Global and Stochastic Analysis
Vol. 12 No. 4 (July, 2025)

Received: 05th May 2025 *Revised: 16th July 2025* *Accepted: 21st July 2025*

BAYES ESTIMATION AND BAYES RISK UNDER DIFFERENT LOSS FUNCTIONS

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Abstract. To analyze the performance of various estimators, this paper compares Bayesian and classical estimate approaches across a variety of loss functions. The work focuses on informative versus non-informative priors in Bayesian estimation. The calculations are based on a number of loss functions, including the Square Error Loss Function (SELF), Quadratic Loss Function (QLF), Precautionary Loss Function (PLF), and Entropy Loss Function (ELF). The methodology uses simulation techniques as well as real-world datasets to test the performance and robustness of the proposed estimators. The Bayesian estimators, classical estimators, and related Bayes hazards using empirical analysis and extensive simulations to computed. The findings show that the Minimum Mean Square Error estimator (MiniMSE) is more accurate and reliable than other estimators in a variety of settings. Furthermore, for both types of priors, it is demonstrated that the Bayes risk under the Quadratic Loss Function (QLF) is the lowest among all loss functions considered, including SELF, PLF, and ELF. This suggests that, when compared to other loss functions, QLF is a more effective and balanced criterion for making estimation decisions. The study's findings provide vital new insights into how to choose the optimum estimate techniques and loss functions for statistical decision theory and practical data processing.

1. Introduction

Statistical inference plays a critical role in the decision-making process, particularly in the estimation of unknown parameters based on observed data. Among the various approaches to statistical estimation, Bayesian inference has emerged as a powerful and flexible framework. Unlike classical methods, which rely solely on the observed data, Bayesian methods incorporate prior knowledge or beliefs about the parameters using prior distributions. This integration of prior and likelihood information enables more robust and adaptive estimation, particularly in situations involving uncertainty or limited data.

Bayesian estimation seeks to minimize the expected loss, leading to the concept of Bayes estimators. These estimators rely on the selection of a loss function, which measures the penalty paid for differences between the estimated and true values, in addition to the shape of the prior distribution. Divergent perspectives on choice risk and estimating mistakes are reflected in several loss functions. The Squared Error Loss Function (SELF), Quadratic Loss Function (QLF), Precautionary Loss

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2000 *Mathematics Subject Classification.* 62C10, 62F15, 62C12 58E17.
Key words and phrases. Bayes Estimator, Bayes Risk, Classical Estimation, Informative and noninformative prior, Loss function and Pareto distribution.

Function (PLF), and Entropy Loss Function (ELF) are often employed loss functions. Different Bayes estimators and related hazards result from each of these functions, which each reflect a distinct viewpoint on punishing estimate mistakes. A key metric for assessing an estimator's performance is the Bayes risk, which is the smallest predicted loss in relation to the posterior distribution. The Bayes risk and, thus, the estimation's effectiveness are directly impacted by the choice of suitable priors (informative or non-informative) and loss functions. This study uses both informative and non-informative priorities to examine how well Bayesian and classical estimators perform under different loss functions. The study evaluates the effectiveness of estimators in terms of their mean square errors and Bayes hazards using simulated methods and real-world data applications. This research finds the best estimate methods and loss functions for real-world inference tasks by comparing estimators across several decision frameworks.

2. Preliminaries

It is suggested that the Pareto distribution be estimated using the Bayesian and classical methods. In order to analyze stock prices and instability, this model is frequently used in business and economics, biological research, insurance firms, population migration, survival time in a quadratic system, geophysical phenomena in society, dependability, and life testing. Al-Kutubi, Hadecelsalim, Noor Akmaibrhim, and Al Omari Ahmed (2010) examined the distinctions between Bayesian estimation and maximum likelihood. Al Omari Mohammed Ahmed and Noor Akmaibrhim (2011) used a non-informative prior and right-censored data to examine the effectiveness of Bayes' survival function and MLE. Sankudey and Sudhansu A. Maiti (2012) used extended Jeffrey's priors with both symmetric and asymmetric loss functions to study the Bayes estimators of the Rayleigh parameter, and the risk associated with it. In 2015, R.K. Radha used an informative prior to study the Bayesian analysis of exponential distributions. The Bayes estimate of the shape parameter of the exponentiated moment exponential distribution with informative and non-informative priors under different loss functions was investigated by Kawsar Fatima and S.P. Ahmad (2018). The fundamental ideas and mathematical formulas required to comprehend Bayes estimate and Bayes risk under various loss functions are presented in this section. The framework for the creation and evaluation of the estimators employed in this investigation is established by these initial phases.

