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THE TWO-WAREHOUSE INVENTORY MODEL FOR NON-INSTANTANEOUS DETERIORATING ITEMS WITH THE EFFECT OF INFLATION AND PRESERVATION TECHNOLOGY

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Abstract. This study establishes a two-warehouse inventory model for deteriorating items, incorporating preservation technology, shortages, and inflation. Deterioration is considered a natural, non-instantaneous, constant process, which is mitigated through preservation technology. The model operates under the influence of inflation and allows shortages in the own warehouse. A hybrid demand function, dependent on both selling price and stock level, is applied to capture demand variations. The two-warehouse system comprises an own warehouse storing a fixed quantity and a rented warehouse (RW) for excess inventory. To demonstrate the model's optimality, sensitivity analysis and graphical presentations are conducted. A numerical example is solved using Mathematica Software 7.0 to illustrate the model's application.

1. Introduction

Deterioration means damage, decay, vaporization, spoilage, obsolescence, and loss of utility of the commodity. Deterioration play vital role in the inventory system and we discussed some types of the deterioration in this manuscript such as- (a) Physical deterioration: physical deterioration means decrease in physical quantity or quality of commodity due to some factors like damage, decay and spoilage. (b) Obsolescence: in this type usefulness of inventory items due to technical advancements, changes in consumers preferences, or new items releases. (c) Seasonal deterioration: decrease in customers demand or value of inventory items due to seasonal fluctuations. All these are the types of the deterioration and deterioration affected in inventory system by the following factors (a) Time: longer storage times increase the likelihood of deterioration. (b) Environmental conditions: humidity, temperature and exposure to light can affect deterioration rates. (c) Items characteristics: perishable items like pharmaceutical are more prone to deterioration than non-perishable items. (d) Storage: improper storage or packing may be lead to physical deterioration. Further we discussed the examples of deterioration in inventory system in this order (a) Food inventory: perishable items, like fruits or dairy products deteriorate over time due to spoilage. (b) Fashion inventory: apparel items deteriorate due to seasonal changes, fashion trends, or storage conditions. (c) Pharmaceutical commodity: medicines or vaccines deteriorate due to

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temperature, expiration date and exposure to light. So for the deteriorating items many prominent researchers and authors have done work in this way like- Rana and Kumar [1] analyzed own warehouse and rented warehouse system with the preservation technology in rental warehouse. They used hybrid type demand function. Pal et al.[2] determined two-warehouse inventory problem for non-instantaneous bad quality items with impact of inflation and delay in payment. Mondal et al.[3] established two-warehouse inventory model for decaying products with partially backlogging and trade credit policy policies. Ahmad et al. [4] proposed a two-warehouse inventory control model for low quality items with limited life time period. And they used shortages and deterioration in the both warehouse system. Sharma et al. [5] established two-warehouse inventory model for decaying goods with time dependent demand function, carbon emission and variable holding cost. Rathore et al. [6] presented an inventory control model for decaying items in the two-warehouse system and time dependent demand function applied in the warehouse. Limi et al. [7] examined the impact of advertisement frequency and selling price on demand. Also, they worked on the shortages of the items to improve the patience level. Faveto et al. [8] reviewed with two-warehouse system and deteriorating items. Sharma and Rathore [9] determined inventory model for bad quality products with selling price and stock level dependent demand. Preservation technology (PT) implemented to control the degradation of the items. Handa et al. [10] investigated reverse logistics model for degrading items in the two-warehouse environment. Carbon tax policy applied in the transportation and two-warehouse environment. In the two-warehouse system preservation technology plays vital role to maintain the quality and freshness of the deteriorating items. The major utilization of preservation technology is to minimize deterioration and reduce loss due to spoilage. So, in this article we applied preservation technology. And preservation technology works as in the warehouse system- (a) own warehouse: items are stored in the own warehouse for shorter period. PT is applied to maintain optimal storage conditions. (b) Rental warehouse: commodities are transferred to rental warehouse for long time storage. (PT) preservation technology is applied to maintain optimal storage conditions. There are many benefits to implement the preservation technology such as to reduce deterioration, improve quality, increase efficiency, cost saving etc. So, for the preservation technology various researchers and authors have done work in the following manner Bhawaria and Rathore [11] developed production inventory model for decaying products with selling price and stock dependent demand. Preservation technology implemented to control deterioration of the products. Bhawaria and Rathore [12] described EPQ model for degrading items with the impact of inflation. And they also worked on preservation technique. Bhawaria et al. [13] established reverse logistics model with preservation technology and impact of inflation and deterioration type was non-instantaneous. Salmasnia and Kohan [14] developed decreasing pricing policy for deteriorating items and preservation technology applied for both quality and quantity deteriorating products. Liao et al. [15] investigated preservation technology cost problem for the non-instantaneous bad quality items with time dependent holding cost. Ogbonna et al. [16] determined deterministic inventory model for deteriorating items with impact of the inflation. And they allowed the shortages with stock level dependent demand function. Zaidi et al.[17] established production model for deteriorating items and carbon emission. Preservation technology implemented to reduce the carbon emission and deterioration of the items. Shah and Joshi [18] determined EOQ model for fragment quality items with the fuzzy environment and impact of inflation. And preservation technology implemented to control the degradation of the items. Saxena et al. [19] proposed inventory model with preservation technology and this technology applied to control the on the greenhouse gases. And entire study carried out under the impact of inflation. Shahu et al. [20] discussed for seasonal deteriorating items with time and price dependent demand. Preservation technology applied during the items deterioration period and effect also carried. In the present time inflation is the key factor in inventory system. In the inventory system inflation refers to the increase in the cost or value of items over time due to changes in market prices, demand, or other economic factors. And inflation divided in three types (a) price inflation (b) cost inflation (c) value inflation. Inventory system affected by the inflation in the following ways (a) increased holding cost: inflation increase the holding cost, making it more expensive to maintain inventory levels (b) reduced profit margins: inflation may reduce benefit margins if the selling price of items doesn't keep pace with increasing costs. (c) Inventory obsolescence: inflation may to items obsolescence if the value of items decreases over time. So, for the inflation research work studied in the following manner-Siriwardena et al. [21] examined hyperinflation on inventory system with machine learning system. And major target of this model was to find the optimum solution of the total cost. Kumar et al.[22] determined inventory control model for decaying goods in the inflationary environment and demand function was ramp type. The major aim of this paper was to decrease the total cost of the inventory. Patro et al.[23] derived a two-warehouse inventory model for imperfect items and variable discount. PSO technology used to find the optimal solution of the total cost with different decision parameters. Chakraborthy [24] established supply chain model (SCM) for bad quality items with the impact of inflation and delay in payment under carbon emission regulation. Xu et al. [25] developed price and ordering policy with trapezoidal demand under the impact of inflation and partial backlogging shortages. Babazadeh and Mirzazadeh [26] proposed multi item inventory model considering imperfect production and partial backlogging under the impact of inflationary environment. They applied (SA) simulated annealing and (WCA) water cycle algorithm to find the solution. Attia et al. [27] developed economic recycle quantity model with piecewise constant demand and shortage. Specially, they focused on production system that produces recyclable defective items. Aliabadi et al. [28] developed supply chain model for varying fragment quality products with the trade-credit policy and impact of inflation. Also they focused on selling price dependent demand and carbon tax policy. Drezner and Barron [29] studied supply chain policy for inventory system with partially backlogging shortages. Specially, they focused on the (S, S) policy keeping in mind stock level. So for the inflation many prominent researcher and authors have done work such as- Yu et al. [30], Hossain et al. [31], Bhardwaj and agarwal [32], Singh et al. [33], Shan and Kittner [34], Zeng et al. [35], Rami and Allouhi [36], Bhawaria et al. [37], Bhatanagr et al. [38], Zhang et al.

