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HAMMING DISTANCE OF A SEMIGRAPH

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Abstract. Suppose that $A(G)$ represents the adjacency matrix of a graph. Let $s(v)$ represent the row elements of $A(G)$ that correspond to vertex v of G. The number of places where the elements of the strings $s(u)$ and $s(v)$ differ from one another is known as the Hamming distance between u and v. The total sum of all Hamming distances between every pair of strings is the graph's hamming index. A semigraph G is a generalization of a graph G. In a semigraph, an edge can contain more than two vertices. The hamming distance and hamming index of a semigraph G are defined in this article. Also, we determine the hamming distance and hamming index of some classes of semigraph G generated by $A(G)$.

1. Introduction

Any number of errors up to a certain degree at the receiving end can be removed and/or fixed in a representation theory of information in binary form without extra information being presented. The encoder will convert the input message, which is made up of a string of letters, characters, or symbols from one set, one at a time, into a string of characters or symbols from another set one to one passion. Hamming distance is one of the strategies used to encode an input message. For the binary channel, the encoder will convert a message's input into a binary string consist of the set's symbols 0 and 1 from the set $\mathbb{Z}_2 = \{0, 1\}$.

The set $\mathbb{Z}_2$ is a group under binary operation $\bigoplus$ addition modulo 2. For any positive integer n,

$$\begin{aligned}\mathbb{Z}_2^n &= \mathbb{Z}_2 \times \mathbb{Z}_2 \times \mathbb{Z}_2 \times ... \times \mathbb{Z}_2 (n \ factors)\\ &= \{x_1 x_2 ... x_n \mid x_1, x_2, ..., x_n\}\end{aligned}$$

Therefore, let $x = x_1x_2...x_n$ where every x_i is either 0 or 1 and is know as a word or string denoted by $s(x)$. The number of 1's (or sum of all 1's) in a string x is called the weight of x, represented as $wt(x)$. If $x = x_1x_2...x_n$ and $y = y_1y_2...y_n$ are two strings, then the $wt(x + y)$(weight of x and y) is computed by adding the corresponding components of $x \ and \ y$ under addition modulo 2. That is $x_i+y_i = 0$ if $x_i = y_i$ and $x_i + y_i = 1$ if $x_i \neq y_i$ for $i = 1, 2, 3, ..., n$ and it is called hamming distance between two strings x and y (also it is the number of $i's$ such that $x_i \neq y_i$, $1 \leq i \leq n$) denoted by $H_d(x, y) = wt(x + y)$[1].The total sum of the Hamming distances between all string pairs produced by the adjacency matrix of a graph G

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is referred to as Hamming index of G, denoted by $H_A(G)$. Let $s(v_i)$ and $s(v_j)$ be the strings of vertices v_i and v_j generated by $A(G)$. Then

$$H_A(G) = \sum_{1 \le i < j \le n} H_d\big(s(v_i), s(v_j)\big)$$

If each vertex $v \in V(G)$ can be labeled by a string $s(v)$ of a particular length such that $H_d(s(u), s(v)) = d_G(u, v)$ for all $u, v \in V(G)$, where $d_G(u, v)$ is the length of the shortest path connecting u and v in G,

A semigraph is a generalization of a graph. E. Sampath Kumar [3] proposed the idea of a semigraph. According to Frank Harrary, an edge is a 2-tuple of vertices in a graph that satisfies the condition that two edges, (a, b) and (a', b'), are equivalent if and only if either $a = a'$ and $b = b'$ or $a = b'$ and $b = a'$. Using this concept, E. Sampath Kumar defined a semigraph as a pair (V, X), where V is a non-empty set whose components are known as the vertices of G and X is a set of n-tuples known as the edges of G of different vertices, for various $n \ge 2$ matching the conditions:

SG-1 Any two edges of G can have at most one vertex in common.

SG-2 Two edges $(a_1, a_2, a_3, ..., a_p)$ and $(b_1, b_2, b_3, ..., b_q)$ are said to be equal if and only if,
- Number of vertices in both edges must be equal, i.e $p = q$.
- Either $a_i = b_i$ for $1 \le i \le p$ or $a_i = b_{p-i+1}$, $1 \le i \le p$.

If $E_1 = (u_1, u_2, ..., u_k)$ and $E_2 = (u_k, u_{k-1}, ..., u_1)$ are two edges, then by $SG - II$, it is noted that $E_1 = E_2$. The size of an edge is denoted by $\mid E \mid$ is the number of vertices in an edge E.

