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ON THE CONVERGENCE OF THE FRACTIONAL PART OF CONVOLUTIONS OF POISSON DISTRIBUTED RANDOM VARIABLES WITH UNEQUAL PARAMETERS

ALEXANDER E. CONDRATENKO ET AL.

Abstract. The article provides an explicit formula for the distribution of remainders modulo an arbitrary natural number of the convolutions of unequly distributed Poisson random variables and shows their convergence in law to the uniform distribution.

Introduction

The law of large numbers and the central limit theorem which describe the asymptotic behaviour of the suitably normalised convolutions are not only well known within the probability theory but also are applied far beyond the boundaries of pure mathematics [10, 12, 13].

However, it proves to be of interest to study to consider the behaviour of convolutions in a slightly different normalisation — the fractional part. Such problems on the sums of the independent random variables are known. For instance, in [7] the result for the Gaussian random variables was obtained. Y.V. Prokhorov as well as his students investigated problems that involved convergence to the uniform distribution. It is known that the uniform distribution maximises entropy. Therefore, the convergence to the uniform distribution of sum of the increasing number of the independent random variables may be interpreted as a system tending towards the state of the maximal entropy. The problems concerning the maximisation of entropy arise in pure mathematics as well as in applications (for instance, [2, 8]).

Let us consider $\xi_1, \xi_2, ...$ — independent Poisson distributed random variables with the parameter $\lambda > 0$, that is for each $n \in \mathbb{N}$ $\xi_n \sim \Pi(\lambda)$:

$$\mathrm{P}(\xi_n = k) = \frac{\lambda^k}{k!} e^{-\lambda}, \quad k \in \mathbb{Z}_+ .$$

In [5] (see also [3, 4, 6]) convergence to the uniform distribution of remainders modulo $m = 2, 3, 4$ of the convolutions of identically ditributed Poisson random variables was proved and the explicit formula for the distributions was obtained.

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Here follows the formula for the distribution of the remainders modulo 2:

$$P\big(\{\xi_1+\ldots+\xi_n\}_2=0\big)=\frac{1}{2}+\frac{e^{-2\lambda n}}{2}\to\frac{1}{2},\quad n\to\infty\,,$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_2=1\big)=\frac{1}{2}-\frac{e^{-2\lambda n}}{2}\to\frac{1}{2},\quad n\to\infty\,.$$

Modulo 3 as $n\to\infty$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=0\big)=\frac{1}{3}+\frac{2}{3}\cdot e^{-\frac{3\lambda n}{2}}\cos\tfrac{\sqrt{3}\lambda n}{2}\to\frac{1}{3},$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=1\big)=\frac{1}{3}-\frac{1}{3}\cdot e^{-\frac{3\lambda n}{2}}\cos\tfrac{\sqrt{3}\lambda n}{2}+\frac{1}{\sqrt{3}}\cdot e^{-\frac{3\lambda n}{2}}\sin\tfrac{\sqrt{3}\lambda n}{2}\to\frac{1}{3},$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=2\big)=\frac{1}{3}-\frac{1}{3}\cdot e^{-\frac{3\lambda n}{2}}\cos\tfrac{\sqrt{3}\lambda n}{2}-\frac{1}{\sqrt{3}}\cdot e^{-\frac{3\lambda n}{2}}\sin\tfrac{\sqrt{3}\lambda n}{2}\to\frac{1}{3}.$$

And modulo 4:

$$P\big(\{\xi_1+\ldots+\xi_n\}_4=0\big)=\frac{1}{4}+\frac{1}{4}\cdot e^{-2\lambda n}+\frac{1}{2}\cdot e^{-\lambda n}\cdot\cos\lambda n\to\frac{1}{4},\quad n\to\infty\,,$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=1\big)=\frac{1}{4}-\frac{1}{4}\cdot e^{-2\lambda n}+\frac{1}{2}\cdot e^{-\lambda n}\cdot\sin\lambda n\to\frac{1}{4},\quad n\to\infty\,,$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=2\big)=\frac{1}{4}+\frac{1}{4}\cdot e^{-2\lambda n}-\frac{1}{2}\cdot e^{-\lambda n}\cdot\cos\lambda n\to\frac{1}{4},\quad n\to\infty\,,$$

$$P\big(\{\xi_1+\ldots+\xi_n\}_3=3\big)=\frac{1}{4}-\frac{1}{4}\cdot e^{-2\lambda n}-\frac{1}{2}\cdot e^{-\lambda n}\cdot\sin\lambda n\to\frac{1}{4},\quad n\to\infty\,.$$