2.1. Bayesian Estimation.

Assume X is the observed data with the likelihood function f(x—), where is an unknown parameter associated to a statistical model. The Bayesian framework treats as a random variable by using the prior distribution (), which represents preexisting views or information about before data observation.

The Bayes theorem provides the posterior distribution of θ after seeing data x :

$$\pi(\theta \mid x) = [f(x \mid \theta)\pi(\theta)] / \left[\int f(x \mid \theta)\pi(\theta)d\theta \right]$$

The value that minimizes the posterior anticipated loss is the Bayes estimator $\delta(\mathrm{x})$ of θ :

$$\delta(\mathrm{x}) = \mathrm{argmin}_{\mathrm{a}} \int \mathrm{L}(\theta, \mathrm{a})\pi(\theta \mid \mathrm{x})\mathrm{d}\theta$$

2.2. Loss Functions.

The shape of the Bayes estimator is determined by the choice of loss function $\mathrm{L}(\theta, \mathrm{a})$. Typical loss functions consist of:
SELF(Squared Error Loss Function) : $\mathrm{L}(\theta, a) = (\theta - a)^2$
Bayes estimator: posterior mean $\mathrm{E}[\theta \mid \mathrm{x}]$
QLF (Quadratic Loss Function): $\mathrm{L}(\theta, \mathrm{a}) = \mathrm{w}(\theta)(\theta - \mathrm{a})^2$
PLF(Precautionary Loss Function) : $\mathrm{L}(\theta, \mathrm{a}) = (\theta - \mathrm{a})^2 * \exp(\lambda(\theta - \mathrm{a}))$
ELF(Entropy Loss Function): $\mathrm{L}(\theta, \mathrm{a}) = (\mathrm{a}/\theta - \log(\mathrm{a}/\theta) - 1)$

2.3. Bayes Risk.

The Bayes risk of an estimator $\delta(\mathrm{x})$ is the expected loss averaged over both the data and the prior distribution:

$$\mathrm{r}(\pi, \delta) = \iint \mathrm{L}(\theta, \delta(\mathrm{x}))\mathrm{f}(\mathrm{x} \mid \theta)\mathrm{d}\mathrm{x}\pi(\theta)\mathrm{d}\theta$$

2.4. Priors.

Non-informative priors, like Jeffreys or uniform priors, reflect minimal past knowledge. Informative Priors: Include a significant amount of prior information based on historical data or professional judgment. Umesh Chandra, Vinod Kumar, and Gaurav Shukla developed and tested the risk function expression under three different loss functions in 2020. The lack of progress in building proper Bayesian inference tools is striking. Bayesian inference for Pareto type I distributions with known scale parameters.

The pdf of Pareto type - I distribution is defined as

$$\mathrm{f}(\mathrm{t}; \alpha, \theta) = \begin{cases} \frac{\theta\alpha^\theta}{t^{\theta+1}}; \mathrm{t} > \alpha; \theta > 0; \alpha > 0 \end{cases} \tag{2.1}$$

where θ represents the shape parameter, α is the known scale parameter, and t is a random variable. The moments of the Pareto type I distribution were supplied by

Mean,

$$\mathrm{E}(\mathrm{t}) = \frac{\alpha\theta}{\theta - 1}; \theta > 1$$

Variance,

$$\mathrm{V}(\mathrm{t}) = \frac{\theta\alpha^2}{(\theta - 1)^2(\theta - 2)}; \theta > 2$$