If $E = (v_1, v_2, v_3, ..., v_n)$ is an edge of a semigraph $G = (V, X)$, then we call v_1 and v_n end vertices, v_i, $2 \le i \le n - 1$ as middle vertices and if a vertex is middle vertex in one edge and end vertex in another edge, then it is called middle end vertex. In a semigraph G, if two vertices are in same edge, and consecutive in order, then they are adjacent and consecutive adjacent respectively.

A subedge of an edge $E = (v_1, v_2, v_3, ..., v_n)$ is a k-tuple $E' = (v_{i_1}, v_{i_2}, ..., v_{i_k})$, $1 \le i_1 < i_2 < ... < i_k \le n$. We say that E' is the subedge induced by the set of vertices $\{v_{i_1}, v_{i_2}, ..., v_{i_k}\}$. A partial edge of E is a $(j - i + 1)$ tuple $E(v_i, v_j) = (v_i, v_{i+1}, v_{i+2}, ..., v_j)$, where $1 \le i \le n$.

A semigraph is complete if any two of its vertices are adjacent, and it is strongly complete if every vertex is the end vertex of an edge. An edge with cardinality n forms a complete semigraph on n vertices, and this edge is denoted by the notation E_n^c. The strongly complete semigraph with one edge of cardinality $n - 1$ and all other edges of cardinality 2 is denoted as T_{n-1}^1. If the cardinality of each edge in a semigraph G is r, then the semigraph is said to be $r - uniform$. We obtain a semigraph that is $(m + 2) - uniform$ and represented by $C_{n,m}$ by adding m number of middle vertices to each edge of the graph C_n, where C_n is the cycle with n vertices. Similarly, we obtain a semigraph that is $(m + 2) - uniform$ and denote it by $K_{n,m}$ by adding m number of middle vertices to each edge of the graph K_n, where K_n is the full graph with n vertices. More generally, given a graph G, we may create a semigraph that is $(m + 2) - uniform$ by adding m middle vertices to each edge of the graph G.

Example 1.1. Consider a semigraph G with vertex set V=$\{v_1, v_2, v_3, ..., v_8\}$ and edges $E_1 = \{v_1, v_2, v_3, v_5\}$, $E_2 = \{v_1, v_6\}$, $E_3 = \{v_2, v_8, v_7\}$, $E_5 = \{v_6, v_8, v_3, v_4\}$ as shown in Figure 1.

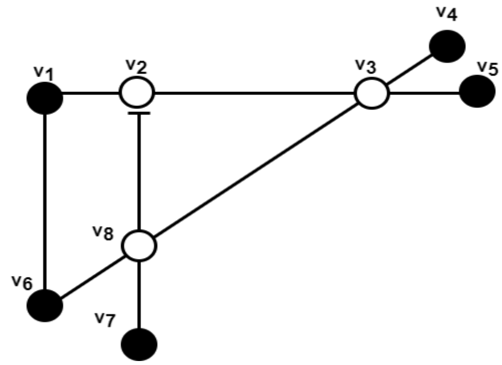

FIGURE 1. Semigraph $G(V, X)$

It is observed that v_1, v_4, v_5, v_6, v_7 are end vertices, v_3 and v_8 are middle vertices and v_2 is middle end vertex of G. Among the edges E_1 and E_5, the sets $\{v_1, v_2, v_5\}$ and $\{v_6, v_3, v_4\}$ are subedges. Also, the set $\{v_8, v_3, v_4\}$ is a partial edge of E_5.

Gaidhani Y.S. et al.[4] have defined adjacency matrix of semigraphs. For a semigraph $G(V, X)$ with a vertex set $V = \{v_1, v_2, v_3, ..., v_n\}$ and edge set $X = \{E_1, E_2, ..., E_m\}$, an adjacency matrix $A(G) = [a_{ij}]$ of G is a square matrix of order m, defined as follows,

(i) For every edge $E_i = (v_1, v_2, ..., v_k)$, if $v_r \in E_i$ then
  (a) $a_{v_1 v_r} = r - 1$
  (b) $a_{v_k v_r} = k - r$, for $r = 1, 2, 3, ..., k$
(ii) All the remaining entries of $A(G)$ are zero.