In this work the results mentioned are generalised to an arbitrary natural m. It turns out that in the general case the explicit formula for computing probabilities $P\big(\{\xi_1+\ldots+\xi_n\}_m=l\big)$ may be established for arbitrary $n,m\in\mathbb{N}$ and $l\in\{0,1,2,\ldots,m-1\}$. It is worth mentioning that the probability distributions obtained are expressed via the m-th roots of unity, of which there are exactly m, all distinct.

From the explicit expressions for the probabilities $P\big(\{\xi_1+\ldots+\xi_n\}_m=l\big)$ follows the convergence in law (hereafter referred to as simply convergence) of the convolution of the Poisson distributed random variables modulo m to the uniform distribution .

Let us recall the convolution formula for two integer-valued random variables ξ and η:

$$P\left(\xi+\eta=k\right)=\sum_{s=-\infty}^{+\infty}P\left(\xi=s\right)\cdot P\left(\eta=k-s\right)\,.\tag{0.1}$$

1. The case of the equal distribution

We formulate the following theorem.

Theorem 1.1. *Let $\xi_1, \ldots, \xi_n$ be independent identically distributed Poisson random variables with the parameter $\lambda > 0$.*

Then for any $n, m \in \mathbb{N}$ and $l \in \{0, 1, 2, \ldots, m-1\}$ the following holds true

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_m = l\big) = \frac{e^{-\lambda n}}{m} \cdot \sum_{k=0}^{m-1} u_k^{\{m-l\}_m} e^{u_k \lambda n} = \frac{1}{m} + \sum_{k=1}^{m-1} \frac{u_k^{\{m-l\}_m}}{m} e^{-\lambda n (1-u_k)},$$

where $\{a\}_m$ denotes the remainder of dividing the integer $a \in \mathbb{Z}$ by a natural $m \in \mathbb{N}$, $i = \sqrt{-1}$ is the imaginary unit and $u_k = e^{\frac{2\pi k}{m} i}$ are the m-th roots of unity.

Proof. It is known that the sum of the independent Poisson random variables also follows the Poisson distribution with the parameter equal to the sum of the individual parameters, that is

$$\xi_1 + \ldots + \xi_n \sim \Pi\left(\lambda n\right).$$

It can easily be noticed that for $a \geq 0$ and for any $l \in \{0, 1, 2, \ldots, m-1\}$ it holds true that

$$\big\{\{a\}_m = l\big\} = \bigcup_{k=0}^{\infty} \big\{a = l + mk\big\},$$

where the outer brackets denote sets and the inner ones denote the fractional part of a number. Therefore,

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_m = l\big) = \mathrm{P}\left(\bigcup_{k=0}^{\infty} \{\xi_1 + \ldots + \xi_n = l + mk\}\right) = \sum_{k=0}^{\infty} \mathrm{P}\left(\xi_1 + \ldots + \xi_n = l + mk\right) = e^{-\lambda n} \cdot \sum_{k=0}^{\infty} \frac{(\lambda n)^{mk+l}}{(mk+l)!}.$$

Let us define a function

$$G\left(x\right) = G_m\left(x\right) = \sum_{k=0}^{\infty} \frac{x^{mk}}{(mk)!},$$

so that for any $l \in \{1, 2, \ldots, m-1\}$ the derivatives exist.

$$G^{(l)}\left(x\right) = \sum_{k=1}^{\infty} \frac{x^{mk-l}}{(mk-l)!} = \sum_{k=0}^{\infty} \frac{x^{mk+m-l}}{(mk+m-l)!}.$$

Thus, all the above probabilities can be expressed as

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_m = l\big) = e^{-\lambda n} \cdot G^{(m-l)}\left(\lambda n\right).$$