3. Classical Estimation

Classical estimation is an important estimation techique in statistics. In this section a specfic method of estimation such as Maximaum Liklihood Estimation(MLE),Uniformly Minimum Variance Unbiased Eestimation(UMVUE), MiniMean Sequare Error Estimation (MiniMSE), are considered to estimate the shape parameter of Parato model proper in the study

3.1. Maximum Likelihood Estimator. Let $t_1, t_2, \ldots..t_n$ be a collection of n random variables taken from a Pareto Type I distribution, using the likelihood function, parameters θ and α, and the probability density function.

$$\mathrm{L} = \prod_{i=1}^{n} \mathrm{f}\left(t_i; \alpha, \theta\right)$$

In case of frequency distribution

$$\widehat{\theta} = \left[\frac{\sum_{i=1}^{n} f_i \log t_i}{N} - \log \alpha\right]^{-1} \tag{3.1}$$

3.2. Exponential family and Uniformly Minimum Variance Unbiased Estimator.

The distribution has the following general form for its density functions is $f(t, \theta) = a(\theta) \cdot b(t) e^{[c(\theta) d(t)]}$ is defined as an exponential family of distributions with a single parameter. The Pareto distribution, which has a density function and belongs to the exponential distribution family, can be expressed as,

$$\begin{aligned} \mathrm{f}(\mathrm{t}, \theta) &= \theta e^{\theta \log \alpha - (\theta+1) \log_e t} \\ &= \theta e^{\theta \log_e \alpha - \theta \log_e t - \log_e t} \\ &= \theta e^{-\log_e \mathrm{t}} e^{-\theta \log_e (t/\alpha)} \end{aligned}$$

$$\text{Where, } \mathrm{a}(\theta) = \theta, \mathrm{b}(\mathrm{t}) = e^{\log_e t}, \quad \mathrm{c}(\theta) = -\theta, \mathrm{d}(\mathrm{t}) = \log_e \left(\frac{t}{\alpha}\right)$$

Therefore, the given statistic is a complete sufficient statistic for θ of

$$\mathrm{P} = \textstyle\sum_{i=1}^{n} \log_e \left(\frac{t_i}{\alpha}\right).$$

With parameters n and θ, it is simple to demonstrate that the statistic P is distributed as a Gamma distribution.

If T $\sim$ Pareto (α, θ), then $\log_e(t/\alpha) \sim$ exponential(θ) and
$\mathrm{P} = \sum_{i=1}^{n} \log_e \left(\frac{t_i}{\alpha}\right) \sim$ Gamma(n, θ) with pdf

$$\mathrm{g(p)} = \frac{\theta^n}{\Gamma n} p^{n-1} e^{-\theta \mathrm{p}}; \quad \mathrm{p} \geq 0, \quad \theta > 0$$

Let us consider,

$$\mathrm{E}\left(\frac{1}{p}\right) = \int_0^\infty \frac{1}{p} g(p) dp \tag{3.2}$$

$$= \int_0^\infty \frac{1}{p} \frac{\theta^n}{\ulcorner n} p^{n-1} e^{-\theta \mathrm{p}} dp \tag{3.3}$$

$$= \frac{\theta^n}{\ulcorner n} \frac{\ulcorner n-1}{\theta^{n-1}} \tag{3.4}$$

$$\mathrm{E}\left(\frac{1}{p}\right) = \frac{\theta}{n-1} \tag{3.5}$$

$$\mathrm{E}\left(\frac{n-1}{p}\right) = \theta \tag{3.6}$$

$\frac{n-1}{p}$ is a P is an objective estimate of θ, and P is the whole statistic for θ. The Lehmann-Scheffe theorem is used to get the UMVUE of θ, which is

$$\hat{\theta}_{UMVUE} = \frac{n-1}{p} \tag{3.7}$$

The class of estimators of the type includes the Minimum Mean Squared Error estimator (MinMSE) is $\frac{c}{p}$.