Example 1.2. The adjacency matrix of a semigraph in Fig.1 is given below.

$$A(G) = \begin{array}{c} \\ v_1 \\ v_2 \\ v_3 \\ v_4 \\ v_5 \\ v_6 \\ v_7 \\ v_8 \end{array} \begin{array}{c} \begin{array}{cccccccc} v_1 & v_2 & v_3 & v_4 & v_5 & v_6 & v_7 & v_8 \end{array} \\ \begin{bmatrix} 0 & 1 & 2 & 0 & 3 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 2 & 0 & 2 & 1 \\ 2 & 1 & 0 & 1 & 1 & 2 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 1 \\ 3 & 2 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 3 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 2 & 0 & 1 & 1 & 0 \end{bmatrix} \end{array}$$

The Hamming distance is used in telecommunications for error detection and correction[5]. In coding theory, this can be used to compare data words of equal length. In biology, it is also used to calculate genetic distance. For more on semigraph one can refer [6, 7].

The paper is organized as follows. We define Hamming distance and hamming index of a semigraph generated by its adjacency matrix and obtain some related results in section 3. We compute hamming index of some standard classes of semigraph in section 4.

2. Hamming distance of a semigraph

In this section, we define Hamming distance and Hamming index of a semigraph $G(V, X)$ generated by the adjacency matrix $A(G)$.

Let $G(V, X)$ be a semigraph with vertex set $V = \{v_1, v_2, v_3, ..., v_n\}$ and edge set $X = \{E_1, E_2, ..., E_m\}$. For a set $\mathbb{Z}_p = \{0, 1, 2, ..., p-1\}$, where $p = \max\{\mid E_1 \mid, \mid E_2 \mid, ..., \mid E_m \mid\}$ and a positive integer n, we have,

$$\begin{aligned}\mathbb{Z}_p^n =& \mathbb{Z}_p \times \mathbb{Z}_p \times ... \times \mathbb{Z}_p \ (n\ factors)\\ =& \{(x_1 x_2 x_3 ... x_n) \mid x_1, x_2, x_3, ... x_n \in Z_p\}\end{aligned}$$

Thus every element $x \in \mathbb{Z}_p^n$ can be expressed as $n-tuple$ $x = x_1, x_2, x_3, ..., x_n$ is called a string. The number of $x_i's$ greater than 0 is called weight of x and it is denoted by $wt(x)$.

$$wt(x) = \sum_{i=1}^{n} x_i, \ x_i > 0 \tag{2.1}$$

The Hamming distance between strings, $x = x_1 x_2 ... x_n$ and $y = y_1 y_2 ... y_n$ of a semigraph G is the number of positions $i's$ such that $x_i \neq y_i$, $1 \leq i \leq n$, it is denoted by $H_{ds}(x, y)$.

The sum of Hamming distances between all pairs of strings generated by the adjacency matrix of a semigraph G is called Hamming Index, denoted as $H_{As}(G)$. Let $s(v_i)$ and $s(v_j)$ be the strings of vertices v_i and v_j generated by $A(G)$. Then

$$H_{As}(G) = \sum_{1 \leq i,\ j \leq n} H_{ds}\big(s(v_i), s(v_j)\big)$$

Theorem 2.1. *Consider a semigraph $G(V, X)$, where $V = \{v_1, v_2, ..., v_n\}$ and $X = \{E_1, E_2, ..., E_m\}$. If there exists a vertex v_r adjacent to any two vertices v_i and v_j such that $\mid j - r \mid = \mid i - r \mid$, then*

$$H_{ds} = \begin{cases} \mid V \mid -p - q, & if\ v_i \sim v_j \\ \mid V \mid -p - q - 2, & if\ v_i \nsim v_j \end{cases}$$

Where p is the number of common neighbors satisfying the condition $\mid j - r \mid = \mid i - r \mid$ and q is the number of non-common neighbors.

Proof. (i) Suppose $v_i, v_j \in E_l$. Let $s(v_i) = x_1 x_2 ... x_n$; $s(v_j) = y_1 y_2 ... y_n$. If vertex $v_r \in E_l$ such that $\mid i - r \mid = \mid j - r \mid$ then, $x_r = y_r$. Let p be the number of vertices for which $x_r = y_r$ and q be the number of vertices for which both v_i, v_j are non-adjacent. Then, $s(v_i)$ and $s(v_j)$ differ at $\mid V \mid -p - q$ places. Hence $H_{ds}(s(v_i), s(v_j)) = \mid V \mid -p - q$.

