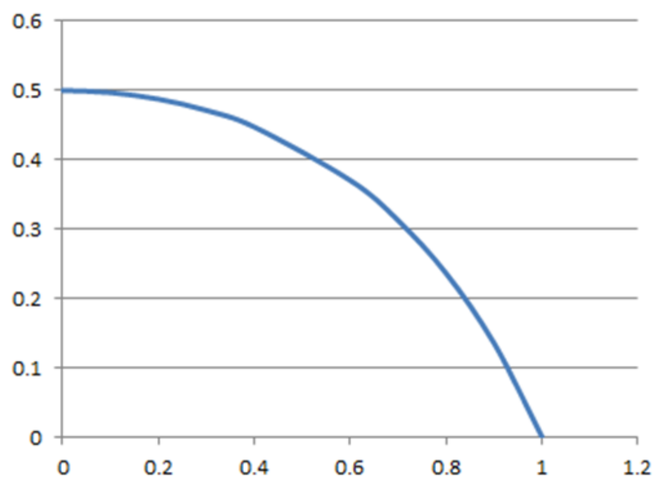

Figure-1

3.2. Application of Fuzzy Divergence Measure in Decision Making. In real-world applications, multi-criteria decision making is a highly effective and commonly used approach. This technique allows for the selection of the best option based on given criteria. Over the past few years, many scholars have extensively utilized fuzzy directed divergence in multi-criteria decision-making processes. This paper investigates the multi-criteria decision-making issue within a fuzzy context.

For such problems, we consider a set of strategies

$$A_1, A_2, A_3, \ldots, A_n,$$

with each strategy exhibiting varying degrees of effectiveness in relation to a cost set

$$C_1, C_2, C_3, \ldots, C_m.$$

Step 1. Initially, we organize the preferences of decision makers into a fuzzy decision-making matrix for each available option A_j $(j = 1, 2, \ldots, n)$ with respect to the cost set C_k $(k = 1, 2, \ldots, m)$ as follows:

$$\mathbf{D_{n\times m}} = [a_{ij}] = \begin{pmatrix} a_{11} & a_{12} & \dots & a_{1m} \\ a_{21} & a_{22} & \dots & a_{2m} \\ \vdots & \vdots & \ddots & \vdots \\ a_{n1} & a_{n2} & \dots & a_{nm} \end{pmatrix}_{n\times m}$$

Step 2. From all alternatives corresponding to their cost set, we identify the optimal solution as:

$$A^* = \{R_1^*, R_2^*, \dots, R_n^*\},$$

where

$$R_l^* = \max\{R_i^*\}.$$

Step 3. Now, we calculate the divergence using:

$$\mathbb{D}(\mathbb{A} : \mathbb{B}) = \frac{2}{n}\sum_{i=1}^{n} \frac{\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}{1+\cos\left(\frac{\pi}{2}\left|\tilde{\mu}_{\mathbb{A}}(\tau_i) - \tilde{\mu}_{\mathbb{B}}(\tau_i)\right|\right)}.$$

Step 4. For sorting, we take:

$$\min\left\{\mathbb{D}(\mathbb{A}_j \parallel \mathbb{A}^*)\right\}, \quad \text{where } 1 \le j \le n.$$

Finally, we choose the most favorable option by ranking their functions in descending order.

3.3. Numerical Example. This example demonstrates a fuzzy multi-criteria approach to evaluate a student's admission preferences for postgraduate medical programs. The scenario involves a student seeking admission to the All India Institute of Medical Sciences (*AIIMS*) and considering five potential institutes for enrollment:

- K_1 = AIIMS Delhi
- K_2 = AIIMS Bhubaneswar
- K_3 = AIIMS Guwahati
- K_4 = AIIMS Raipur
- K_5 = AIIMS Patna

These represent some of the most highly regarded medical educational institutions. Students aim to select an institution based on the following criteria:

- L_1 = Research
- L_2 = Ranking
- L_3 = Faculty
- L_4 = Facility
- L_5 = Fee

Step 1. Arranging the fuzzy decision matrix

$$M_{n\times m} = \left[m_{ij}\right]$$

	L_1	L_2	L_3	L_4	L_5
K_1	0.3	0.4	0.8	0.5	0.7
K_2	0.5	0.5	0.6	0.2	0.8
K_3	0.6	0.7	0.2	0.4	0.7
K_4	0.8	0.9	0.4	0.5	0.5
K_5	0.2	0.3	0.5	0.7	0.9

Table 1: Fuzzy Decision Matrix

Step 2. The optimal solution from the above matrix is:

$$K^* = \{0.8, 0.9, 0.8, 0.7, 0.9\}$$

Step 3. The divergence of K^* with respect to each alternative is given as:

	$\mathbb{D}(K_1, K^*)$	$\mathbb{D}(K_2, K^*)$	$\mathbb{D}(K_3, K^*)$	$\mathbb{D}(K_4, K^*)$	$\mathbb{D}(K_5, K^*)$
Value	0.931879	0.856868	0.919559	0.975986	0.847023

Table 2: Divergence Values of Alternatives

Step 4. Based on the information in Table 2, we observe that K_5 exhibits the lowest divergence value among all alternatives. This indicates that K_5 is the optimal choice. Therefore, the student should enroll in **AIIMS Patna**.

This demonstrates that the newly introduced fuzzy divergence measure is an excellent tool for addressing multi-criteria decision-making problems.

4. Conclusions

This paper introduces a directed divergence measure for fuzzy sets and examines its characteristics. The study explores the practical application of this measure in decision-making processes. To illustrate its use, a numerical example is provided, demonstrating how the proposed measure can be applied to support optimal decision-making.

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