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ON POSITIVE SOLUTIONS OF THE LOGISTIC EQUATION WITH RANDOM PERTURBATIONS

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Abstract. The paper establishes the conditions of positivity and boundedness of solutions of the stochastic logistic equation. It is important to emphasize that the stochastic perturbation makes the logistic equation globally stable.

1. Introduction

Deterministic population dynamics of the simplest biological species is described using ordinary differential equations

$$\dot{X} = f(X), \tag{1}$$

which admits an explosion (i.e., an infinite population size in finite time) provided that $f(X)$ satisfies some constraints.

For example, consider the logistic equation

$$\dot{X}(t) = X(t)[m + nX(t)] \tag{2}$$

on $\mathbb{R}^+ = t \geq 0$ with an initial value $X(0) = X_0 > 0$. It is well known that equation (2) has a solution

$$X(t) = \frac{m}{-n + e^{-nt}(m + nX_0)/X_0}.$$

As the variable $X(t)$ means population size, then we will only be interested in positive solutions. In the case where $m > 0$, $n < 0$ equation (2) has a global solution on $\mathbb{R}^+$, which is not only positive and bounded, but has, in addition, an asymptotic property, which is

$$\lim_{t\to\infty} X(t) = m/-n.$$

On the contrary, if we take $n > 0$, preserving $m > 0$, then equation (2) has only a local solution on the segment $[0, T]$, that goes to infinity in a finite amount of time

$$T = -\frac{1}{m} log\left(\frac{nX_0}{m + nX_0}\right).$$

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However, very often the environment in which a population system is defined is subject to random noise (see e.g. [1,2]). In this case, it is important to find out when the presence of such noise affects the result. In [3], the parameter n stochastically perturbed, i.e. instead of n is taken $n + \sigma\dot{B}(t)$, where $\dot{B}(t)$ – white noise, and $\sigma > 0$ represents the intensity of the noise. Thus the solution to the Ito equation is of the form

$$dX(t) = X(t)[m + nX(t)]dt + \sigma X(t)dB(t)$$

with probability of one does not admit the solution-explosion at any finite moment of time if $\sigma > 0$. which is not only positive and bounded, but has, in addition, an asymptotic property, which is $m + \sigma X^{\theta}(t)\dot{B}(t), \theta \in (0, 1/2)$, which is not only positive and bounded, but has, in addition, an asymptotic property, which is $m > 0, n < 0$ and $\sigma > 0$, then the solution of the following equation

$$dX(t) = X(t)[m + nX(t)]dt + nX^{\theta}(t)dB(t)$$

with probability one does not admit a solution-explosion in finite time. In addition, his p-th moment is bounded, stochastically persistent and globally stable, i.e., the stochastic noise environment retains good properties.

Further, in the paper we will consider the equation

$$dX(t) = X(t)[(a - bX(t)]dt + \sigma X^{\theta}(t)dB(t), 0 < \theta < 1/2, \tag{3}$$

where parameters a, b, σ – are positive.

2. Positive and global solutions

Everywhere else in the future, through $(\Omega, \mathcal{F}, \ \mathcal{F}_{tt\geq 0}, P)$ denote the complete probability space with filtering $\mathcal{F}_{tt\geq 0}$ satisfying the usual conditions (i.e., it is continuous on the right and $\mathcal{F}_o$ contains all P-zero sets). Let $B(t)$ a given one-dimensional standard Brownian motion defined on a probability space. For $p \in (0, \infty), letL^p = L^p(\Omega, \mathbb{R}^d)$ family of $\mathbb{R}^d$-of significant random variables x on the condition $\mathbb{E}[x^p] < \infty$.

Consider equation (3)

$$dX(t) = X(t)[(a + bX(t)]dt + \sigma X^{\sigma}(t)dB(t)$$

and further everywhere assume that $a > 0, b > 0, \sigma > 0$ and $\theta \in (0, 1/2)$. In order to have a single global solution of a stochastic equation, for any initial value the equation coefficients must satisfy, in general, the sublinear growth condition and the Lipschitz condition (see e.g. [4], [5]). However, the coefficients of the equation do not satisfy the sublinear growth condition, although they are locally Lipschitzable. We will show that the solution of equation (3) is positive and global.