(ii) Suppose $v_i \nsim v_j$ and $v_i \in E_l$, $v_j \in E_k$. Let $s(v_i) = x_1 x_2 ... x_n$ and $s(v_j) = y_1 y_2 ... y_n$. For any vertex $v_r \in E_l, E_k$, $\mid i - r \mid = \mid j - r \mid$, then $x_r = y_r$. Since $v_i \nsim v_j$, the entries $a_{v_i v_i} = a_{v_i v_j} = a_{v_j v_i} = a_{v_j v_j} = 0$. Hence $H_{ds}(s(v_i), s(v_j)) = \mid V \mid -p - q - 2$. □

Lemma 2.2. *If $u, v \in V$ are any two vertices where $u \in E_l$, $v \in E_k$ and there no common vertex between E_l and E_k (i.e $E_l \cap E_k = 0$), then $H_{ds}(s(u), s(v)) = wt(u) + wt(v)$.*

Proof. Let $u \in E_l, v \in E_k; E_l, E_k \in X$ and $u \not\sim v$. Then u is non-adjacent to every vertex of edge E_k and v is non-adjacent to every vertex of edge E_l. Thus the entries of $A(G)$ corresponding to the row $u \backslash v$ and columns of $V(E_k) \backslash V(E_l)$ are all zero. Let $s(u) = x_1 x_2 ... x_n$ and $s(v) = y_1 y_2 ... y_n$. If $E_l = \{v_1, v_2, ..., v_k\}$ and $E_k = \{v_i, v_{i+1}, ..., v_n\}$ then $a_{ut} \neq a_{vt}$, $v_1 \leq t \leq v_k$; $v_i \leq t \leq v_n$ and $t \neq u, v$. Therefore,

$$H_{ds}(s(u), s(v)) = wt(u) + wt(v)$$

□

Lemma 2.3. *Let $u, v \in E_l$ be the end vertices of edge E_l of a semigraph $G = (V, X)$ such that $u, v \notin X - E_l$. Then*

$$H_{ds}(s(u), s(v)) = \begin{cases} \mid E_l \mid, & when \ \mid E_l \mid \ is\ even \\ \mid E_l \mid -1, & when \ \mid E_l \mid \ is\ odd \end{cases}$$

Proof. Let u, v be the end vertices of an edge $E_l = \{v_i = u, v_{i+1}, ..., v_k = v\}$. Let $s(u) = x_1 x_2 ... x_n$ and $s(v) = y_1 y_2 ... y_n$. Since u and v are adjacent only to $V(E_l)$, by the definition of adjacency matrix,

$$s(u) : x_1 = 0, x_2 = 0, ..., x_i = 0, x_{i+1} = 1, x_{i+2} = 2, ..., x_{k-1} = k-2, x_k = k-1$$
$$s(v) : y_1 = 0, y_2 = 0, ..., y_i = k-1, y_{i+1} = k-2, y_{i+2} = k-3, ..., y_{k-1} = 1, y_k = 0$$

(i) When $\mid E_l \mid$ is even, $x_r \neq y_r$, $i \leq r \leq k$. Hence $H_{ds}(s(u), s(v)) = \mid E_l \mid$

(ii) When $\mid E_l \mid$ is odd, $x_r = y_r$, for $r = \frac{\lceil \mid E_l \mid \rceil}{2}$ otherwise $x_r \neq y_r$, for $i \leq r \leq k$. Therefore, $H_{ds}(s(u), s(v)) = \mid E_l \mid -1$ □

3. Hamming Index of Semigraph

Now we find Hamming index of some well known classes of semigraph.

Theorem 3.1. *Hamming index of complete semigraph E_n^c on n vertices is*

$$H_{As} = \begin{cases} \frac{2n^3 - 3n^2 + 2n}{4}, & when\ n\ is\ even \\ \frac{2n^3 - 3n^2 + 2n - 1}{4}, & when\ n\ is\ odd. \end{cases}$$

Proof. Consider a complete semigraph E_n^c on n vertices. Let $v_1, v_2, ..., v_n$ be the vertices of E_n^c. Then,

$$H_{As} = \sum_{i=1}^{n-1} \sum_{j=i+1}^{n} (s(v_i), s(v_j))$$

$$= \text{Hamming distance of } \left(\sum_{k=n-1}^{1} k \right) \text{ pairs.}$$

Let v_1 be the initial vertex in E_n^c. Then there will be $n-1$ pairs of vertices of the form (v_1, v_i), $2 \leq i \leq n$. If $n-1$ is odd, then $\frac{n+1}{2}$ number of pairs among $n-1$ pairs have hamming distance n and remaining $\frac{n-1}{2}$ number of pairs of vertices have hamming distance $n-1$. If $n-1$ is even, then hamming distance between

