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ON SIGNED ADMISSIBLE MARTINGALE DEFORMATIONS

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Abstract. Currently, the development of the theory of Haar interpolations of financial markets with the use of martingale measures continues. The existence of martingale measures of discounted stock prices means that this kind of interpolation can only be used in arbitrage-free markets (c.f. the survey [1]). However, real financial markets often contain elements of arbitrage opportunities. Therefore, it is important to develop techniques for interpolating processes that do not admit martingale measures. The work [2] is devoted to just this problem, where signed martingale measures serve as the main interpolation tool. In this paper, we proceed to consider admissible signed martingale deformations (in particular, their existence). Various other facts about processes on deform structures are contained in the works [3]–[13]).

1. Introduction

Consider a filtered space $\big(\Omega, \mathbf{F}\big)$, where Ω is any set and $\mathbf{F} = (\mathcal{F}_n)_{n=0}^{\infty}$ is an increasing sequence of σ-algebras on it (filtration). We say that $\big(\Omega, \mathbf{F}\big)$ has a finite horizon $N < \infty$, if $\forall n \geq N$ $\mathcal{F}_n = \mathcal{F}_N$. Denote by $\mathcal{F}_\infty = \bigvee\limits_{n=0}^{\infty} \mathcal{F}_n$ the least σ-algebra, containing the algebra $\bigcup\limits_{n=0}^{\infty} \mathcal{F}_n$. We suppose that $\mathcal{F}_0 = \{\Omega, \emptyset\}$. We will also use the notion $\mathbb{N} = \{0, 1, 2, \dots\}$.

Definition 1.1. A family $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^{\infty}$ of probability measures $Q^{(n)}$ defined on $\mathcal{F}_n$ is called

1) deformation of the 1-st kind (D1) if $\forall n \in \mathbb{N}$ the restriction of measure $Q^{(n+1)}$ on σ-algebra $\mathcal{F}_n$ is absolutely continuous with respect to $Q^{(n)}$, i.e.

$$Q^{(n+1)}|_{\mathcal{F}_n} << Q^{(n)}; \tag{1.1}$$

2) deformation of the 2-nd kind (D2) if $\forall n \in \mathbb{N}$

$$Q^{(n)} << Q^{(n+1)}|_{\mathcal{F}_n}; \tag{1.2}$$

3) weak deformation (WD) if $\mathbf{Q}$ is simultaneously a deformation of 1-st and 2-nd kind, i.e. if $\forall n \in \mathbb{N}$

$$Q^{(n+1)}|_{\mathcal{F}_n} \sim Q^{(n)}; \tag{1.3}$$

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4) consistent sequence of probability measures (CSPM) if $\forall n \in \mathbb{N}$

$$Q^{(n+1)}|_{\mathcal{F}_n} = Q^{(n)}; \tag{1.4}$$

5) probability measure (PM) if there exists a probability Q on $\mathcal{F}_\infty$ such that $\forall n \in \mathbb{N}$

$$Q|_{\mathcal{F}_n} = Q^{(n)}. \tag{1.5}$$

We use the notions of deformations of the 1-st and the 2-nd kind with density processes $(h_n, \mathcal{F}_n)_{n=0}^\infty$ and $(h^{(n)}, \mathcal{F}_n)_{n=0}^\infty$ respectively:

- for deformations of the 1-st kind: $dQ^{(n+1)}|_{\mathcal{F}_n} = h_n dQ^{(n)}$;
- for deformations of the 2-nd kind: $dQ^{(n)} = h^{(n)} dQ^{(n+1)}|_{\mathcal{F}_n}$.

A deformed stochastic basis (DSB) is a filtered space equipped with a deformation $\mathbf{Q}$. Depending on the type of deformation $\mathbf{Q}$, deformed stochastic bases of the 1st and 2nd kind (DSB1 and DSB2) and weakly deformed stochastic bases (WDSB) are naturally defined. DSB is denoted by $(\Omega, \mathbf{Q})$.

DSB with a finite horizon $N < \infty$ is $(\Omega, \mathbf{Q})$ such that $\forall n \geq N$ $\mathcal{F}_n = \mathcal{F}_N$ and $Q^{(n)} = Q^{(N)}$, i.e. $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^N$.

It is clear that PM is CSPM, CSPM is WD, and WD is simultaneously D1 and D2.