Theorem 2.1. *There exists a single solution of the equation on $\mathbb{R}^+$ with any given initial value $X_o > 0$.*

Proof. Since the coefficients of the equation are locally Lipschitzable and continuous, for any $X_o \in \mathbb{R}^+$ there exists a single local solution $X(t)$ on $t \in [0, \tau_e]$, where τ_e-explosion time [4]. In order to prove that the solution is global, we must show that $\tau_e = \infty$-almost certainly. Let $k_0 > 0$ large enough for X_o to belong to the segment $[1/k_0, k_0]$. For each integer $k \geq k_0$ determine the stop time

$$\tau_k = inf\left\{t \in [0, \tau_e) : X(t) \notin \left(\frac{1}{k}, k\right)\right\}$$

.

Everywhere else we'll put $inf\emptyset = \infty$ (as usual $\emptyset$ means an empty set). More precisely, τ_k grows with $k \to \infty$. Taking $\tau_\infty = \lim_{k\to\infty} \tau_k$, we have $\tau_\infty \leq \tau_e$ a.c. If we can prove that $\tau_\infty = \infty$ a.c., then $\tau_e = \infty$ a.c. and $x(t) \in \mathbb{R}^+$ a.e. for all $t \geq 0$. In other words, to complete the proof we must show that the $\tau_\infty = \infty$ a.e. If this statement is not true, then there is a pair of constants $T > 0$ and $\varepsilon \in (0, 1)$ such that $P[\tau_\infty \leq T] > \varepsilon$.

Hence, there exists an integer $k_1 \geq k_0$ such that

$$P[\tau_k \leq T] \geq \varepsilon : \forall\, k \geq k_1. \tag{4}$$

Let's determine C^2- of function $V : \mathbb{R}^+ \to \mathbb{R}^+$ given by

$$V(x) = \sqrt{x} - 1 - 1/2\ log\ x.$$

Obviously, when $x > 0$, this function will reach its minimum value 0 at $x = 1$, so it's non-negative. If $x\,(t) \in R^+$, it follows from Ito's formula that

$$dV(x) = \left(1/2\ x^{1/2} - \frac{1}{2}x^{-1}\right) dx +\ 1/2 \left(-1/4\ x^{-1/2} + 1/q\ x^{-1/2}\right)(dx)^2 =$$

$$= \frac{1}{2}\left[\left(x^{1/2} - 1\right)(a - bx) + \left(-\frac{1}{4}x^{1/2} + 1/2\right)\sigma^2 x^{2\theta}\right] dt + \frac{1}{2}\left(x^{1/2} - 1\right)\sigma x^\theta dB(t) =$$

$$= \left(-\frac{a}{2} + ax^{1/2} + \frac{\sigma^2}{4}x^{2\theta} + \frac{b}{2}x - \frac{\sigma^2}{8}x^{2\theta+1/2} - \frac{b}{2}x^{3/2}\right) dt +$$

$$+\frac{\sigma}{2}\left(-x^\theta + x^{\theta+1/2}\right) dB(t),$$

where $X(t) = x$.

In the case where $\theta \in (0, 1/2)$ we'll see that

$$-\frac{a}{2} + \frac{\sigma}{2}x^{1/2} + \frac{\sigma^2}{4}x^{2\theta} + \frac{b}{2}x - \frac{\sigma^2}{8}x^{2\theta+1/2} - \frac{b}{2}x^{3/2}$$

is bounded from above, say by the number K in $\mathbb{R}^+$. Then we get

$$\int_0^{\tau_k \wedge T} dV(x(t)) = \int_0^{\tau_k \wedge T} K dt + \int_0^{\tau_k \wedge T} \frac{\sigma}{2}(-x^\theta + x^{\theta+1/2}) dB(t)$$

since $x(\tau_k \wedge T) \in \mathbb{R}^+$. Taking the mathematical expectation we obtain

$$EV(x(\tau_k \wedge T)) \leq V(x_0) + KE(x(\tau_k \wedge T)) \leq V(x_0) + KT. \tag{5}$$