$\frac{n-1}{2}$ pairs of vertices is n and that of $\frac{n-1}{2}$ number of pairs of vertices is $n-1$. This argument holds for all other pairs of vertices. Therefore hamming index of a graph E_n^c can be calculated as follows.
(i) If n is even, then there are $\frac{n^2}{4}$ pairs of vertices with hamming distance n and $\frac{n(n-2)}{4}$ pairs of vertices with hamming distance $n-1$. Thus,

$$\begin{aligned} H_{As}(E_n^c) =&\Big(\frac{n^2}{4}\Big)(n) + \Big(\frac{(n-2)n}{4}\Big)(n-1) \\ =&\frac{n^3}{4} + \frac{n^3-3n^2+2n}{4} \\ =&\frac{2n^3-3n^2+2n}{4} \end{aligned}$$

(ii) If n is odd, then there are $\frac{n^2-1}{4}$ pairs of vertices having hamming distance n and $\frac{n^2-2n+1}{4}$ pairs of vertices with hamming distance $(n-1)$. Hence,

$$\begin{aligned} H_{As}(E_n^c) =&\Big(\frac{n^2-1}{4}\Big)(n) + \Big(\frac{n^2-2n+1}{4}\Big)(n-1) \\ =&\frac{n^3-n}{4} + \frac{n^3-3n^2+3n-1}{4} \\ =&\frac{2n^3-3n^2+2n-1}{4} \end{aligned}$$

□

Theorem 3.2. *Hamming index of a semigraph $C_{n,m}$ is*

$$H_{As} = \begin{cases} \frac{(10m^3+33m^2+30m+C')n}{4}, & when\ n=3 \\ m^2n^2(m+4) - \frac{m^2n}{4}(2m+15) + \frac{mn}{2}(6n+1) + B', & when\ n=4 \\ m^2n^2(m+4) - \frac{m^2n}{4}(2m+15) + \frac{5}{2}mn(2n-3) + A', & when\ n \geq 5 \end{cases}$$

where,

$$A' = \begin{cases} n(8n-15), m\ is\ odd \\ 2n(n-2), m\ is\ even. \end{cases} \quad B' = \begin{cases} 3(n-4), m\ is\ even \\ \frac{23n}{4} - 8, m\ is\ odd. \end{cases}$$

$$C' = \begin{cases} 7, m\ is\ odd \\ 8, m\ is\ even. \end{cases}$$

Proof. A semigraph $C_{n,m}$ has $n(m+1)$ vertices, where m is the number of middle vertices in an edge. Let $E_l = \{l_1, l_2, ..., l_{n_1}\}$ and $E_k = \{k_1, k_2, ..., k_{n_2}\}$ be the two edges of $C_{n,m}$. Hamming distance between pair of vertices of $C_{n,m}$ can be calculated using following cases.

1) Hamming distance between pair of vertices of an edge.

i) When $n \geq 4$.
From Theorem 3.1 we have,

$$H_{As}(E^c_{m+2}) = \begin{cases} \frac{2(m+2)^3-3(m+2)^2+2(m+2)}{4}, & \textit{when } (m+2) \textit{ is even} \\ \frac{2(m+2)^3-3(m+2)^2+2(m+2)-1}{4}, & \textit{when } n \textit{ is odd.} \end{cases}$$

Since every end vertex of $C_{n,m}$ is adjacent to exactly two edges, the weight of an end vertex in $C_{n,m}$ is equal to weight of end vertex in $E^c_{m+2}+(m+1)$. An edge in $C_{n,m}$ has $2m$ pairs of middle and end vertices together with a pair of end vertices. Therefore by using Theorem 3.1, sum of Hamming distance of all pairs of vertices of an edge is,
When m is even,

$$=\left(\frac{2(m+2)^3-3(m+2)^2+2(m+2)}{4}+2(m+1)^2\right)n$$
$$=\frac{(2m^3+17m^2+30m+16)n}{4}.$$