And as

$$\sum_{l=0}^{m-1} \mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_m = l\big) = G\,(\lambda n) + \sum_{l=1}^{m-1} e^{-\lambda n} \cdot G^{(m-l)}\,(\lambda n) = 1,$$

$G\,(x)$ satisfies the following inhomogeneous differential equation

$$G\,(x) + G^{(1)}\,(x) + \ldots + G^{(m-1)}\,(x) = e^x$$

with the initial conditions

$$G\,(0) = 1 + \sum_{k=1}^{\infty} \frac{0^{mk}}{(mk)!} = 1\,, \tag{1.1}$$

$$G^{(l)}\,(0) = \sum_{k=1}^{\infty} \frac{0^{mk-l}}{(mk-l)!} = 0\,, \tag{1.2}$$

where $l \in \{1, 2, \ldots, m-1\}$. We consider the homogeneous differential equation

$$G\,(x) + G^{(1)}\,(x) + \ldots + G^{(m-1)}\,(x) = 0\,.$$

Its characteristic equation $u^{m-1} + \ldots + u + 1 = 0$ for $u \neq 1$ can be reduced to the form

$$\frac{u^m - 1}{u - 1} = 0\,, \quad u^m = 1\,.$$

And since $u = 1$ is not a solution of the given homogeneous equation, its distinct solutions are precisely all the remaining roots of unity

$$u_k = e^{\frac{2\pi \cdot k}{m} i}\,, \quad k \in \{1, 2, \ldots, m-1\}\ . \tag{1.3}$$

Therefore, the general solution of the homogeneous equation can be expressed as follows

$$G_{\text{g.h.}}\,(x) = \sum_{k=1}^{m-1} C_k e^{u_k x}\,,$$

where C_k are some constants. If we now seek the particular solution of the inhomogeneous equation in the following form

$$G_{\text{p.i.}}\,(x) = A \cdot e^x\,,$$

than after substituting it into the original homogeneous equation and performing some straightforward algebraic manipulations we obtain that

$$G_{\text{p.i.}}\,(x) = \frac{1}{m} \cdot e^x\,.$$

Combining the general and particular solutions yields the following expression

$$G\,(x) = \sum_{k=0}^{\infty} \frac{x^{mk}}{(mk)!} = \frac{1}{m} \cdot e^x + \sum_{k=1}^{m-1} C_k e^{u_k x}\,, \tag{1.4}$$

where C_k are some constants as it has been mentioned before.

Let us find the values of the constants C_k which correspond with the conditions (1.1) and (1.2). to do this we substitute the initial conditions (1.1) and (1.2) into the general form of the function $G(x)$. This substitution leads to the system of $m-1$ equations which can be written in a matrix form as follows

$$\begin{pmatrix} 1 & \dots & 1 \\ u_1 & \dots & u_{m-1} \\ \dots & \dots & \dots \\ u_1^{m-2} & \dots & u_{m-1}^{m-2} \end{pmatrix} \begin{pmatrix} C_1 \\ C_2 \\ \dots \\ C_{m-1} \end{pmatrix} = \begin{pmatrix} 1-\frac{1}{m} \\ -\frac{1}{m} \\ \dots \\ -\frac{1}{m} \end{pmatrix}. \tag{1.5}$$

It is not hard to notice that the matrix in the system given

$$\begin{pmatrix} 1 & \dots & 1 \\ u_1 & \dots & u_{m-1} \\ \dots & \dots & \dots \\ u_1^{m-2} & \dots & u_{m-1}^{m-2} \end{pmatrix}$$

is a $(m-1, m-1)$ Vandermonde matrix. As it is widely known the determinant of the Vandermonde matrix $V(u_1, \dots, u_{m-1})$ can ve expressed explicitly in terms of the values $u_1, \dots, u_{m-1}$:

$$V(u_1, \dots, u_{m-1}) = \prod_{1 \le i < j \le m-1} (u_j - u_i) .$$

This expression shows that in our case due to (1.3) the determinant $V(u_1, \dots, u_{m-1})$ is not equal to zero, hence the matrix equation (1.5) has a unique solution. Due to the uniqueness of the solution $(C_1, \dots, C_{m-1})$ it is sufficient to present it explicitly.