Let's $\Omega_k = \tau_k \leq T$ for $k \geq k_1$ and with the help of (4) $P(\Omega_k) \geq \varepsilon$. Note that for each $\omega \in \Omega_k, x(\tau_k,\ \omega)$ equal to k either $1/k, henceV(x(\tau_k,\ \omega))$ no less, or

$$\sqrt{k} - 1 - 1/2\ log\ k$$

either

$$\sqrt{\frac{1}{k}} - 1 - \frac{1}{2}\ log\ \left(\frac{1}{k}\right) = \sqrt{\frac{1}{k}} - 1 + \frac{1}{2}\ log\ k.$$

It follows that

$$V\left(x\left(0\right)\right) + KT \geq \mathbb{E}\left[I_{\Omega_k(\omega)} V\left(x\left(\tau_k, \omega\right)\right)\right] \geq$$

$$\geq \varepsilon \left[\left(\sqrt{k} - 1 - 1/2\ log\ k\right) \bigwedge \left(\sqrt{\frac{1}{k}} - 1 + 1/2\ log\ k\right)\right],$$

where I_{Ω_k} is an indicator-function Ω_k. Addressing $k \to \infty$ we come to a contradiction

$$\infty > V(x(0)) + KT = \infty.$$

Thus, we must $\tau_\infty = \infty$ have a.c.

The proof of Theorem 2.1 is complete. □

3. Stochastic resilience

Resilience is a crucial property in population dynamics, meaning that each species is unlikely to go extinct. The most natural analog of stochastic population dynamics is that a species will never go extinct with probability 1. To be precise, let us give a definition.

Definition 3.1. A solution of equation (3) is stochastically persistent if for any $\delta \in (0, 1)$ will find positive constants H_1 and H_2 ($H_1 < H_2, H_1, H_2$ depending on δ) such that for any initial given $X > 0$, the solution of equation (3) has the following property

$$\lim_{t\to\infty} sup\ PX(t) < H_1 < \delta$$

and

$$\lim_{t\to\infty} sup\ PX(t) > H_2 < \delta.$$

Lemma 3.2. *Let $X(t,\ X_0)$ solution of equation (3) for any initial value $X_0 > 0$. Then we have*

$$\lim_{t\to\infty} supE[Xp(t)] \leq L(p)\ \forall\ p \geq 1,$$

where

$$L(p) = \left[\frac{2a + m\sigma^2(p-1)\varepsilon^{-1/m}}{2b - n\sigma^2(p-1)\varepsilon^{1/n}}\right]^p,\ 0 < \varepsilon < \left[\frac{2b}{n\sigma^2(p-1)}\right]^n,\ m = 1-2\theta,\ n = 2\theta.$$

Proof. For simplicity we write $X(t, x_0)$ as $X = x$. Using Ito's formula we have

$$d\left(x^p\right) = px^{p-1}dx + 1/2p\left(p-1\right)x^{p-2}\left(dx\right)^2 =$$

$$= px^p\left[(a - bx) + \sigma x^b dB\left(t\right)\right] + 1/2p\left(p-1\right)x^p\sigma^2 x^{e\theta}dt =$$

$$= [apx^p + 1/2\sigma^2 p(p-1)x^{p-2\theta} - bpx^{p+1}]dt + \sigma px^{p+\theta}dB(t) \quad (6)$$

and

$$x^p(t) = x_0^p + \int_0^t \left[apx^p(s) + \tfrac{\sigma^2}{2}p(p-1)x^{p+2\theta}(s) - bpx^{p+1}(s)\right] ds + \\ +\sigma px^{p+\theta}(s)db(s). \quad (7)$$