$$\hat{\theta}_{\text{MinMSE}} = \frac{n-2}{p} \tag{3.8}$$

$$\mathrm{MSE}\left(\hat{\theta}_{MLE}\right) = \frac{\theta^2(n+2)}{(n-1)(n-2)} \tag{3.9}$$

$$\mathrm{MSE}\left(\hat{\theta}_{UMVUE}\right) = \frac{\theta^2}{(n-2)} \tag{3.10}$$

$$\mathrm{MSE}\left(\hat{\theta}_{MinMSE}\right) = \frac{\theta^2}{(n-1)} \tag{3.11}$$

$$\tag{3.12}$$

Therefor the combination is

$$\mathrm{MSE}\left(\hat{\theta}_{MinMSE}\right) \leq \mathrm{MSE}\left(\hat{\theta}_{UMVUE}\right) \leq \mathrm{MSE}\left(\hat{\theta}_{MLE}\right)$$

It has been demonstrated that the Uniformly Minimum Variance Unbiased Estimator (UMVUE) and Maximum Likelihood Estimator (MLE) are less accurate than the Minimum Mean Squared Error (MinMSE).

4. Bayesian Estimation and Bayes Risk using Non - informative prior

The Bayesian estimation approach minimizes the estimated loss for all X observations by estimating an unknown parameter, θ. The Bayes approach is an average case analysis since it calculates an estimator's average risk for each parameter in the distribution under study. The average risk is defined as follows, given the prior probability distribution π on the parameter space ω :

$$R_\pi(\hat{\theta}) = E_{\theta,x}[L(\theta,\hat{\theta})]$$

Thus, the lowest that the average risk may reach is the Bayes risk for a previous π.

$$\hat{R}_\pi = {}_{\theta}^{inf}\left[R_\pi(\hat{\theta})\right]$$

The non-informative prior can be used in Bayesian analysis when there is no prior information of the parameter. Because we do not know the parameters, we may utilize Jeffrey's prior, which is the square root of the Fisher information matrix for each observation.

The Jeffrey's prior is defined as

$$\mathrm{g}_1(\theta) \propto \sqrt{\mathrm{I}(\theta)} = b\sqrt{I(\theta)}$$

Where $I(\theta) = -nE\left(\frac{\partial^2 \log L}{\partial\theta^2}\right)$
Therefore

$$g(\theta) = b\sqrt{-nE\left(\frac{d^2 \log L}{d\theta^2}\right)}$$
$$\log L = \theta \log\alpha - (\theta+1)\log x$$
$$\frac{d\log L}{d\theta} = \frac{1}{\theta} + \log\alpha - \log x$$
$$\frac{d^2\log L}{d\theta^2} = -\frac{1}{\theta^2}$$
$$E\left(\frac{d^2\log L}{d\theta^2}\right) = -\frac{1}{\theta^2}$$
$$\mathrm{g}(\theta) = \frac{b}{\theta}\sqrt{n}, \theta > 0$$

Assuming that $t_1, t_2, \ldots.., t_n$ be the n independent observation which follows the Pareto Type I Distribution with probability density function, given in(1.1) the value of α is known and θ is the only unknown parameter, we shall obtain the posterior probability density function for θ using Jeffrey's prior distribution.

Posterior probability density (pdf) function for θ is

$$h\left(\theta/t_1, t_2, \ldots., t_n\right) = \frac{\mathrm{g}(\theta)L\left(\theta; t_1, t_2, \ldots., t_n\right)}{\int_0^\infty \mathrm{g}(\theta)L\left(\theta; t_1, t_2, \ldots., t_n\right)d\theta}$$
$$= \frac{\left(\frac{1}{\theta}b\sqrt{n}.\theta^n e^{n\theta\log\alpha}e^{-(\theta+1)\Sigma\log x}\right)}{\int_0^\infty \frac{1}{\theta}b\sqrt{n}.\theta^n e^{n\theta\log\alpha}e^{-(\theta+1)\Sigma\log x}d\theta} \text{ using(2.1.1)}$$
$$= \frac{\left(\theta^{-1}\theta^n e^{n\theta\log\alpha-(\theta+1)\Sigma\log x}\right)}{\int_0^\infty \theta^{-1}\theta^n e^{n\theta\log\alpha-(\theta+1)\Sigma\log x}d\theta}$$
$$= \frac{\left(\theta^{n-1}e^{-\theta(-n\log\alpha+\Sigma\log x}\right)}{\int_0^\infty \theta^{n-1}e^{-\theta(-n\log\alpha+\Sigma\log x}d\theta}$$