[39], Frolov [40]. This model aims to develop the two-warehouse inventory system for degrading items under the effect of inflation. We implemented preservation technology to control the deterioration and shortages are permitted and are completely backlogged. Finally to verify theoretical results the numerical examples and graphs are solved.

2. Notations and Assumptions

To develop the mathematical model, the following notations and assumptions are used.

Notations	**Discription.**
A	Ordering cost per cycle
a	Stock level
C_p	Purchasing cost of items per unit
w	Own warehouse capacity
$D(s)$	Selling price dependent demand per unit time
Q_F	Order quantity per cycle
S_F	Inventory level per cycle
h	Holding cost per unit time in own warehouse
f	Holding cost per unit time in rental warehouse ($f > h$)
σ	Shortage cost per unit per unit time
τ_θ	Deterioration rate in rental and own warehouse
r	Rate of inflation
$I_o(t)$	Inventory level in own warehouse at time t
$I_R(t)$	Inventory level in rental warehouse at time t
S_t	Maximum amount of backlogged demand per cycle
$B(t)$	Backlogged demand at time t
t_D	Time period during which no deterioration occurs
t_W	Time at which inventory level reaches zero in own warehouse
t_R	Time at which inventory level reaches zero in rental warehouse
OC	Ordering cost
HC	Holding cost
DC	Deterioration cost
PTC	Preservation technology cost
SC	Shortage cost
T	Total cycle time length

Assumptions.

- The rented warehouse has unlimited capacity, while the own warehouse has a fixed capacity of w units.
- Lead time is negligible.
- Replenishment rate is instantaneous.
- The planning horizon is infinite.
- Shortages are allowed and are completely backlogged.

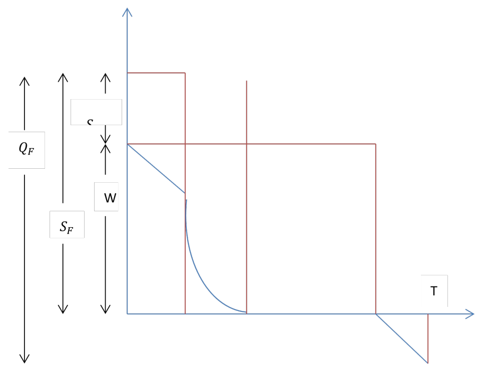

FIGURE 1. Two-warehouse inventory system when $t_D < t_w$

- The holding cost per unit time in the rental warehouse is taken as an exponential function $h(t) = e^{ct}$ where $c > 0$, and it is higher than the holding cost h in the own warehouse.
- The deterioration rate in the rental warehouse is less than that in the own warehouse. Deterioration is constant and non-instantaneous, given by:

$$\tau_\theta = \theta - m(\xi), \quad \text{where} \quad m(\xi) = e^{-b\xi}, \quad 0 \le \theta < 1$$

- The hybrid type demand function depends on selling price s and inventory level $I(t)$:

$$f(s, I(t)) = D(s) + aI(t), \quad a > 0$$

where the price-dependent demand function $D(s)$ is defined as:

$$D(s) = \tau(x_1 - ys) + (1-\tau)x_2 s^{-\gamma}$$

with the conditions: $0 \le \tau \le 1$, $x_1 > 0$, $x_2 > 0$, $\gamma > 0$, $\frac{x_1}{y} \ge s$, and $y > 1$.