Taking the mathematical expectation from all parts of equation (7), we obtain

$$\mathbb{E}(x^p(t)) = x_0^p + \int_0^t E\left[apx^p(s) + \frac{\sigma^2}{2}p(p-1)x^{p+2\theta}(s) - bpx^{p+1}(s)\right] ds, \quad (8)$$

and we have

$$\frac{dE(x^p)}{dt} = apE(x^p) + \frac{\sigma^2}{2}p(p-1)E(x^{p+1}) - bpE(x^{p+1}) =$$

$$= apE(x^p) + \frac{\sigma^2}{2}p(p-1)E[(x^p)^m(x^{p+1})^n] - bpE(xp+1) \leq$$

$$\leq apE(x^p) + \frac{\sigma^2}{2}p(p-1)[E(xp)]m[E(xp+1)]n - bpE(xp+1) =$$

$$= apE(x^p) + \frac{\sigma^2}{2}p(p-1)[\varepsilon^{-1}E(xp)]m[\varepsilon 1/nE(xp+1)]n - bpE(xp+1) \leq$$

$$\leq apE(x^p) + \frac{\sigma^2}{2}p(p-1)[m\varepsilon^{-1/m}E(x^p) + \varepsilon^{1/n}E(x^{p+1})] - bpE(xp+1) =$$

$$= \left[ap + \frac{\sigma^2}{2}p(p-1)me^{-1/m}\right]E(x^p) - \left[bp + \frac{\sigma^2}{2}p(p-1)n\varepsilon^{1/n}\right]E(x^{p+1}) \quad (9)$$

where $m = 1-2\theta,\ n = 2\theta,\ m+n = 1,\ \varepsilon > 0$. The first inequality is obtained using Gelder's inequality, and the second inequality using Young's inequality, which has the following form

$$ab \leq (1/p)(\varepsilon a)^p + (1/q)(b/\varepsilon)^q,\ 1/p + 1/q = 1,\ \forall\varepsilon > 0,\ p > 1, a > 0, \sigma > 0.$$

We can take ε so small, which is the following

$$0 < \varepsilon < [2b/n\ \sigma^2(p-1)^n]$$

in order to

$$bp - \frac{\sigma^2}{2}p(p-1)m\varepsilon^{1/n} > 0.$$

Thus from (9) we obtain

$$\frac{dE(x^p(t))}{dt} \leq \left[ap + \tfrac{\sigma^2}{2}p(p-1)me^{-1/m}\right]E(x^p(t)) - \\ -\left[bp - \tfrac{\sigma^2}{2}p(p-1)ne^{1/n}\right][E(x^p(ts))]^{(p+1)\ p}.$$

Let's $y(t) = E(x^p(t))$, then we have

$$\frac{dy(t)}{dt} \leq py(t)\left[a + \frac{\sigma^2}{2}p(p-1)m\varepsilon^{-1/m} - \left(b + \frac{\sigma^2}{2}p(p-1)ne^{1/n}\right)y^{1/p}(t)\right].$$

Note that the solution of Eq.

$$\frac{dz(t)}{dt} \le pz(t)\left[a + \frac{\sigma^2}{2}p(p-1)m\varepsilon^{-1/m} - \left(b + \frac{\sigma^2}{2}p(p-1)ne^{1/n}\right)z^{1/p}(t)\right]$$

is

$$z(t) = [X_0^{-1}e^{-(a+(\frac{\sigma^2}{2}(p-1))m\varepsilon^{-1/m})t} +$$

$$+\frac{2b - \sigma^2 n(p-1)\varepsilon^{1/n}}{2a + \sigma^2 m(p-1)\varepsilon^{-1/m}}[1 - e^{(a+(\frac{\sigma^2}{2})(p-1)m\varepsilon - 1/n)t}] - p.$$

Pointing $t \to \infty$, we get

$$z(t) \to \left[\frac{2a + \sigma^2 m(p-1)\varepsilon^{-1/m}}{2b - \sigma^2 n(p-1)\varepsilon^{1/n}}\right].$$

Thus, using the comparison principle, we obtain

$$\lim_{t\to\infty} sup\ y(t) \le \left[\frac{2a + \sigma^2 m(p-1)\varepsilon^{-1/m}}{2b - \sigma^2 n(p-1)\varepsilon^{1/n}}\right]^p.$$

Let's

$$L(p) = \left[\frac{2a + \sigma^2 m(p-1)\varepsilon^{-1/m}}{2b - \sigma^2 n(p-1)\varepsilon^{1/n}}\right]^p$$

then we have

$$\lim_{t\to\infty} sup\ E(x^p(t)) \le L(p).$$

The proof of Lemma 3.1 is complete. □

Observation 3.1. From lemma 3.1 we find $T > 0$ such that

$$E(x^p(t)) \le 2L(p)\ \forall\ t \ge T$$

in addition to the fact that $E(x^p(t))$ continuous, and there is $\widetilde{L}(p) > 0$ such that

$$E(x^p(t)) \le \widetilde{L}(p)\ fort \in [0, T].$$

Let's

$$K = max2L(p),\ \widetilde{L}(p).$$

Then we have

$$E(x^p(t)) \le K(p)\ \forall\ t \in [0, \infty).$$

In other words, the p-th moment of the solution is bounded.