For that we will be using the results acquired in [5, 6] and provided at the beginning of this paper. Ergo, for $m = 2$ the substitution of the function

$$G(x) = \frac{1}{2} \cdot e^x + C_1 e^{u_1 x} = \frac{1}{2} \cdot e^x + C_1 e^{\pi i x}$$

into

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_2 = 0\big) = e^{-\lambda n} \cdot G_2^{(0)}(\lambda n) = \frac{1}{2} + C_1 e^{(\pi i - 1)\lambda n} = \frac{1}{2} + C_1 e^{-2\lambda n}$$

must coincide with

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_2 = 0\big) = \frac{1}{2} + \frac{e^{-2\lambda n}}{2},$$

Consequently, $C_1 = \dfrac{1}{2}$ will be a solution.

Similar reasoning shows that for $m = 3$ we will obtain a vector $(C_1, C_2) = \left(\dfrac{1}{3}, \dfrac{1}{3}\right)$ and for $m = 4$ the resulting vector will be $(C_1, C_2, C_3) = \left(\dfrac{1}{4}, \dfrac{1}{4}, \dfrac{1}{4}\right)$.

Therefore we assume that in the general case

$$C_1 = C_2 = \ldots = C_{m-1} = \frac{1}{m}$$

is the desired solution

To verify this assumption we write out the system (1.5) as a system of linear equations

$$\begin{cases} C_1+C_2+\ldots+C_{m-1}=1-\frac{1}{m} \\ C_1u_1+C_2u_2+\ldots+C_{m-1}u_{m-1}=-\frac{1}{m} \\ C_1u_1^2+C_2u_2^2+\ldots+C_{m-1}u_{m-1}^2=-\frac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_1u_1^k+C_2u_2^k+\ldots+C_{m-1}u_{m-1}^k=-\frac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_1u_1^{m-2}+C_2u_2^{m-2}+\ldots+C_{m-1}u_{m-1}^{m-2}=-\frac{1}{m} \end{cases}.$$

Since for $s\in\{1,2,\ldots,m-1\}$ from the equalities $u_s=u_1^s$ it follows that

$$u_s^k=(u_1^s)^k=\left(u_1^k\right)^s=u_k^s,$$

the system can be rewritten as follows

$$\begin{cases} C_1+C_2+\ldots+C_{m-1}=1-\frac{1}{m} \\ C_1u_1+C_2u_1^2+\ldots+C_{m-1}u_1^{m-1}=-\frac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_1u_k+C_2u_k^2+\ldots+C_{m-1}u_k^{m-1}=-\frac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ C_1u_{m-1}+C_2u_{m-1}^2+\ldots+C_{m-1}u_{m-1}^{m-1}=-\frac{1}{m} \end{cases}.$$

Substituting the values $C_1=C_2=\ldots=C_{m-1}=\dfrac{1}{m}$ into the latter system and using the well known equality for the unity roots $u_k^0+u_k^1+\ldots+u_k^{m-1}=0$ we see that the resulting system

$$\begin{cases} \dfrac{1}{m}\cdot(m-1)=1-\dfrac{1}{m} \\ \dfrac{1}{m}\cdot\left(1+u_1+\ldots+u_1^{m-1}-1\right)=-\dfrac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{1}{m}\cdot\left(1+u_k+\ldots+u_k^{m-1}-1\right)=-\dfrac{1}{m} \\ \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots \\ \dfrac{1}{m}\cdot\left(1+u_{m-1}+\ldots+u_{m-1}^{m-1}-1\right)=-\dfrac{1}{m} \end{cases}$$

consists entirely of identities. It means that substituting the values $C_1=C_2=\ldots=C_{m-1}=\dfrac{1}{m}$ into the original system leads to the equalities that hold true. Thus, the assumption that these value constitute the solution is confirmed.