2. Construction of weak deformations

Consider $(\Omega, \mathbf{F})$, where $\mathbf{F} = (\mathcal{F}_n)_{n=0}^N$, $N < \infty$. Let P be a probability measure on $\mathcal{F}_N$ and $(h^{(n)}, \mathcal{F}_n)_{n=0}^N$ be a finite adapted sequence of random variables. Put the following question: under what conditions on $(h^{(n)})_{n=0}^N$ does a WD $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^N$ exist such that $Q^{(N)} = P$ and the equalities $dQ^{(n)} = h^{(n)} dQ^{(n+1)}|_{\mathcal{F}_n}$ are satisfied for $n = 0, 1, \dots, N$? It should be noted that similar questions were posed in works [6] and [13]. But in [6] this task was solved under condition that σ-algebra $\mathcal{F}_n$ is generated for any $n \in \mathbb{N}$ by a partition of Ω into an at most countable number of atoms. And in [13] to find $\mathbf{Q}$ no boundary condition (in the form of an arbitrary probability P) was set. In this paper, the σ-algebras $\mathcal{F}_n$ are arbitrary and there is a boundary condition P. In our opinion, this formulation is more suitable for application in financial mathematics.

In what follows, we use the symbol E to denote the mathematical expectation for probability P and use the notation: $P_n = P|_{\mathcal{F}_n}$.

Theorem 2.1. *Let* $h^{(N)} = 1$,

$$P\left(\prod_{n=0}^{N} h^{(n)} > 0\right) = 1 \tag{2.1}$$

and

$$E\left(\prod_{k=n}^{N} h^{(k)}\right) = 1, \quad \forall n = 0, 1, \dots, N. \tag{2.2}$$

Then $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^{\infty}$ *is WD with density process* $(h^{(n)}, \mathcal{F}_n)_{n=0}^{N}$*, where*

$$dQ^{(n)} = E\left(\prod_{k=n}^{N} h^{(k)} | \mathcal{F}_n\right) dP_n, \quad 0 \leq n \leq N, \tag{2.3}$$

and this WD satisfies the equality $Q^{(N)} = P$*.*

Proof. Since $h^{(N)} = 1$, we have $Q^{(N)} = P$. From (2.3) and (2.2) we obtain

$$Q^{(n)}(\Omega) = E\left[E\left(\prod_{k=n}^{N} h^{(k)} | \mathcal{F}_n\right)\right] = E\left(\prod_{k=n}^{N} h^{(k)}\right) = 1.$$

Next, write down the formula (2.3), replacing n with $n+1$:

$$dQ^{(n+1)} = E\left(\prod_{k=n+1}^{N} h^{(k)} | \mathcal{F}_{n+1}\right) dP_{n+1} :$$

Using the well-known formula, let us express $dQ^{(n+1)}|_{\mathcal{F}_n}$ through $dP_{n+1}|_{\mathcal{F}_n} = dP_n$:

$$dQ^{(n+1)}|_{\mathcal{F}_n} = E\left[E\left(\prod_{k=n+1}^{N} h^{(k)} | \mathcal{F}_{n+1}\right) |_{\mathcal{F}_n}\right] dP_n = E\left(\prod_{k=n+1}^{N} h^{(k)} | \mathcal{F}_n\right) dP_n.$$

From this and from formula (2.3) it follows:

$$h^{(n)} dQ^{(n+1)}|_{\mathcal{F}_n} = dQ^{(n)}.$$

Since (2.1), $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^{\infty}$ is WD with density process $(h^{(n)}, \mathcal{F}_n)_{n=0}^{N}$. □

3. Deformed deflators

Let $L_1(\Omega, \mathbf{Q})$ denote the set of classes of $\mathbf{Q}$-indistinguishable adapted processes satisfying the condition $\|Z_n\|_{L_1(\Omega, \mathcal{F}_n, Q^{(n)})} < \infty$, $\forall n = 0, 1, \ldots$.