When m is odd,

$$=\left(\frac{2(m+2)^3-3(m+2)^2+2(m+2)-1}{4}+2(m+1)^2\right)n$$
$$=\frac{(2m^3+17m^2+30m+15)n}{4}.$$

ii) When $n = 3$.
Since all edges are adjacent, the hamming distance between end vertices of edge in $C_{n,m}$ is equal to hamming distance of end vertices of $E^c_{m+2}+2m$. Also, weight of end vertex in $C_{n,m}$ is equal to weight of end vertex in $E^c_{m+2}+(m+1)$. Therefore, when m is even, hamming distance,

$$=\left(\frac{2(m+2)^3-3(m+2)^2+2(m+2)}{4}+2m(m+1)+2m\right)n$$
$$=\frac{(2m^3+17m^2+30m+8)n}{4}.$$

When m is odd,

$$=\left(\frac{2r^3-3r^2+2r-1}{4}+2m(r-1)+2(r-2)\right)n$$
$$=\frac{(2m^3+17m^2+30m+7)n}{4}.$$

2) Hamming distance of remaining pair of vertices is given in the following table.

Type of pair of vertices (v_i, v_j)	**n**	**p**	**q**	**Number of Pairs**	**Hamming distance of pair of vertices**
$v_i = l_1$, $v_j = k_{n_2}$; $l_{n_1} = v_r = k_1$; $\mid E_l \cap E_k \mid= 1$; $\mid i - n_1 \mid=\mid n_2 - 1 \mid$	when $n \geq 5$	1	$n(m+1) - (4m+5)$	n	$2(2m+1)$
	when $n = 4$	2	0	2	$n(m+1)-4$
$v_i \in E_l$, $v_j \in E_k$, v_i, v_j are end vertices, $\mid E_l \cap E_k \mid= 0$	$n > 5$	0	$n(m+1) - 4(m+1) - 2$	$\frac{n(n-5)}{2}$	$4(m+1)$
	$n \leq 5$	all end vertices satisfy $\mid E_l \cap E_k \mid= 1$			
v_i is the end vertex and v_j is the middle vertex, $\mid E_l \cap E_k \mid= 1$	$n \geq 4$	0	$n(m+1)-3m-4$	$2mn$	$3m+2$
	$n = 3$	0	0	mn	$3m+1$
v_i is the end vertex and v_j is the middle vertex, $\mid E_l \cap E_k \mid= 0$	$n \geq 4$	0	$n(m+1)-3m-5$	$mn(n-4)$	$3(m+1)$
v_i, v_j are middle vertices; $v_i = l_i$, $v_j = k_j$; $l_{n_1} = v_r = j_1$; $\mid E_l \cap E_k \mid= 1$; $\mid i - n_1 \mid=\mid j - 1 \mid$	$n \geq 3$	1	$n(m+1) - (2m+3)$	mn	$2m$
v_i, v_j are middle vertices; $v_i = l_i$, $v_j = k_j$; $l_{n_1} = v_r = j_1$; $\mid E_l \cap E_k \mid= 1$; $\mid i - n_1 \mid\neq\mid j - 1 \mid$	$n \geq 3$	0	$n(m+1) - (2m+3)$	$mn(m-1)$	$2m+1$
v_i, v_j are middle vertices; $v_i = l_i$, $v_j = k_j$; $\mid E_l \cap E_k \mid= 0$;	$n \geq 4$	0	$n(m+1) - 2(m+2)$	$\frac{m^2 n(n-3)}{2}$	$2(m+1)$
	$n = 3$	all end vertices satisfy $\mid E_l \cap E_k \mid= 1$			

Thus hamming index of semigraph $C_{n,m}$ is,

i) When $n \geq 5$.

$$\begin{aligned}H_{As}(C_{n,m}) =& \frac{(2m^3+17m^2+30m+A_1)n}{4} + 2n(2m+1) + 2n(n-5)(m+1) + \\ & 2mn(3m+2) + 3mn(n-4)(m+1) + 2m^2n + mn(m-1)(2m+1) \\ & + m^2n(m+1)(n-3). \\ H_{As}(C_{n,m}) =& m^2n^2(m+4) - \frac{m^2n}{4}(2m+15) + \frac{5}{2}mn(2n-3) + A'\end{aligned}$$

ii) When $n = 4$

$$\begin{aligned}H_{As}(C_{4,m}) =& \frac{(2m^3+17m^2+30m+A_1)n}{4} + 2(n(m+1)-4) + 2mn(3m+2) + \\ & 3mn(n-4)(m+1) + 2m^2n + mn(m-1)(2m+1) + \\ & m^2n(m+1)(n-3). \\ H_{As}(C_{4,m}) =& m^2n^2(m+4) - \frac{m^2n}{4}(2m+15) + \frac{mn}{2}(6n+1) + B'\end{aligned}$$

iii) When $n = 3$.