3. Mathematical Model Formulation

3.1. Case-1 when $t_D < t_w$. In the interval $[0, t_D]$, there is no deterioration. So, the inventory level in the own warehouse decreases due to demand, whereas in the rental warehouse, the rest inventory remains the same. During the interval $[t_D, t_w]$, the inventory level of OW reaches zero due to demand and deterioration of the items, and in the RW, inventory depletes due to deterioration. Now, in the interval $[t_w, t_R]$, the inventory level decreases due to demand and deterioration and reaches zero at time $t = t_R$ for the rental warehouse. Further, in the interval $[t_R, T]$, demand is backlogged. So, it is denoted by $B(t)$ and represents the negative inventory level at time t. The overall inventory effect is shown in Figure-1.

So, the governing differential equations are as follows:

$$\frac{dI_0(t)}{dt} = -D(s), \qquad 0 \le t \le t_D, \tag{1}$$

$$\frac{dI_0(t)}{dt} + \tau_\theta a I_0(t) = -D(s), \qquad t_D \le t \le t_w, \tag{2}$$

$$\frac{dI_R(t)}{dt} + \tau_\theta a I_R(t) = 0, \qquad t_D \le t \le t_w, \tag{3}$$

$$\frac{dI_R(t)}{dt} + \tau_\theta a I_R(t) = -D(s), \qquad t_w \le t \le t_R, \tag{4}$$

$$\frac{dB(t)}{dt} = D(s), \qquad t_R \le t \le T. \tag{5}$$

Solution of equations (1) to (5) with the boundary conditions $I_0(0) = w$, $I_0(t_w) = 0$, $I_R(t_w) = S_F - w$, $I_R(t_R) = 0$, $B(t_R) = 0$:

$$I_0(t) = -D(s) + w, \tag{6}$$

$$I_0(t) = \frac{D(s)}{a\tau_\theta}\left\{-1 + e^{a\tau_\theta(t_w - t)}\right\}, \tag{7}$$

$$I_R(t) = (S_F - w)e^{-\tau_\theta a(t + t_D)}, \tag{8}$$

$$I_R(t) = \frac{D(s)}{a\tau_\theta}\left\{-1 + e^{a\tau_\theta(t_w - t)}\right\}, \tag{9}$$

$$B(t) = D(s)\{t - t_R\}. \tag{10}$$

Considering continuity of $I_0(t)$ at $t = t_D$:

$$t_w = t_D + \frac{D(s)}{a\tau_\theta}\log\left[1 + \frac{a\tau_\theta}{D(s)}(w - D(s))\right], \tag{11}$$

Considering continuity of $I_R(t)$ at $t = t_w$:

$$S_F = w + \frac{D(s)}{a\tau_\theta}e^{a\tau_\theta(t_w + t_D)}\left\{-1 + e^{a\tau_\theta(t_R + t_w)}\right\}. \tag{12}$$

Maximum amount of backlogged demand is:

$$B(t) = D(s)(T - t_R). \tag{13}$$

Now the order quantity is:

$$Q_F = s_t + S_F, \tag{3.1}$$

$$Q_F = \left[D(s)(T - t_R) + w + \frac{D(s)}{a\tau_\theta}e^{a\tau_\theta(t_w + t_D)}\left\{-1 + e^{a\tau_\theta(t_R + t_w)}\right\}\right]. \tag{14}$$

3.2. Measure Performance.

Ordering Cost.

$$OC = A \tag{15}$$

Holding Cost in OW.

$$HC_{ow} = h\left[\int_0^{t_D} e^{-rt} I_o(t)\,dt + \int_{t_D}^{t_W} e^{-rt} I_o(t)\,dt\right]$$

$$\begin{aligned}HC_{ow} = h\Bigg[&\frac{1}{r}\left\{\frac{1}{r} - e^{-rt_D}\left(t_D + \frac{1}{r}\right)\right\} + \frac{w}{r}\left(1 - e^{-rt_D}\right)\\ &+\frac{D(s)}{a\tau_\theta}\left\{\frac{1}{r}\left(e^{-rt_w} - e^{-rt_D}\right) + \frac{1}{r + a\tau_\theta}\left(e^{a\tau_\theta(t_w - t_D)} - e^{-rt_w}\right)\right\}\Bigg]\end{aligned} \tag{16}$$

Holding Cost in RW.