Definition 3.1. 1) Let $(\Omega, \mathbf{Q})$ be a DSB1. A process

$$\mathbf{Z} = (Z_n, \mathcal{F}_n, Q^{(n)})_{n=0}^{\infty} \in L_1(\Omega, \mathbf{Q})$$

is called deformed martingale of the 1-st kind (DM1) if $\forall n \in \mathbb{N}$ the following equality is fulfilled:

$$E^{Q^{(n+1)}}[Z_{n+1} | \mathcal{F}_n] = Z_n \quad Q^{(n+1)}|_{\mathcal{F}_n} - a.s. \tag{3.1}$$

2) Let $(\Omega, \mathbf{Q})$ be a DSB2. A process $\mathbf{Z} = (Z_n, \mathcal{F}_n, Q^{(n)})_{n=0}^{\infty} \in L_1(\Omega, \mathbf{Q})$ is called deformed martingale of the 2-nd kind (DM2) if $\forall n = 0, 1, \ldots$

$$E^{Q^{(n+1)}}[Z_{n+1} | \mathcal{F}_n] = Z_n \quad Q^{(n)} - a.s. \tag{3.2}$$

3) Let $(\Omega, \mathbf{Q})$ be a WDSB. A process $\mathbf{Z} = (Z_n, \mathcal{F}_n, Q^{(n)})_{n=0}^{\infty}$ is called weakly deformed martingale (WDM) if it is both DM1 and DM2. If $(\mathbf{Q})$ is CSPM, then we will call WDM martingale with respect CSPM (MCSPM).

Remark 3.2. If $(\Omega, \mathbf{Q})$ is a WDSB and a process $\mathbf{Z} = (Z_n, \mathcal{F}_n, Q^{(n)})_{n=0}^{\infty}$ is DM1 or DM2, then $\mathbf{Z}$ is WDM.

The following definition is new.

Definition 3.3. Let $Z = (Z_n, \mathcal{F}_n)_{n=0}^N$ be an adabted process that can take any real values. A DM $D = \left(D_n, \mathcal{F}_n, Q^{(n)}\right)_{n=0}^N$ is said a deformed deflator of the process Z if D_0=1 and the process $DZ = \left(D_n Z_n, \mathcal{F}_n, Q^{(n)}\right)_{n=0}^N$ is a DM of the same kind as D.

Introduce the family of measures $(\mu^{(n)})_{n=0}^N$, where

$$\mu^{(n)}(A) = \int\limits_A D_n dQ^{(n)}, \quad A \in \mathcal{F}_n. \tag{3.3}$$

It is obvious that if $\mathbf{Q}$ is CSPM, then $(\mu^{(n)})_{n=0}^N$ is CSPM too.

Denote $\nu^{(n)} = \frac{\mu^{(n)}}{E^{Q^{(n)}}(D_n)}$.

Theorem 3.4. *1) Let* $\mathbf{Q}$ *be D1, deformed deflator* D *be DM1 and* $D_n > 0$ $Q^{(n)}$*-a.s. Then* $(\nu^{(n)})_{n=0}^N$ *is D1 and* $(Z_n, \mathcal{F}_n, \nu^{(n)})_{n=0}^N$ *is DM1.*

2) Let $\mathbf{Q}$ *be WD, deformed deflator* D *be WD and* $D_n > 0$ $Q^{(n)}$*-a.s. Then* $(\nu^{(n)})_{n=0}^N$ *is WD and* $(Z_n, \mathcal{F}_n, \nu^{(n)})_{n=0}^N$ *is WDM.*

Proof. 1) Prove first that $(\nu^{(n)})_{n=0}^N$ is D1. Let $A \in \mathcal{F}_n$ and $\nu^{(n)}(A) = 0$. Then

$$\mu^{(n)}(A) = \int\limits_A D_n dQ^{(n)} = 0 \Leftrightarrow Q^{(n)}(A) = 0 \Rightarrow Q^{(n+1)}(A) = 0$$

and hence

$$\mu^{(n+1)}(A) = \int\limits_A D_{n+1} dQ^{(n+1)} = 0 \Rightarrow \nu^{(n+1)}(A) = 0.$$

This means that $(\nu^{(n)})_{n=0}^N$ is D1.