$$\begin{aligned}H_{As}(C_{3,m}) =& \frac{(2m^3+17m^2+30m+C_1)n}{4} + mn(3m+1) + 2m^2n + \\ & mn(m-1)(2m+1) \\ H_{As}(C_{3,m}) =& \frac{(10m^3+33m^2+30m+C')n}{4}\end{aligned}$$

where,

$$A_1 = \begin{cases} 16, \textit{when } m \textit{ is even} \\ 15, \textit{ when } m \textit{ is odd.} \end{cases} \quad A' = \begin{cases} n(8n-15), \textit{when } m \textit{ is odd} \\ 2n(n-2), \textit{when } m \textit{ is even.} \end{cases}$$

$$B' = \begin{cases} 3(n-4), \textit{when } m \textit{ is even} \\ \frac{23n}{4} - 8, \textit{when } m \textit{ is odd.} \end{cases} \quad C' = C_1 = \begin{cases} 7, \textit{when } m \textit{ is odd} \\ 8, \textit{when } m \textit{ is even.} \end{cases}$$

□

Theorem 3.3. *Hamming index strongly complete semigraph* (T^1_{k-1}) *is,*

$$H_{As}(T^1_{k-1}) = \begin{cases} \frac{2m^3+13m^2+26m+24}{4}, & \textit{when } m \textit{ is even} \\ \frac{2m^3+13m^2+26m+23}{4}, & \textit{when } m \textit{ is odd} \end{cases}$$

Proof. In a semigraph T^1_{k-1}, a vertex v_k is connected to all the vertices of E^c_{k-1}. Therefore

$$H_{As}(T_{k-1}^1) = H_{ds}(E_{k-1}^c) + \sum_{2\le j\le k-2} H_{ds}(v_j, v_k) + \sum_{j=1,k-1} H_{ds}(v_j, v_k)$$

By Theorem 3.1 we have,

$$H_{As}(E_{k-1}^c) = \begin{cases} \frac{2(k-1)^3-3(k-1)^2+2(k-1)}{4}, & \textit{when } k-1 \textit{ is even} \\ \frac{2(k-1)^3-3(k-1)^2+2(k-1)-1}{4}, & \textit{when } k-1 \textit{ is odd.} \end{cases}$$

The vertex v_k and middle vertex v_j are adjacent to both v_{j-1} and v_{j+1} with $a_{(j-1)k} = a_{(j-1)j}$ and $a_{k(j+1)} = a_{j(j+1)}$. Therefore by Theorem 2.1 we have $|V| = k$, $p = 2$, $q = 0$. Hence hamming distance of m pairs of (v_k, v_j), $2 \le j \le k-2$, is $m(k-2)$. Also v_k, v_1 are adjacent to v_2 and v_k, v_{k-1} are adjacent to v_{k-2} with $a_{2k} = a_{12}$ and $a_{k(k-2)} = a_{(k-2)k-1}$. Therefore hamming distance between the pairs (v_k, v_1) and (v_k, v_{k-1}) is $2(k-1)$. Thus,

when m is even,

$$H_{As} = \frac{2(k-1)^3 - 3(k-1)^2 + 2(k-1)}{4} + m(k-2) + 2(k-1)$$

By replacing k-1=m+2,

$$= \frac{2(m+2)^3 - 3(m+2)^2 + 2(m+2)}{4} + m(m+1) + 2(m+2)$$

Therefore,

$$H_{As} = \frac{2m^3 + 13m^2 + 26m + 24}{4}$$

when m is odd,

$$H_{As} = \frac{2(k-1)^3 - 3(k-1)^2 + 2(k-1) - 1}{4} + m(k-2) + 2(k-1)$$

By replacing k-1=m+2,

$$= \frac{2(m+2)^3 - 3(m+2)^2 + 2(m+2) - 1}{4} + m(m+1) + 2(m+2)$$

Therefore,

$$H_{As} = \frac{2m^3 + 13m^2 + 26m + 23}{4}$$

□

Theorem 3.4. *Hamming Index of a semigraph $K_{n,m}$ is*

$$H_{As}(K_{n,m}) = \frac{(m^3+3m^2+2m)}{4}(n^4-2n^3) + \frac{(3m^2+10m+D_1)n^2}{8} + \frac{(2m^3+3m^2-6m-D_1)n}{8}$$

Where,

$$D_1 = \begin{cases} 7, \text{when } m \text{ is odd} \\ 8, \text{when } m \text{ is even.} \end{cases}$$

Proof. An $(m+2)$ uniform $K_{n,m}$ semigraph has $\frac{nm(n-1)+2n}{2}$ vertices, where m is the number of middle vertices in an edge. Let $E_l = \{l_1, l_2, ..., l_{n_1}\}$ and $E_j = \{j_1, j_2, ..., j_{n_2}\}$ be the edges of $K_{n,m}$. Then the hamming distance of pair of vertices can be calculated using following cases.