$$\begin{aligned}HC_{Rw} = f\Bigg[&\int_0^{t_D} e^{a\tau_\theta(t+t_D)+t(c-r)}(S_F - w)\,dt\\ &+ \frac{D(s)}{a\tau_\theta}\int_{t_D}^{t_w} e^{t(c-r)}\left(-1 + e^{a\tau_\theta(t_w - t)}\right)\,dt\\ &+\frac{D(s)}{a\tau_\theta}\int_{t_w}^{t_R} e^{t(c-r)}\left(-1 + e^{a\tau_\theta(t_w - t)}\right)\,dt\Bigg]\end{aligned}$$

$$\begin{aligned}HC_{Rw} = f\Bigg[&\frac{(S_F - w)}{c - r - a\tau_\theta}\left\{e^{-2a\tau_\theta(t+t_D)+\tau_\theta(c-r)} - e^{-a\tau_\theta t_D}\right\}\\ &+ \frac{D(s)}{a\tau_\theta}\Bigg\{\frac{1}{c-r}\left(e^{t_w(c-r)} - e^{t_R(c-r)}\right)\\ &+\frac{1}{a\tau_\theta + c - r}\left(e^{t_R(c-r)} - e^{a\tau_\theta(t_R - t_w)+t_w(c-r)}\right)\Bigg\}\\ &+ \frac{D(s)}{a\tau_\theta}\Bigg\{\frac{1}{c-r}\left(e^{t_D(c-r)} - e^{t_W(c-r)}\right)\\ &+\frac{1}{a\tau_\theta + c - r}\left(e^{t_w(c-r)} - e^{a\tau_\theta(-t_D + t_w)+t_D(c-r)}\right)\Bigg\}\Bigg]\end{aligned} \tag{17}$$

Shortage Cost.

$$SC = \sigma\int_{t_R}^{T} S_t\,dt$$

$$SC = \sigma D(s)\frac{(T - t_R)}{2} \tag{18}$$

Deterioration Cost.

$$DC = C_p(Q_F - D(s)T)$$

$$DC = C_p\left[D(s)(T - t_R) + \frac{D(s)}{a\tau_\theta}\left(e^{a\tau_\theta(t_w - t_D)}\right)\left(-1 + e^{a\tau_\theta(t_R - t_w)}\right)\right] \tag{19}$$

Preservation Technology Cost.

$$PTC = \int_{t_D}^{t_R} \xi\,dt$$

$$PTC = \xi(t_R - t_D) \tag{20}$$

The total cost.

$$TC = [OC + HC_{OW} + HC_{RW} + SC + DC + PTC] \tag{21}$$

4. Solution Process

To minimize the total cost we differentiate $TC(P, t_1, \xi)$ with respect to P, ξ, and t_1. And for optimum value necessary conditions are:

$$\frac{\partial TC(T, t_R, \xi)}{\partial T} = 0, \quad \frac{\partial TC(T, t_R, \xi)}{\partial \xi} = 0, \quad \frac{\partial TC(T, t_R, \xi)}{\partial t_R} = 0$$

$$\text{Det.}(H_1) > 0, \quad \text{Det.}(H_2) > 0, \quad \text{Det.}(H_3) > 0;$$

where H_1, H_2, & H_3, are the principle minor of the Hessian matrix. Hessian Matrix of the total cost function is as follows:

$$TC(T, t_R, \xi) = \begin{bmatrix} \frac{\partial^2(TC)}{\partial \xi^2} & \frac{\partial^2(TC)}{\partial \xi \partial t_R} & \frac{\partial^2(TC)}{\partial \xi \partial T} \\ \frac{\partial^2(TC)}{\partial t_1 \partial \xi} & \frac{\partial^2(TC)}{\partial t_R^2} & \frac{\partial^2(TC)}{\partial t_R \partial T} \\ \frac{\partial^2(TC)}{\partial T \partial \xi} & \frac{\partial^2(TC)}{\partial T \partial t_R} & \frac{\partial^2(TC)}{\partial T^2} \end{bmatrix}$$

4.1. Numerical Example for case $t_D < t_w$. We have taken the appropriate values of the following parameters with proper units to find the optimal values of the parameters and total cost which are as:

$a = 70$, $b = 0.3$, $c = 0.5$, $\theta = 0.6$, $f = 600$, $h = 5$, $A = 8$, $t_D = 3$, $x_1 = 100$, $x_2 = 1$, $C_p = 7$, $y = 0.6$, $s = 40$, $\tau = 2$, $\gamma = 0.3$, $r = 0.4$, $t_W = 3$, $w = 20$, $\sigma = 2$.

Now the optimum values of the decision parameters and total cost are as:

$T = 8.62789 \times 10^{17}$, $t_R = 5.91118 \times 10^{14}$, $\xi = 1.51603$, $TC = 2445.47$.

4.2. Table-1 Sensitivity Analysis for case $t_D < t_w$.

tableSensitivity Analysis for case $t_D < t_w$

Parameters	Changes	$T \times 10^{17}$	$t_R \times 10^{14}$	ξ	TC
b	0.2	8.6281800	5.911470	2.27404	2445.47
	0.3	8.6278900	5.911180	1.51603	2445.47
	0.4	8.6277500	5.911040	1.13702	2445.47
θ	0.5	8.6280200	5.911310	1.84936	2445.46
	0.6	8.6278900	5.911180	1.51603	2445.47
	0.7	8.6277600	5.911050	1.18269	2445.48
r	0.39	8.6327200	6.541720	1.51602	2448.78
	0.40	8.6278900	5.911180	1.51603	2445.47
	0.41	8.6230800	5.288090	1.51604	2442.18
σ	1.5	0.1520870	0.104286	1.51588	2407.58
	2.0	8.6278900	5.911180	1.51603	2445.47
	2.5	0.2424900	0.153509	1.51617	2483.36
x_2	0.99	0.2408500	0.165009	1.51603	2445.50
	1.00	8.6278900	5.911180	1.51603	2445.47