Now prove that $(Z_n, \mathcal{F}_n, \nu^{(n)})_{n=0}^N$ is DM1. We have:

$$\int\limits_A Z_{n+1} d\mu^{(n+1)} = \int\limits_A Z_{n+1} D_{n+1} dQ^{(n+1)} = \int\limits_A Z_n D_n dQ^{(n+1)} =$$

$$= \int\limits_A Z_n D_n dQ^{(n+1)} |_{\mathcal{F}_n} = \int\limits_A Z_n E^{Q^{(n+1)}}[D_{n+1}|\mathcal{F}_n] dQ^{(n+1)} |_{\mathcal{F}_n} =$$

$$= \int\limits_A Z_n d\mu^{(n+1)} |_{\mathcal{F}_n} = \int\limits_A Z_n d\mu^{(n+1)},$$

i.e. $\int\limits_A Z_{n+1} d\nu^{(n+1)} = \int\limits_A Z_n d\nu^{(n+1)}$.

2) The proof follows from Remark 3.2 and part 2) of this theorem. □

Remark 3.5. Since Definition 3.3 does not assume that the deformed deflator is a positive process, the measures in formula (3.3) can also take negative values. The calculation at the end of the proof of Theorem 3.4 shows that signed martingale deformations can also be suitable for use in financial mathematics.

4. Admissible signed martingale deformations

Consider a filtration $\big(\Omega, \mathbf{F} = (\mathcal{F}_n)_{n=0}^N\big)$, where $\forall n \in \mathbb{N}$ $\mathcal{F}_n$ is finite.

Definition 4.1. A family $\mathbf{Q} = (Q^{(n)}, \mathcal{F}_n)_{n=0}^N$ of signed measures $Q^{(n)}$, defined on $\mathcal{F}_n$, is called signed deformation if $\forall n = 0, 1, \ldots, N$ $Q^{(n)}(\Omega) = 1$. A signed deformation is called **F**-admissible if $\forall n = 0, 1, \ldots, N$ and for any atom A of σ-algebra $\mathcal{F}_n$ $Q^{(n)}(A) \neq 0$. It is called admissible if $\forall A \in \mathcal{F}_n$ $(A \neq \emptyset)$ the inequality $Q^{(n)}(A) \neq 0$ holds. If **F**-admissible (or admissible) deformation **Q** is also martingale deformation for a process $Z = (Z_n, \mathcal{F}_n)_{n=0}^N$, then we call **Q** **F**-admissible (or admissible) signed martingale deformation.

The rest of this article will be devoted to constructing **F**-admissible signed martingale deformation on a binary filtration with an infinite horizon. We will need the following lemma, borrowed from the article [14]. It does not use any specific properties of numbers h_n^k and p_n^k.

Lemma 4.2. *Let the numbers q_n^k, $n \in \mathbb{N}$, $k = 1, 2, \ldots, 2^n$, be given by recurrent formulas:*

$$\begin{cases} q_0^1 = 1 \\ q_{n+1}^{2k-1} = h_n^k q_n^k p_{n+1}^{2k-1} \\ q_{n+1}^{2k} = h_n^k q_n^k p_{n+1}^{2k}, \end{cases} \tag{4.1}$$

where $h_0^1 = 1$ and the numbers h_n^k and p_n^k are arbitrary. Then the following formula is valid:

$$h_n^k q_n^k = \prod_{m=1}^{n} h_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} p_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} \tag{4.2}$$

(we assume that $\prod_{m=1}^{0} h_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} p_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} = 1$ and $[a]$ is the integer part of the number a).

Consider the filtration $(\Omega, \mathbf{F} = (\mathcal{F}_n)_{n=0}^\infty)$ such that each σ-algebra $\mathcal{F}_n$ is generated by the partition of Ω into atoms $A_1^n, A_2^n, \ldots, A_{2^n}^n$ $(A_1^0 := \Omega)$ and $A_k^n = A_{2k-1}^{n+1} + A_{2k}^{n+1}$ $\forall n \in \mathbb{N}$ and $k = 1, 2, \ldots, 2^n$. This filtration is called binary filtration. For a deformation **Q** we will use the notation $Q^{(n)}(A_n^k) = q_n^k$.