Where,

$$p = \sum_{i=1}^{n} \log(t_i/\alpha)$$

$$= \frac{\left(\theta^{n-1} e^{-\theta P}\right)}{\int_0^\infty \theta^{n-1} e^{-\theta P} d\theta}$$

$$h(\theta/t_1, t_2, \ldots, t_n) = \frac{P^n}{\Gamma n} \theta^{n-1} e^{-\theta p}$$

The posterior density function of Jeffreys prior is
$h(\theta/t_1, t_2, \ldots, t_n) = \frac{P^n}{\Gamma n} \theta^{n-1} e^{-\theta p}$

5. Experimental Study

Experiment-1

We employed sample sizes of n = 25, 50, and 100 to represent the small, median, and large data sets in this investigation. The simulation results for the traditional estimate of the Pareto type I distribution's shape parameter using MLE, UMVUE, and MinMSE are displayed in Table 1.

TABLE 1. Classical estimation of the shape parameter

Distribution	n	shape = 0.1					
		scale = 0.2			scale = 0.4		
		MLE	UMVUE	MINMSE	MLE	UMVUE	MINMSE
PARETO	25	0.005	0.004	0.001	0.019	0.018	0.003
	50	0.001	0.001	0.001	0.007	0.006	0.001
	100	0.001	0.0018	0.001	0.003	0.003	0.001

Form the table 1, it is observed that MiniMSE is the best among the other proposed estimators such as MLE and UMVUE.

Experiment-2

Risks and Bayesian estimates for the Pareto type-I distribution's form parameter. To represent small, medium, and big data sets, we utilize samples of n = 25, 50, 75, and 100. The informative prior (Exponential prior) and non-informative prior (Jefferey's prior) are used to calculate the Bayes risk of the shape parameter for the Pareto type-I distribution with different loss functions. The shape parameter values are $\theta = 1, 2, 3$ and $\alpha = 4, 5, 6$.

TABLE 2. Bayes Estimation under Square Error Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	θ								
		$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	2.48	2.46	2.11	1.96	1.95	1.94	1.93	1.92	1.91
	50	2.32	2.32	2.11	1.95	1.94	1.93	1.92	1.91	1.90
	75	1.98	1.97	1.96	1.94	1.93	1.92	1.91	1.90	1.89
	100	1.97	1.96	1.95	1.93	1.92	1.91	1.90	1.89	1.88
Exponential	25	2.31	2.31	2.10	1.95	1.94	1.93	1.92	1.91	1.90
	50	2.22	2.21	2.00	1.94	1.93	1.92	1.91	1.90	1.88
	75	1.97	1.96	1.93	1.93	1.92	1.91	1.90	1.89	1.87
	100	1.96	1.95	1.92	1.92	1.91	1.90	1.90	1.89	1.86

TABLE 3. Bayes Risk under Square Error Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	θ								
		$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	1.54	1.52	0.98	0.97	0.98	0.97	1.10	0.90	0.89
	50	1.49	1.48	0.98	0.96	0.98	0.97	1.09	0.89	0.88
	75	0.98	0.98	0.96	0.99	0.96	0.95	0.98	0.87	0.84
	100	0.97	0.96	0.95	0.95	0.95	0.94	0.97	0.86	0.83
Exponential	25	1.42	1.51	0.97	0.96	0.95	0.94	1.10	0.80	0.82
	50	1.41	1.48	0.96	0.95	0.94	0.93	1.08	0.72	0.81
	75	0.98	0.96	0.94	0.93	0.93	0.92	0.97	0.73	0.73
	100	0.96	0.95	0.94	0.93	0.92	0.90	0.92	0.72	0.72