Continued on next page

Table 1 – continued from previous page

Parameters	Changes	$T \times 10^{17}$	$t_R \times 10^{14}$	ξ	TC
	1.10	31178.000	21365.40	1.51603	2445.52
x_1	95	0.1936110	0.141891	1.51618	2365.44
	100	8.6278900	5.911180	1.51603	2445.47
	105	9.1811700	5.905550	1.51589	2525.49
c	0.45	8.6037500	2.868200	1.51608	2429.20
	0.50	8.6278900	5.911180	1.51603	2445.47
	0.60	8.6780000	0.125695	1.51592	2479.39
y	0.5	9.0212200	5.874520	1.51592	2509.49
	0.6	8.6278900	5.911180	1.51603	2445.47
	0.7	0.1934640	0.139834	1.51615	2381.45
τ	1.8	0.0162475	0.011493	1.15651	2260.81
	2.0	8.6278900	5.911180	1.51603	2445.47
	2.5	0.00157429	0.0865315	1.51560	2748.23

4.3. Analysis. The following observations can be made from Table 1:

- When b and r increased, then the total cycle length T, t_R and preservation technology investment increases, and total cost shows no changes for b and for t_R total cost slightly decreases. Conversely, if b decreased then T and t_R slightly increased but preservation cost increased at high rate, and for r, T, TC and preservation cost were slightly affected but t_R was highly affected.
- If increment and decrement in θ then slightly change in all decision parameters.
- If increment and decrement in shortage parameter σ then T and t_R are sensitively increases but PTC and TC both minor affected and during decrement T and t_R both are at high level affected and PTC and TC slightly affected.
- Now we discussed about the holding cost parameter c: if increment and decrement in c then T, PTC and TC slightly affected but t_R sensitively affected.
- Further we discussed about the demand parameters: if increment and decrement in x_1 and x_2 then T, t_R and TC increased and decreased with highly sensitive rate.
- And increment and decrement in y then minor increment in PTC but in remaining parameters decrement and in decrement T and TC increased but other remaining decision parameters slightly decreased.
- And finally discussed about demand parameter τ: if increased and decreased the value of τ then during increment only TC increased but in remaining all decision parameters decreased.