1) Sum of Hamming distance of pair of vertices of an edge.

Let (v_i, v_j) be any two vertices in an edge of $K_{n,m}$. There are $2m$ pairs of middle and end vertices together with a pair of end vertices. The weight of an end vertex in $K_{n,m}$ is $(n-2)(m+1)$ more than weight of an end vertex in E^c_{m+2}. Therefore sum of hamming distances of all pairs of vertices (v_i, v_j) of an edge is,

When m is even,

$$= \left(\frac{2(m+2)^3 - 3(m+2)^2 + 2(m+2)}{4} + 2m(n-2)(m+1) + 2m(n-2) \right)(n^2 - n)$$

$$= \frac{2m^3 - 7m^2 + 8m^2n + 16mn - 18m + 8}{8}$$

When m is odd,

$$= \left(\frac{2(m+2)^3 - 3(m+2)^2 + 2(m+2) - 1}{4} + 2m(n-2)(m+1) + 2m(n-2) \right)(n^2 - n)$$

$$= \frac{2m^3 - 7m^2 + 8m^2n + 16mn - 18m + 7}{8}.$$

2) Hamming distance of remaining pair of (v_i, v_j) is given below,

Type of pair of vertices (v_i, v_j)	**p**	**q**	**Number of Pairs**	**Hamming distance of pair of vertices**
$v_i \in E_l$ is a end vertex, $v_j \in E_j$ is a middle vertex, $v_i \not\sim v_j$ $\mid E_i \cap E_j \mid = 0$	0	$\frac{mn(n-3)}{2}$	$\frac{mn(n-1)(n-2)}{2}$	$n(m+1)-2$
$v_i \in E_l$ is a end vertex, $v_j \in E_j$ is a middle vertex, $v_i \not\sim v_j$, $\mid E_i \cap E_j \mid = 0$	0	$\frac{mn(n-3)}{2}$	$\frac{mn(n-1)(n-2)}{2}$	$n(m+1)-2$
v_i and v_j are middle vertices $\mid i - n_1 \mid = \mid j-1 \mid$ $E_i \cap E_j = 1$	0	$\frac{n(n-1)-4}{2} + n - 3$	$\frac{nm(m-1)(n-1)}{2} + \frac{(n-2)}{2}$	$2m+1$
v_i and v_j are middle vertices $E_i \cap E_j = 0$	0	$\frac{m(n^2-n-4)}{2} + \frac{2(n-4)}{2}$	$2(m+1)$	$\frac{n(n-1)(n-2)}{8} + \frac{(n-3)m^2}{8}$
v_i and v_j are middle vertices $\mid i - n_1 \mid \neq \mid j-1 \mid$, $E_i \cap E_j = 1$	1	$\frac{(n(n-1)-4)m}{2} + n - 3$	$\frac{nm}{2}(n-2)(n-1)$	$2m$

Thus hamming sum of semigraph $K_{n,m}$ is,

$$=\frac{2m^3 - 7m^2 + 8m^2n + 16mn - 18m + 8}{8} + \frac{mn(n-1)(n-2)(n(m+1)-2)}{2} + \frac{(2m+1)(nm(m-1)(n-1)n-2)}{2} + nm^2(n-1)(n-2) + \frac{nm^2(m+1)(n-1)(n-2)(n-3)}{4}$$

$$=\frac{(m^3+3m^2+2m)}{4}(n^4-2n^3) + \frac{(3m^2+10m+D_1)n^2}{8} + \frac{(2m^3+3m^2-6m-D_1)n}{8}$$

Where,

$$D_1 = \begin{cases} 7, when\ m\ is\ odd \\ 8, when\ m\ is\ even. \end{cases}$$

□

4. Conclusion

This article presents a Hamming distance in a semigraph generated by its adjacency matrix. Hamming distance in a semigraph is based on the type of adjacency in a particular edge. We also computed Hamming index of some known classes of semigraph. There is a huge scope to compute Hamming index of semigraphs.

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