Experiment-3 Bayes Estimation and Bayes Risk of the Shape parameter (Pareto model) by using real life problem Under various loss functions, informative (Exponential prior) and non-informative (Jeffrey's) priors are used to estimate the Bayes risk of the shape parameter for the Pareto type-I distribution. The survival data are displayed in tables and utilized to estimate the Bayes risk and Bayes estimation of the Pareto Type-I distribution's shape parameter.

In Table 10 denoted U-up:Year of follow-up, NAI:Number alive at beginning of interval and NDI: Number dying interval.

TABLE 4. Bayes Estimation under Quadratic Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	θ								
		$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	1.42	1.31	1.30	1.28	1.26	1.25	1.27	1.25	1.25
	50	1.40	1.28	1.27	1.26	1.24	1.25	1.26	1.24	1.24
	75	1.21	1.11	1.10	1.09	1.01	1.01	1.09	1.01	1.01
	100	0.98	0.98	0.97	0.95	0.94	0.93	0.94	0.93	0.92
Exponential	25	1.31	1.28	1.28	1.26	1.25	1.24	1.26	1.25	1.23
	50	1.30	1.12	1.27	1.25	1.22	1.21	1.25	1.24	1.22
	75	1.18	1.10	1.10	1.00	1.01	1.01	1.00	1.01	1.00
	100	0.98	0.97	0.96	0.94	0.93	0.93	0.93	0.92	0.91

TABLE 5. Bayes Risk under Quadratic Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	0.82	0.79	0.77	0.69	0.65	0.62	0.61	0.59	0.42
	50	0.80	0.78	0.76	0.68	0.62	0.61	0.59	0.57	0.41
	75	0.70	0.70	0.69	0.67	0.61	0.59	0.58	0.56	0.40
	100	0.69	0.68	0.67	0.66	0.59	0.58	0.57	0.55	0.40
Exponential	25	0.73	0.73	0.71	0.60	0.60	0.59	0.59	0.58	0.40
	50	0.72	0.72	0.70	0.68	0.60	0.58	0.57	0.56	0.39
	75	0.61	0.70	0.67	0.65	0.58	0.57	0.55	0.54	0.38
	100	0.60	0.67	0.67	0.65	0.57	0.57	0.54	0.53	0.37

TABLE 6. Bayes Estimation under Precautionary Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	2.98	1.98	1.97	1.96	1.96	1.95	1.94	1.92	1.90
	50	1.99	1.97	1.96	1.95	1.94	1.93	1.92	1.91	1.89
	75	1.97	1.96	1.95	1.93	1.93	1.92	1.90	1.89	1.87
	100	1.82	1.80	1.80	1.71	1.80	1.80	1.79	1.88	1.87
Exponential	25	2.16	1.98	1.96	1.95	1.95	1.94	1.92	1.90	1.89
	50	1.99	1.97	1.95	1.93	1.93	1.92	1.91	1.89	1.88
	75	1.92	1.95	1.94	1.92	1.92	1.91	1.89	1.88	1.88
	100	1.78	1.80	1.80	1.80	2.00	1.90	1.88	1.88	1.87

TABLE 7. Bayes Risk under Precautionary Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	1.41	1.21	0.98	0.96	0.92	0.90	0.89	0.79	0.76
	50	1.21	1.09	0.97	0.95	0.90	0.89	0.88	0.78	0.75
	75	0.98	0.97	0.95	0.92	0.89	0.88	0.87	0.78	0.74
	100	0.97	0.96	0.94	0.90	0.88	0.87	0.82	0.76	0.73
Exponential	25	1.21	1.20	0.97	0.96	0.89	0.87	0.87	0.73	0.75
	50	1.12	1.01	0.96	0.95	0.89	0.88	0.86	0.72	0.74
	75	0.97	0.96	0.95	0.94	0.88	0.87	0.85	0.70	0.73
	100	0.96	0.95	0.94	0.93	0.87	0.86	0.84	0.69	0.72