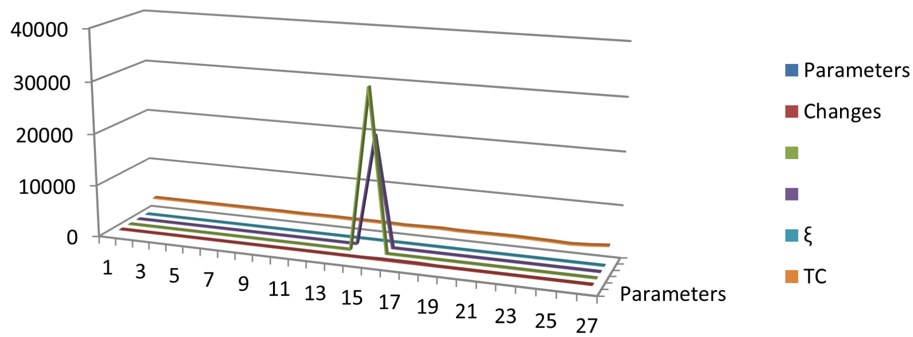

FIGURE 2. Sensitivity Analysis

Graphical Presentation for case $t_D < t_w$

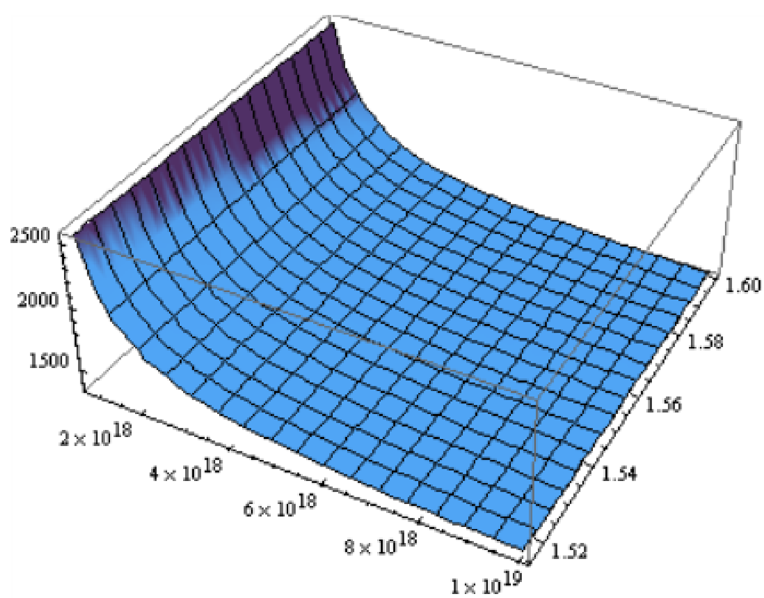

FIGURE 3. Optimality between T & ξ w.r.t TC

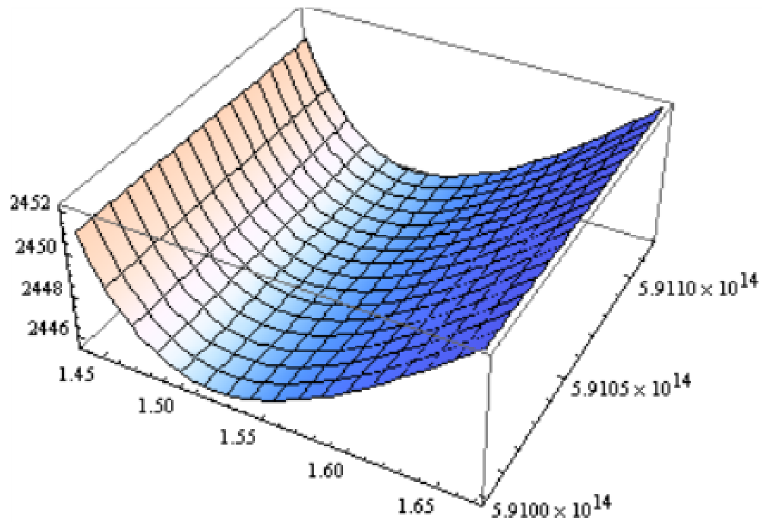

FIGURE 4. Optimality between t_R & ξ w.r.t TC

4.4. Case-2 when $t_D > t_W$. Now for the second case the governed differential equations for RW and OW at the time t over the interval $[0, T]$ are given as-

$$\frac{dI_0(t)}{dt} = -D(s), \qquad 0 \le t \le t_W \qquad (22) \tag{4.1}$$

$$\frac{dI_R(t)}{dt} = -D(s), \qquad t_W \le t \le t_D \qquad (23) \tag{4.2}$$

$$\frac{dI_R(t)}{dt} + a\tau_\theta I_R(t) = -D(s), \qquad t_D \le t \le t_R \qquad (24) \tag{4.3}$$

$$\frac{dB(t)}{dt} = D(s), \qquad t_R \le t \le T \qquad (25) \tag{4.4}$$

Solution of the equations (22) to (25) with the boundary conditions $I_0(0) = w$, $I_R(t_w) = s_F - w$, $I_R(t_R) = 0$, $B(t_R) = 0$.

$$I_0(t) = -D(s)t + w \qquad (26) \tag{4.5}$$

$$I_R(t) = -D(s)(t - t_R) + (s_F - w) \qquad (27) \tag{4.6}$$

$$I_R(t) = \frac{-D(s)}{a\tau_\theta}\left[-1 + e^{a\tau_\theta(t_R - t)}\right] \qquad (28) \tag{4.7}$$

$$B(t) = D(s)(t - t_R) \qquad (29) \tag{4.8}$$

Now at time $t = t_w$ when $I_0(t) = 0$, we get $t_w = \frac{w}{D(s)}$.
We considering continuity of $I_R(t)$ at time $t = t_D$

$$s_F = w + D(s)\left[(t_D - t_R) + \frac{1 - e^{a\tau_\theta(t_R - t_D)}}{a\tau_\theta}\right] \qquad (30) \tag{4.9}$$

The maximum quantity of demand backlogging per cycle is calculated as put $t = T$ in equation (29).

$$B(t) = D(s)(T - t_R) \qquad (31) \tag{4.10}$$

We determined the order quantity over the replenishment cycle

$$\begin{aligned} Q_F &= s_F + B(t) \\ Q_F &= D(s)(T - t_R) + w + D(s)\left[(t_D - t_R) + \frac{1 - e^{a\tau_\theta(t_R - t_D)}}{a\tau_\theta}\right] \qquad (32) \end{aligned}$$

4.5. Performance Measure for case $t_D > t_W$.

Ordering Cost.

$$OC = A \tag{4.11}$$

Holding Cost in OW.

$$HC_{OW} = h\left[\int_0^{t_W} I_o(t)e^{-rt}dt\right] \tag{4.12}$$

$$HC_{OW} = \frac{-h}{r}\left[D(s)\left(e^{-rt_W}\left(t_W + \frac{1}{r}\right) - \frac{1}{r}\right) - e^{-rt_W} + 1\right] \tag{4.13}$$

Holding Cost in RW.

$$\begin{aligned} HC_{RW} = f\Bigg[&\int_0^{t_W} I_R(t)e^{t(-r+r)}dt + \int_{t_W}^{t_D} I_R(t)e^{t(-r+r)}dt \\ &+ \int_{t_D}^{t_R} I_R(t)e^{t(-r+r)}dt\Bigg] \end{aligned} \tag{4.14}$$

$$\begin{aligned} HC_{RW} = f\Bigg[&\frac{1}{c-r}\left\{w\left(e^{t_w(-r+r)} - 1\right) + D(s)\left\{e^{t_w(-r+r)}\left(t_w + \frac{1}{-r+r}\right) - \frac{1}{-r+r}\right\}\right\} \\ &+ \frac{1}{c-r}\Bigg\{D(s)\left\{e^{t_w(-r+r)}\left(t_w + \frac{1}{-r+r}\right) - e^{t_D(-r+r)}\left(t_D + \frac{1}{-r+r}\right)\right. \\ &\left.+t_R\left(e^{t_D(-r+r)} - e^{t_W(-r+r)}\right)\right\} + e^{(t_D - t_w)(-r+r)}\Bigg\} \\ &-\frac{D(s)}{a\tau_\theta}\left\{\frac{1}{-r+r}\left(e^{t_D(-r+r)} - e^{t_R(-r+r)}\right) + \frac{1}{r-c-a\tau_\theta}\left(e^{t_R(-r+r)} - e^{-t_D(-r+r)+a\tau_\theta(t_R - t_D)}\right)\right\}\Bigg] \end{aligned} \tag{4.15}$$