Thereby

$$G(x)=\frac{1}{m}\cdot e^x+\sum_{k=1}^{m-1}\frac{1}{m}e^{u_kx},$$

$$G(\lambda n)=\frac{1}{m}\cdot e^{\lambda n}+\sum_{k=1}^{m-1}\frac{1}{m}e^{u_k\lambda n}.$$

Therefore

$$\mathrm{P}\big(\{\xi_1+\ldots+\xi_n\}_m=l\big)=e^{-\lambda n}\cdot G^{(\{m-l\}_m)}(\lambda n)=\frac{e^{-\lambda n}}{m}\sum_{k=0}^{m-1}u_k^{\{m-l\}_m}e^{u_k\lambda n}$$

$$=\frac{1}{m}+\sum_{k=1}^{m-1}\frac{u_k^{\{m-l\}_m}}{m}e^{-\lambda n(1-u_k)},$$

where $\{a\}_m$ denotes the remainder of dividing the integer $a\in\mathbb{Z}$ by a natural $m\in\mathbb{N}$, $u_k=e^{\frac{2\pi k}{m}i}$ are the m-th roots of unity, $k\in\{1,2,\ldots,m-1\}$, $l\in\{0,1,2,\ldots,m-1\}$ and i is the imaginary unit. At this point the proof can be considered complete.

□

Corollary 1.2. *Let $\xi_1,\xi_2,\ldots$ be independent identically distributed Poisson random variables with the parameter $\lambda>0$. Then the distribution of the remainders modulo m of the convolution of these random variables that is $\{\xi_1+\ldots+\xi_n\}_m$ converges to the distribution uniform on the set $\{0,1,2,\ldots,m-1\}$. In other words, for any $n,m\in\mathbb{N}$ and $l\in\{0,1,2,\ldots,m-1\}$ it holds true that*

$$\mathrm{P}\big(\{\xi_1+\ldots+\xi_n\}_m=l\big)\to\frac{1}{m},\quad n\to\infty\,. \tag{1.6}$$

Proof. Due to the previous theorem for $l\in\{0,1,2,\ldots,m-1\}$ it holds that

$$\mathrm{P}\big(\{\xi_1+\ldots+\xi_n\}_m=l\big)=\frac{1}{m}+\sum_{k=1}^{m-1}\frac{u_k^{\{m-l\}_m}}{m}e^{-\lambda n(1-u_k)}\,.$$

It is clear that $\left|u_k^{\{m-l\}_m}\right|=1$. Thence for $n\to\infty$

$$\left|e^{-\lambda n(1-u_k)}\right|=\left|e^{-\lambda n\left(1-\cos\frac{2\pi\cdot k}{m}-i\sin\frac{2\pi\cdot k}{m}\right)}\right|=1\cdot\left|e^{-\lambda n\left(1-\cos\frac{2\pi\cdot k}{m}\right)}\right|\to 0\,.$$

Thus, the relation (1.6) is proved.

□

2. The case of the unequal distribution

Let the mean value of the parameter for n random variables $\xi_1,\ldots,\xi_n$ be defined as the arithmetic mean of their parameter values:

$$\overline{\lambda}_n=\frac{\lambda_1+\ldots+\lambda_n}{n}.$$

Then the generalisations of the results obtained above can be proved in a similar way. Moreover, the complete analogy of the formulations of the statements is obvious.

Theorem 2.1. *Let $\xi_1,\ldots,\xi_n$ be independent Poisson distributed random variables with the positive parameters $\lambda_1,\ldots,\lambda_n$ respectively. Then for any $n,m\in\mathbb{N}$ and*

$l \in \{0, 1, 2, \ldots, m-1\}$ it holds true that

$$\mathrm{P}\big(\{\xi_1 + \ldots + \xi_n\}_m = l\big) = \frac{e^{-n\overline{\lambda}_n}}{m} \cdot \sum_{k=0}^{m-1} u_k^{\{m-l\}_m} e^{u_k n \overline{\lambda}_n} = = \frac{1}{m} + \sum_{k=1}^{m-1} \frac{u_k^{\{m-l\}_m}}{m} e^{-n\overline{\lambda}_n (1-u_k)} ,$$

where where $\{a\}_m$ denotes the remainder of dividing the integer $a \in \mathbb{Z}$ by a natural $m \in \mathbb{N}$, $i = \sqrt{-1}$ is the imaginary unit and $u_k = e^{\frac{2\pi k}{m} i}$ are the m-th roots of unity.