Theorem 4.3. *Let a process $\mathbf{Z} = (Z_n, \mathcal{F}_n)_{n=0}^\infty$ be given such that $\forall n \in \mathbb{N}$ and $\forall k = 0, 1, \ldots, 2^n$ $z_{n+1}^{2k-1} \neq z_{n+1}^{2k}$, $z_n^k \neq z_{n+1}^{2k-1}$ and $z_n^k \neq z_{n+1}^{2k}$. On the other hand, let $\mathbf{H} = (h_n, \mathcal{H}_n)_{n=0}^\infty$ be an adapted process with the properties: $h_0 = 1$, $\forall n \in \mathbb{N}$ $h_n \neq 0$ $Q^{(n)}$-a.s. and*

$$\sum_{k=1}^{2^n} \frac{1}{h_n^k} \prod_{m=1}^{n} h_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} p_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} = 1, \quad n \in \mathbb{N}. \tag{4.3}$$

*Then there exists a unique **F**-admissible signed martingale deformation **Q**, for which $dQ^{(n+1)}|_{\mathcal{F}_n} = h_n dQ^{(n)}$, $n \in \mathbb{N}$.*

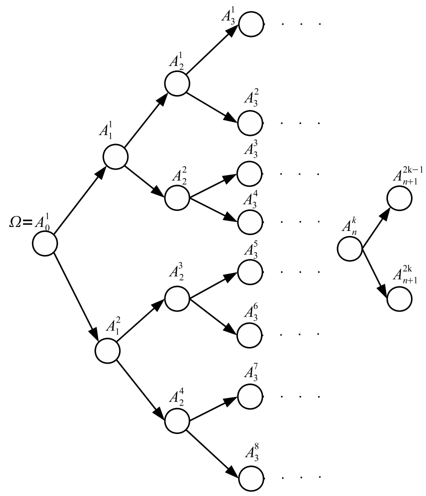

FIGURE 1. Filtration generated by a binary tree

Proof. Let us write out all the conditions for finding the required deformation **Q**. Namely, $\forall n \in \mathbb{N}$ and $\forall k = 0, 1, \ldots, 2^n$ all following systems must be implemented

$$\left\{ \begin{array}{l} q_{n+1}^{2k-1} + q_{n+1}^{2k} = h_n^k q_n^k \\ z_{n+1}^{2k-1} q_{n+1}^{2k-1} + z_{n+1}^{2k} q_{n+1}^{2k} = z_n^k (q_{n+1}^{2k-1} + q_{n+1}^{2k}) \end{array} \right. \tag{4.4}$$

and inequalities

$$q_n^k \neq 0 \tag{4.5}$$

must be satisfied, and $\forall n \in \mathbb{N}$ the equalities

$$\sum_{k=1}^{2^n} q_n^k = 1 \tag{4.6}$$

nust be true. Remark that the first equalities of system (4.4) were obtained from $dQ^{(n+1)}|_{\mathcal{F}_n} = h_n dQ^{(n)}$, the second equalities of (4.4) got from deform martingale equalities, inequalities (4.5) and equalities (4.6) are taken from Definition 4.1.

It is clear that each of the systems (4.4) has a unique solution

$$\left\{ \begin{array}{l} q_{n+1}^{2k-1} = h_n^k q_n^k \frac{z_{n+1}^{2k} - z_n^k}{z_{n+1}^{2k} - z_{n+1}^{2k-1}} \\ q_{n+1}^{2k} = -h_n^k q_n^k \frac{z_{n+1}^{2k-1} - z_n^k}{z_{n+1}^{2k} - z_{n+1}^{2k-1}}. \end{array} \right. \tag{4.7}$$

Denoting $p_{n+1}^{2k-1} := \frac{z_{n+1}^{2k} - z_n^k}{z_{n+1}^{2k} - z_{n+1}^{2k-1}}$, $p_{n+1}^{2k} := \frac{z_{n+1}^{2k-1} - z_n^k}{z_{n+1}^{2k} - z_{n+1}^{2k-1}}$ and applying Lemma 4.2, we obtain the explicit solution of (4.7):

$$q_n^k = \frac{1}{h_n^k} \prod_{m=1}^{n} h_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]} p_m^{\left[\frac{k+2^{n-m}-1}{2^{n-m}}\right]}. \tag{4.8}$$

From assumptions of Theorem it follows that (4.5) and (4.6) hold. The fact that obained deformation **Q** is **F**-admissible follows from the first equation of the system (4.4). □

Remark 4.4. Theorem 4.3 is a generalization of the theorem 6 of the work [14] from ordinary strictly positive martingale deformations to signed deformations.

5. Conclusion

In this article, definitions of signed versions of the concept of deformed martingales, deflators and measures are given. Theorems related to the construction of signed admissible deformations are proved.

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