Shortage Cost.

$$SC = \sigma\int_{t_R}^{T} S_t e^{-rt}dt \tag{4.16}$$

$$SC = \frac{\sigma D(s)e^{-rt}}{-r} \tag{4.17}$$

Deterioration Cost.

$$DC = C_p(Q_F - D(s)T) \tag{4.18}$$

$$DC = C_p(S_F - D(s)t_R) \tag{4.19}$$

4.6. Preservation technology cost.

$$PTC = \int_{t_D}^{t_R} \xi e^{-rt} dt \tag{4.20}$$

$$PTC = \frac{\xi}{-r}\left\{e^{-rt_R} - e^{-rt_D}\right\} \tag{4.21}$$

4.7. Total Cost.

$$TC = [OC + HC_{RW} + HC_{OW} + DC + PTC + SC] \tag{4.22}$$

5. Numerical Example for case $t_D > t_W$

We have taken the appropriate values of the following parameters with proper units to find the optimum values of the parameters and total cost which are as-

$a = 70$, $b = 0.2$, $c = 0.5$, $\theta = 0.09$, $f = 600$, $h = 5$, $A = 8$, $t_R = 220$, $x_1 = 100$, $x_2 = 1$, $C_p = 7$, $y = 1$, $s = 40$, $\tau = 1.5$, $\gamma = 0.01$, $r = 0.4$, $t_W = 9$, $w = 200$, $\sigma = 2$, $S_F = 34$.

Now the optimum values are-

$T = 4.57336 \times 10^6$, $t_D = 2186.91$, $\xi = 4.51893$, $TC = 117.406$.

6. Table-2 Sensitivity Analysis for case $t_D > t_W$

Parameters	Changes	$T \times 10^6$	t_D	ξ	TC
b	0.15	4.57394	2186.93	6.02524	117.4120
	0.20	4.57336	2186.91	4.51893	117.4060
	0.25	4.57300	2186.90	3.61514	117.4070
θ	0.085	4.57337	2186.91	4.54393	117.4060
	0.090	4.57336	2186.91	4.51893	117.4060
	0.095	4.57335	2186.91	4.49393	117.4060
c	0.45	7.41234	2188.53	4.51891	165.3720
	0.50	4.57336	2186.91	4.51893	117.4060
	0.60	3.34293	2181.30	4.51896	115.4768
r	0.39	5.52734	2220.88	4.51987	119.0110
	0.40	4.57336	2186.91	4.51893	117.4060
	0.41	3.52219	2151.57	4.51790	112.2130
w	180	4.61446	2187.17	4.51893	117.9160
	200	4.57336	2186.91	4.51893	117.4060
	220	4.53227	2186.65	4.51892	116.8960
f	590	3.86775	2162.69	4.51876	109.2460
	600	4.57336	2186.91	4.51893	117.4060
	610	5.28077	2210.22	4.57907	123.4110
h	4.5	6.62088	2320.11	4.51933	130.4800
	5.0	4.57336	2186.91	4.51893	117.4060
	5.5	2.98826	2068.88	4.51843	95.96640

6.1. Sensitivity Analysis. Increment and decrement in the inventory parameter b then slightly changes in the decision parameters T, t_D and total cost TC but preservation technology cost ξ is highly affected. We make changes in deterioration parameter θ then ξ is minor affected. In same manner increment and decrement in the holding cost parameter c then total cycle length T and total cost at high rate decrease and increased but ξ and t_D are minor affected.

Increment and decrement in the inflation parameter r then sensitively decrement and increment in all decision parameters. When increment and decrement in w then in all parameters slightly decrement and increment. Further increment and decrement in f then at high rate T, t_D, and TC increased but minor affected ξ. And during decrement all decision parameters at highly rate decreased.

Finally, increment and decrement in h then T, t_D, ξ and TC decreased sensitively and during decrement all decision parameters at high rate increased.