Corollary 2.2. *Let $\xi_1, \xi_2, \ldots$ be independent Poisson distributed random variables with the positive parameters $\lambda_1, \ldots, \lambda_n$ respectively. Then for $n, m \in \mathbb{N}$ and $l \in \{0, 1, 2, \ldots, m-1\}$ (1.6) holds true if for any $n \in \mathbb{N}$ and for some $c > 0$ the inequalities $\overline{\lambda}_n \geq c$ hold.*

As it is well known [10], the entropy $\mathrm{H}\xi$ of a discrete random variable ξ taking on m values $x_1, \ldots, x_m$ with the corresponding probabilities $x_1, \ldots, x_m$ is defined by the following formula

$$\mathrm{H}\xi = \mathrm{H}(p_1, \ldots, p_m) = -\sum_{k=1}^{m} p_k \log_2 p_k ,$$

and reaches its maximum value $\log_2 m$ on the uniform on the set $\{x_1, \ldots, x_m\}$ distribution and only on it, i.e.

$$\max_{p_1, \ldots, p_m} \mathrm{H}(p_1, \ldots p_m) = \mathrm{H}(\frac{1}{m}, \ldots, \frac{1}{m}) = \log_2 m .$$

This remark enables us to reformulate the above corollaries in terms of the entropy of a discrete random variable. Thus, the following statement holds true.

Corollary 2.3. *Let $\xi_1, \xi_2, \ldots$ be independent identically distributed Poisson random variables with the parameter $\lambda > 0$. Then the entropy of the distribution of remainders from dividing the convolution of the given random variables by $m \in \mathbb{N}$, that is the random variables $\{\xi_1 + \ldots + \xi_n\}_m$ maximises with the increase of the number of terms :*

$$\mathrm{H}\big(\{\xi_1 + \ldots + \xi_n\}_m\big) \to \log_2 m, \quad n \to \infty . \tag{2.1}$$

For the Poisson random variables with unequal parameters a similar statement holds true.

Corollary 2.4. *Let $\xi_1, \xi_2, \ldots$ be independent Poisson distributed random variables with the positive parameters $\lambda_1, \ldots, \lambda_n$ respectively. Let the inequalities $\overline{\lambda}_n \geq c$ hold for any $n \in \mathbb{N}$ and for some $c > 0$. Then the entropy of the distribution of remainders from dividing the convolution of the given random variables by $m \in \mathbb{N}$, that is the random variables $\{\xi_1 + \ldots + \xi_n\}_m$ maximises with the increase of the number of summands in the sense of (2.1).*

3. Conclusion

Thus, in the case of independent Poisson random variables, it has been shown that the fractional parts of their convolutions converge in law to the uniform distribution at an exponential rate and, therefore, their entropies converge to their maximum possible value.

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Daria A. Chernyshova: Faculty of Mechanics and Mathematics, M.V. Lomonosov Moscow State University, Moscow, Russia, 119991

Email address: daria.chernyshova@math.msu.ru

Alexander E. Condratenko: Faculty of Mechanics and Mathematics, M.V. Lomonosov Moscow State University, Moscow, Russia, 119991

Email address: ae_cond@mech.math.msu.su

Natalya A. Elizarova: Department of Higher Mathematics, MIREA Russian Technological University, Moscow, Russia, 119454

Email address: elizarova77@mail.ru

Nikolay A. Kondratenko: Faculty of Aeromechanics and Flight Engineering, Moscow Institute of Physics and Technology, Zhukovsky, Moscow Region, Russia, 140180
Email address: kondratenko.na@phystech.edu

Vitaly N. Sobolev: Faculty of Mechanics and Mathematics, M.V. Lomonosov Moscow State University, Moscow, Russia, 119991, Department of Higher Mathematics, MIREA Russian Technological University, Moscow, Russia, 119454
Email address: sobolev_vn@mail.ru