Using Ito's formula from equation (3), we obtain

$$d\frac{1}{x(t)} = \frac{1}{x^2(t)}dx(t) + \frac{1}{x^3(t)}\left(dx(t)\right)^2 =$$

$$-\frac{1}{x(t)}[(a - bx(t))dt + \sigma x^\theta(t)dB(t)] + 1x(t)\sigma^2 x^{z\theta}(t)dt =$$

$$= \left(-\frac{a}{x(t)} + b + \frac{\sigma^2}{x^{1-2\theta}(t)}\right)dt - \frac{\sigma^2}{x^{1-\theta(t)}}dB(t).$$

Let's $y(t) = \frac{1}{x(t)}$ then we have

$$dy(t) = (b+\sigma^2)y^{1-2\theta}(t) - (y(t))dt - \sigma y^{1-\theta}(t)dB(t)$$

and

$$dy^p(t) = py^{p-1}(t)dy(t) + \frac{1}{2}p(p-1)y^{p-2}(t)(dy(t))^2 =$$

$$\left[bpy^{p-1}(t) + \frac{\sigma^2}{2}p(p+1)y^{p-2\theta}(t) - apy^p(t)\right]dt - \sigma py^{p-\theta}(t)db(t). \qquad (10)$$

Lemma 3.3. *Let$x(t,x_0)$ solution of equation (3) for any initial value $x_0 > \theta$. Then we have*

$$\lim_{t\to\infty} supE\left[\frac{1}{x^p(t)}\right] \leq S(p) \;\; \forall \;\; p > 1,$$

where

$$S(p) = \left[\frac{2b + n\sigma^2(p+1)\varepsilon^{-1/n}}{2a - m\sigma^2(p+1)\varepsilon^{1/m}}\right]^p, 0 < \varepsilon < \left[\frac{2a}{m\sigma^2(p+1)}\right]^m, \; m = 1-2\theta, \; n = 2\theta.$$

Proof. From (10) we directly obtain

$$\frac{dE[y^p(t)]}{dt} = bpE[y^p(t)] + \frac{\sigma^2}{2}p(p+1)E[y^{p-2\theta}(t)] - apE[y^p(t)] =$$

$$= bpE[y^{p-1}(t)] + \frac{\sigma^2}{2}p(p+1)E[y^p(t))m(y^p(t))^n - apE[y^p(t)] \leq$$

$$\leq bpE[y^{p-1}(t)] + \frac{\sigma^2}{2}p(p+1)E[(y^p(t))m(y^{p-1}(t))n] - apE[y^p(t)] =$$

$$= bpE[y^{p-1}(t)] + \frac{\sigma^2}{2}p(p+1)(\varepsilon^{1/m}[y^p(t)])^m(\varepsilon^{-1/n}[y^{p-1}(t)])^n - apE[y^p(t)] \leq$$

$$\leq bpE\left[y^{p-1}(t)\right] + \frac{\sigma^2}{2}p(p+1)\left(m\varepsilon^{1/m}\left[y^p(t)\right]\right)^m\left(\varepsilon^{-1/n}\left[y^{p-1}(t)\right]\right)^n - apE\left[y^p(t)\right] =$$

$$= \left[bp + \frac{\sigma^2}{2}p(p+1)n\varepsilon^{-1/n}\right]E[y^{p-1}(t)] + \frac{\sigma^2}{2}p(p+1)m\varepsilon^{1/m} - apE[y^p(t)] \leq$$