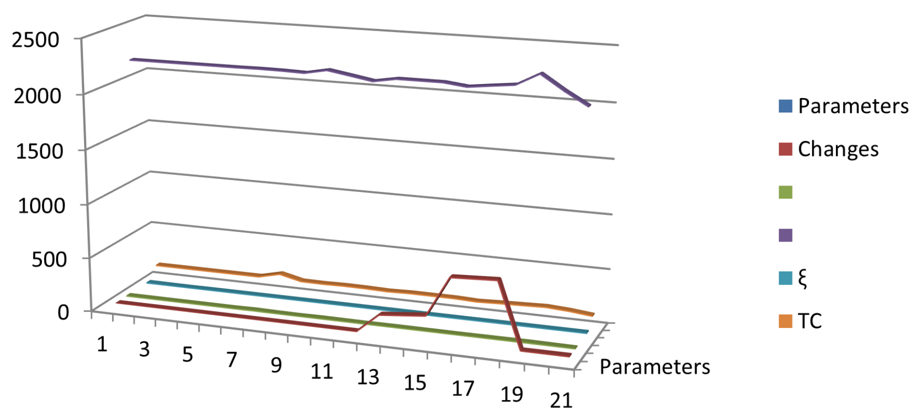

FIGURE 5. Sensitivity Analysis

Graphical Presentation for Case $t_D > t_W$

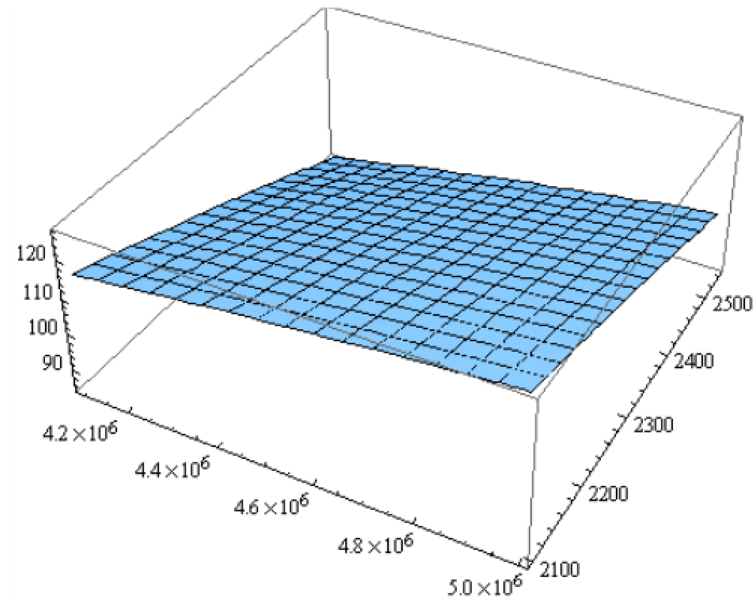

FIGURE 6. Optimality between t_D & T w.r.t TC

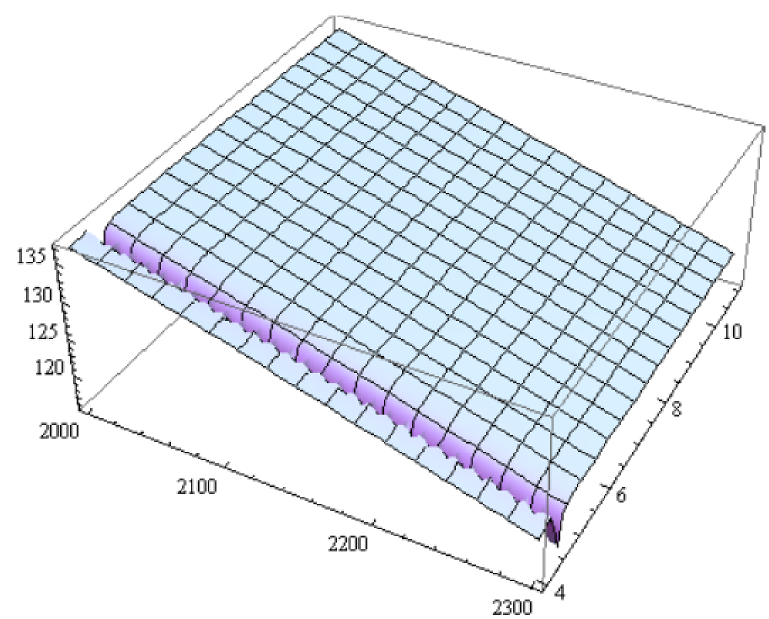

FIGURE 7. Optimality between t_D & ξ w.r.t TC

6.2. Managerial Insights.

- It is clear from the sensitivity analysis that preservation technology significantly affects the total cost in an inflationary environment. Managers need to optimize preservation technology investment to minimize total cost.
- In the present model, we consider selling price and stock-level dependent demand under inflationary conditions. Our analysis reveals that:
  - When inflation increases or decreases, preservation technology costs are only marginally affected
  - Other decision parameters are strongly influenced by inflation fluctuations
  - This highlights the need to compute optimal values for total cost under varying inflationary scenarios

Conclusion

We proposed a two-warehouse inventory model for non-instantaneous deteriorating items with selling price and stock level dependent demand. A hybrid demand function was used to show the variation in the demand parameters, and this variation was shown in the sensitivity analysis. Through the sensitivity analysis and graphs, we concluded that in an inflationary environment, preservation technology is minimally affected in both cases. However, in the inflationary environment, the total cost and other decision parameters are affected sensitively.

We also concluded that the shortage parameter causes minor changes in preservation technology, but other decision parameters are highly affected in the two-warehouse environment. Some numerical examples have been solved to validate the results. The sensitivity analysis shows the variation in the parameters, and the graphs demonstrate the optimality of the model.

The present work may be used to control the deterioration of non-instantaneous deteriorating items, e.g., milk, fruits, fish, meat, etc. This model may be extended in the future to include selling price-dependent demand, time-dependent demand, and fuzzy environments. Furthermore, the model can be extended with other inventory parameters, partially backlogging shortages, and many other realistic assumptions.

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