Experiment-4

Classical Estimation of the Shape parameter of Pareto Type-I distribution

TABLE 8. Bayes Estimation under Entropy Loss Function for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	2.92	2.90	2.89	2.68	2.52	2.46	2.48	2.47	2.45
	50	2.90	2.90	2.79	2.58	2.48	2.32	2.47	2.46	2.12
	75	2.89	2.79	2.68	2.42	2.33	2.31	2.30	2.30	2.28
	100	1.98	1.97	1.96	1.95	1.94	1.93	1.92	1.91	1.90
Exponential	25	2.68	2.89	2.52	2.46	2.48	2.43	2.31	2.33	2.33
	50	2.52	2.79	2.46	2.52	2.46	2.30	2.20	2.12	2.10
	75	2.56	2.68	2.32	2.40	2.28	2.10	2.00	2.00	1.99
	100	1.96	1.96	1.95	1.94	1.93	1.92	1.90	1.92	1.91

TABLE 9. Bayes Risk under Entropy Loss for different values of $\alpha = 4, 5\&6$, $\theta = 1, 2\&3$.

Prior	n	θ								
		$\theta = 1$			$\theta = 2$			$\theta = 3$		
		4	5	6	4	5	6	4	5	6
Jeffrey's	25	1.92	1.90	1.90	1.91	1.90	0.98	0.97	0.95	0.94
	50	1.98	1.97	1.96	1.90	1.89	0.97	0.96	0.94	0.93
	75	0.99	0.98	0.97	0.89	0.98	0.96	0.95	0.93	0.92
	100	0.98	0.97	0.96	0.88	0.87	0.95	0.94	0.92	0.90
Exponential	25	0.98	0.96	0.89	0.88	0.87	0.86	0.85	0.84	0.83
	50	0.97	0.95	0.88	0.87	0.86	0.85	0.84	0.83	0.82
	75	0.96	0.94	0.87	0.86	0.85	0.84	0.83	0.82	0.81
	100	0.95	0.93	0.86	0.85	0.84	0.83	0.82	0.81	0.80

Table 10. Survival time distribution

U-up	NAI	NDI
0–1	1100	240
1–2	860	180
2–3	680	184
3–4	496	138
4–5	358	118
5–6	240	60
6–7	180	52
7–8	128	44
8–9	84	32
≥ 9	52	28

Table 11. Bayes Estimation and Bayes Risk of the Shape parameter

Prior	SELF		QELF		PELF		ELF	
	BE	BR	BE	BR	**BE**	BR	BE	BR
Jeffreys	0.49	0.39	0.39	0.75	0.51	0.71	0.44	0.65
Exponential	0.53	0.25	0.43	0.33	0.43	0.66	0.48	0.56

Table 12. The Classical Estimation of the Shape parameter of Pareto Type-I distribution

Classical Estimation			
	MLE	UMVUE	MinMSE
$\hat{\theta}$	0.1874	0.1405	0.1261

6. Conclusions

This work calculated the form parameter of the Pareto Type-I distribution utilizing traditional estimating techniques such as MLE, UMVUE, and MiniMSE. Informative and noninformative priors were utilized to calculate the Bayes risk of the shape parameter of the Pareto Type-I distribution under various loss functions in both simulation and real-world settings. The MiniMSE beats the MLE and UMVUE estimators when compared to the conventional estimate. The shape parameter has the lowest Bayes risk under QLF when an informative prior is used instead of a non-informative one, both in simulation and in real-world scenarios.

Furthermore, it has been discovered that the simulation approach reduces Bayes risk as sample size increases. Finally, MiniMSE outperforms every other classical estimator that is currently available. Furthermore, it is demonstrated that the Pareto type-I model with informative prior Bayes risk for the shape parameter under the quadratic loss function outperforms simulation and real-world problems.

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