$$\leq \left[bp + \frac{\sigma^2}{2}p(p+1)n\varepsilon^{-1/n}\right]E[yp-1(t)](p-1)/p + \left[\frac{\sigma^2}{2}p(p+1)m\varepsilon^{1/m} - ap\right]E[y^p(t), (11)$$

where $n = 2\theta, m = 1-2\theta$. By taking $\varepsilon > 0$ so small $0 < \varepsilon < [2\alpha/m\sigma 2(p+1)]m$, as

$$ap - \frac{\sigma^2}{2}p(p+1)m\varepsilon^{1/m} > 0.$$

Let's $z(t) = E[y^p(t)]$, then we have

$$\frac{dz(t)}{dt} \leq \left[bp + \frac{\sigma^2}{2} p\,(p+1)\,n\varepsilon^{-1/n}\right]\; z^{(p-1)/p}(t) - \left[ap - \frac{\sigma^2}{2} p\,(p+1)\,m\varepsilon^{1/m}\right] z(t). \tag{12}$$

Obviously, the solution to the equation

$$\frac{du(t)}{dt} = \left[bp + \frac{\sigma^2}{2} p\,(p+1)\,n\varepsilon^{-1/n}\right] u^{(p-1)/p}(t) - \left[ap - \frac{\sigma^2}{2} p\,(p+1)\,m\varepsilon^{1/m}\right] u(t)$$

is

$$u(t) = [x_0^{-2} e^{-(a+(\frac{a^2}{2})p(p+1)m\varepsilon^{-1/n})t}] +$$
$$+ \frac{2b + \sigma^2 n(p-1)\varepsilon^{-1/n}}{2a - m\sigma^2(p+1)\varepsilon^{1/m}} \left[1 - e^{(a+(\frac{a^2}{2})p(p+1)m\varepsilon^{-1/n})t}\right]^p.$$

By aiming $t \to \infty$, we get

$$u(t) \to \left[\frac{2b + \sigma^2 n(p+1)\varepsilon^{-1/n}}{2a - m\sigma^2(p+1)\varepsilon^{1/m}}\right]^p.$$

thus by means of the comparison principle we have

$$\lim_{t\to\infty} sup z(t) \leq \left[\frac{2b + \sigma^2 n(p+1)\varepsilon^{-1/n}}{2a - m\sigma^2(p+1)\varepsilon^{1/m}}\right]^p = S(p)$$

or

$$\lim_{t\to\infty} sup E\left[\frac{1}{x^p(t)}\right] \leq S(p).$$

Lemma 3.2 is proved. □

Remark 3.4. By the power of lemma 3.2, will find $\hat{T} > 0$, such that

$$E\left[\frac{1}{x^p(t)}\right] \leq 2S(p)\ \forall\ t \geq \hat{T}.$$

In addition, $E\left[\frac{1}{x^p(t)}\right]$ continuous and find $\hat{S}(p) > 0$ such that

$$E\left[\frac{1}{x^p(t)}\right] \leq \hat{S}(p)\ \forall\ t \in [0, T].$$

Let's $M(p) = max 2S(p), \hat{S}(p)$, then we have

$$E\left[\frac{1}{x^p(t)}\right] \leq M(p)\ \forall\ t \in [0, \infty).$$

Theorem 3.5. *For any given $x_0 > 0$, the solution of equation (3) is stochastically persistent.*

Proof. It follows from lemmas 3.1 and 3.2 that

$$x(t) \in L^p,\ \frac{1}{x(t)} \in L^p,\ p > 1.$$

On the other hand, it follows from Remarks 3.1 and 3.2 that

$$E[x^p(t)] \le K(p), E[\frac{1}{x^p(t)}] \le M(p), t \in [0,\infty).$$

Then, by the power of Chebyshev's inequality [6], there are

$$H_1 = \left(\frac{\delta}{M(p)}\right)^{1/p}, \; H_2 = \left(\frac{K(p)}{\delta}\right)^{1/p}$$

such that

$$lim t \to \infty sup P[x(t) < H_1] < \delta,$$

$$lim t \to \infty sup P[x(t) < H_2] > \delta.$$

Hence, the solution of equation (2) is persistent.
Theorem 3.1 is completely proved. □

Note that close questions are considered in [7-10].

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